

## **Students' Mathematical Creative Thinking Obstacle and Scaffolding in Solving Derivative Problems**

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*The difficulties experienced by students in studying derivative material are difficulties understanding the definition of derivatives and representations of these derivatives. Derivatives are a material that is quite difficult to develop mathematical creative abilities because they have many functions and symbols. This difficulty indicates a barrier to thinking in students. This research is descriptive qualitative research with five research subjects of Tadris Mathematics at the Curup State Islamic Institute. Each student can think creatively mathematically on the indicators of flexibility, fluency, and originality, but the levels are different. The level of mathematical creative thinking ability can depend on the questions or problems and the material being studied. However, one of the indicators of the ability to think creatively mathematically, which is very weak for students, is originality because students seem rigid with what they have obtained from lecturers and books. Students find it difficult to come up with ideas in solving problems. Barriers to thinking creatively mathematically can occur due to several factors, including 1) due to a lack of prior knowledge of students in understanding problems and determining ideas in planning solutions; 2) a lack of strong concepts possessed by students. One way for lecturers to overcome these obstacles is to provide scaffolding. The provision of scaffolding is carried out using Treffinger learning with several stages, namely basic tools, practice with process, and working with real problems. In practice with process stage, students solve problems by using analogical reasoning to develop their mathematical creative thinking abilities. The stages of analogical reasoning used to consist of the stages of recognition, representation, structuring, mapping, applying, and verifying. This study's findings are that two new stages are added to the early stages of analogical reasoning, namely the stages of recognition and representation.*

*Keywords: obstacle, mathematical creative thinking, scaffolding, derivative problems*

## INTRODUCTION

In an OECD-conducted PISA study, creative thinking is defined as the capacity to actively participate in the generation, assessment, and refinement of ideas, which can result in novel and efficient solutions, developing knowledge and expressions that affect imagination (Azaryahu, Broza, Cohen, Hershkovitz, & Adi-Japha, 2022). Additionally, creativity is acknowledged as being essential to the imaginative capacity of humans across all fields, and it is clear that its impact permeates all aspects of life (Navarrete, 2013). According to a growing body of research in recent years (Beghetto, 2019; Said Metwaly, Van den Noortgate, & Kyndt, 2017; Vygotsky, 2004), creativity is a necessary 21st-century ability that can be fostered and should be incorporated into the curriculum from an early age.

According to (Cahyono, Fauzi, & Rohman, 2021), states creative thinking as an activity in the form of a mentality that is applied by individuals when developing ideas or ideas in new ways. In assessing the ability to think creatively, the indicators used are fluency, flexibility, and originality through various problem-solving. The fluency indicator refers to their ability to answer various questions and finish with the correct answer. The flexibility indicator in problem-solving refers to their ability to solve problems in different ways. The indicator of novelty in problem-solving refers to their ability to answer questions with several different but correct questions or an unusual answer at the stage of their knowledge development (Asmidi, 2021).

In research (Damayanti & Kartini, 2022), concluded that the ability to think creatively in the fluency aspect is 78%, in the flexibility aspect it reaches 58%, and in the novelty aspect, it reaches 25%. Meanwhile, (Kamal, Firmansyah, Rafiah, Rahmawan, & Rejito, 2020) revealed the ability to think creatively in the fluency aspect by 83%, the flexibility aspect by 33.3%, and the novelty aspect by 25% and the elaboration aspect by 16.7%. The preliminary study results indicate that students' ability to think creatively mathematically in solving problems is still low. There are still students who are not fluent in giving answers to the questions given, have not been able to provide varied answers, and have not been able to provide new answers.

Meanwhile, based on the initial research in this article, it was found that no students were with high mathematical creative thinking abilities. Thus, only students with medium and low mathematical creative thinking abilities were involved. This shows that students' mathematical creative thinking abilities are still weak, for example, in the flexibility and fluency of students in answering questions. The ideas that are owned are also still few to solve problems or, in other words, have not been able to provide creative ideas. In addition, in solving problems, students are still focused on one way and have not been able to find or solve problems in a different way or solution other than the examples given by the lecturer during lectures because the questions given are different. Therefore, this paper will explore what hinders the students' mathematical creative thinking abilities in solving derivative problems. As is well known, the derivative is a problem quite common in the field. The derivative problem is a problem that is quite complicated to be studied by students because it is abstract.

A study on the understanding of calculus has shown that a wide range of concepts produces problems for students (White & Mitchelmore, 1996). The study of calculus, with its fundamental concepts of limit, derivative and integral, requires understanding algebraic variables as generalized numbers and functionally related varying quantities (Gray, Loud, & Sokolowski, 2009). (Gray et al., 2009) suggest that calculus instruction should continue to emphasize the differing uses of variables in various contexts and strive to develop students' conceptions of variables as changing and co-varying quantities. He goes on to explain that one of the difficulties in imagining chain rules is the dilemma of whether they can be removed  $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ . Results from other studies show that students struggle to grasp concepts (Orton, 1983; Uygur Kabaal, 2005). Differentiation exacerbates the difficulty of student derivation composite functions requiring the application of chain rules (Maharaj, 2013; Tall, 1992). (Luneta & Makonye, 2010) state that there is an inadequate understanding of factorization, handling of directed arithmetic, equation solving, and indexing. They also noted that errors and misconceptions in calculus were related to learners' over-dependence on procedural knowledge with no conceptual basis. On the other hand, learners sometimes had sound

conceptual knowledge for which they had not acquired the allied procedural knowledge needed to perform questions.

The idea of a derivative is crucial in mathematics and fields like physics, engineering, economics, chemistry, and biology. Unfortunately, students struggle to apply this concept, according to the few studies from the perspectives of science and engineering education that have discussed students' understanding and application of the derivative in their disciplines (Beichner, 1994; Bucy, Thompson, & Mountcastle, 2007; McDermott, Rosenquist, & van Zee, 1987; Sazhin, 1998). The derivative in mathematics can refer to several concepts, including slope, the upper limit of the difference quotient expression, or a rate of change (Zandieh, 2001).

Some calculus education researchers have started to pay more attention to how students understand, apply, and use the derivative concept in contexts outside of mathematics because of the derivative's importance in science and engineering fields of study and the challenges students face when using it in those fields (Berry & Nyman, 2003; Roorda, Vos, & Goedhart, 2007). However, the vast majority of the mathematics education research dealing with applications of the rate of change and the derivative is centered on the contexts of position, velocity, and acceleration (Berry & Nyman, 2003; Bezuidenhout, 1998; Bowers & Doerr, 2001; Hale, 2000; Marrongelle, 2004; Roschelle, 2000; Schwalbach & Dosemagen, 2010; Zandieh, 2001).

The notion that students develop 'misunderstandings' has been the basis of much empirical research on mathematics and science learning over the last 15 years (Smith, Disessa, & Roschelle, 1993). (Smith et al., 1993) highlight that students misunderstand early learning in the classroom (especially for mathematics) or from their interactions with the physical and social world. They further elaborated that misconceptions can be stable and widespread among students, and these misconceptions can be firmly held and resistant to change. (Orton, 1983) asserts that students have problems understanding the meaning of derivatives when written as fractions. His findings cover the various types of errors displayed by students in calculus classes.

This paper is part of a doctoral study to explore errors and obstacles to students' mathematical creative thinking in solving derivative problems. This allows researchers to determine the causes and origins of these errors so that they can develop ways to eliminate errors displayed through scaffolding. The poor performance of students in solving derivative problems using their creative thinking abilities is a major concern in mathematics education. Meanwhile, the ability to think creatively mathematically is needed in all aspects studied in mathematics without exception. Derivatives are quite abstract, so it is not easy for students to use their mathematical creative thinking abilities to study derivatives because they are related to many symbols and functions. This study focuses on the errors displayed by students in the derivative of algebraic functions. Based on the literature reviewed, no research has been conducted on students regarding the barriers to thinking creative mathematical abilities in solving this derivative problem.

## **RESEARCH METHOD**

This research is in the qualitative research paradigm. Qualitative research is an exploratory approach, emphasizing the use of open-ended questions and inquiry, allowing participants to respond in their own words (Devetak et al., 2010). This research begins with the implementation of a preliminary test, an observation activity to find out the initial description of students' thinking barriers. In addition, this observation also makes scaffolding preparations that must be prepared when facing different obstacles. Next, determine the subject based on the results of the initial test and description of student scores, as well as input from the teaching lecturers. Selection of subjects using purposive sampling technique with consideration of communication skills. Then five students were selected: one student with high ability, two with medium ability, and two with low ability. Students with moderate and low abilities were chosen by two people each because it was hoped that researchers would find out more barriers to thinking when conducting interviews. After determining the subject, a data collection test was carried out and continued with interviews with the five students to reveal the work results using the think-aloud method. Students are then given scaffolding for the thought barriers they experience. An analysis will be carried out if students can improve their answers after scaffolding. However, if you still cannot fix it or don't give a response from

the scaffolding, then the next scaffolding will be given. The scaffolding that will be provided refers to research by (Anghileri, 2006), namely, scaffolding level 1 (environmental provisions), scaffolding level 2 (explaining, reviewing, and restructuring), and scaffolding level 3 (developing conceptual thinking).

In qualitative research, the main instrument is the researcher. Researchers will act as planners, executors of data collection, and analysis, data interpreters, and reporters of research results. The supporting instruments used by researchers are test questions. The test questions given in this study were in the form of assignment sheets completed by students individually. This is intended to describe students' thinking barriers in solving derivative problems before getting help (scaffolding) from researchers.

## RESULT AND DISCUSSION

Based on the analysis of the results of the test questions and interviews, it was found that there were differences in the thinking barriers experienced by students in solving derivative problems. Thinking barriers occur in several indicators of students' mathematical creative thinking abilities, which include fluency, flexibility, and originality through various problem-solving.

**TABLE 1**  
**INDICATORS IF MATHEMATICAL CREATIVE THINKING ABILITY**

Aspects/Component Measured	Indicator	Material Achievement Indicators	Problems
Fluency	<ol style="list-style-type: none"> <li>1. Plan and use various resolution strategies when facing complex problems and deadlocks.</li> <li>2. Changing the settlement strategy when the chosen one experiences a deadlock in solving the problem.</li> </ol>	<ol style="list-style-type: none"> <li>1. Can write down what is known and what will be resolved.</li> <li>2. Can write equation <math>P(x) = f(x)</math>. <math>g(x)</math>.</li> <li>3. Can write down the derivative of the equation function <math>P(x) = f(x)</math>. <math>g(x)</math>.</li> </ol>	<p><i>Known function</i> <math>f(x) = x(2x + 5)</math> and <math>g(x) = 3x - 1</math> and <math>P(x) = f(x)</math>. <math>g(x)</math>.</p> <p>a. <i>Determine <math>P'(x)</math> in two different ways and include an explanation for each step of the process.</i></p>
Flexibility	<ol style="list-style-type: none"> <li>1. Have various interpretations of the problem given.</li> <li>2. Have a different point of view in looking at the problem.</li> <li>3. If you are given a problem, you usually think of different ways to solve it.</li> <li>4. Answer questions in a variety of ways.</li> </ol>	<ol style="list-style-type: none"> <li>1. Can solve problems using other ways of solving.</li> <li>2. Be able to interpret the results of the settlement.</li> </ol>	<p>b. <i>Compare the results, which one is easier to implement, and explain why.</i></p>

Fluency	<ol style="list-style-type: none"> <li>1. Plan and use various resolution strategies when facing complex problems and deadlocks.</li> <li>2. Changing the settlement strategy when the chosen one experiences a deadlock in solving the problem.</li> </ol>	<ol style="list-style-type: none"> <li>1. Can write down what is known and what will be resolved.</li> <li>2. Can write the derivative of the function <math>w(t) = 0,2t^2 - 0,09t</math>.</li> <li>3. Can substitute variable values into the derivative function.</li> </ol>	<p><i>The weight in grams of a malignant tumor at time <math>t</math> is <math>w(t) = 0,2t^2 - 0,09t</math>, where <math>t</math> is measured in weeks. Determine the tumor growth rate when <math>t = 10</math>.</i></p>
Flexibility	<ol style="list-style-type: none"> <li>1. Have various interpretations of the problem given.</li> <li>2. Have a different point of view in looking at the problem.</li> <li>3. If you are given a problem, you usually think of different ways to solve it.</li> <li>4. Answer questions in a variety of ways.</li> </ol>	<ol style="list-style-type: none"> <li>1. Can solve the problem using other solutions such as using the definition of a derivative.  <math display="block">f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}</math> </li> <li>2. Be able to interpret the results of the settlement.</li> </ol>	
Originality	<ol style="list-style-type: none"> <li>1. Finding other unusual strategies in solving problems.</li> <li>2. Have a different way of thinking than others.</li> </ol>	Finding solutions to problems by trial and error (unusual solutions).	

The obstacles to mathematical creative thinking experienced by students in this study occurred when they thought fluently, flexibly, and were original. The results of student work in solving derivative problems using mathematical creative thinking skills.

**TABLE 2**  
**STUDENT WORK RESULTS**

**Problems 1**

*Known function*

$f(x) = x(2x + 5)$  and  $g(x) = 3x - 1$  and  $P(x) = f(x) \cdot g(x)$ .

- Determine  $P'(x)$  in two different ways and include an explanation for each step of the process.
- Compare the results, which one is easier to implement, and explain why.

Student Work Results MP (high ability)

**FIGURE 1**  
**MP'S STUDENT WORK**

1. A? Cara Pertama

→ tentukan dulu antara  $f(x)$  dan  $g(x)$

$$\begin{aligned} \hookrightarrow f(x) &= x(2x+5) \\ &= 2x^2 + 5x \end{aligned}$$

→  $f(x) \cdot g(x)$

$$(2x^2 + 5x)(3x-1)$$

$$= 6x^3 + 15x^2 - 5x$$

→ turunkan nilai dari hasil hasil diatas

$$\begin{aligned} \hookrightarrow p'(x) &= f(x) \cdot g(x) \\ p'(x) &= 6x^3 + 15x^2 - 5x \\ p'(x) &= 18x^2 + 30x - 5 \quad \checkmark \end{aligned}$$

Cara kedua

→ turunkan terlebih dulu nilai  $f(x)$  dan  $g(x)$   
Suhu pertama, untuk menentukan  $u'$  &  $v'$

$$\begin{aligned} \hookrightarrow f(x) &= 2x^2 + 5x & g(x) &= 3x-1 \\ f'(x) &= 4x + 5 & g'(x) &= 3 \end{aligned}$$

→ turunkan dua hasil diatas untuk mencari nilai  $p'(x)$  dgn menggunakan rumus  $u'v + uv'$

$$\begin{aligned} u' &= 4x + 5 & u &= 2x^2 + 5x \\ v &= 3x - 1 & v' &= 3 \end{aligned}$$

$$\begin{aligned} p'(x) &= (4x+5)(3x-1) + (2x^2+5x)(3) \\ &= 12x^2 + 11x - 5 + 6x^2 + 15x \\ &= 18x^2 + 26x - 5 \quad \checkmark \end{aligned}$$

Student Work Results MU (medium ability)

**FIGURE 2**  
**MU'S STUDENT WORK**

a. Tentukan  $f(x)$  dengan dua cara berbeda dan sertakan penyelesaian pada setiap langkah pengerjaannya.

I.  $f(x) = x(2x+5)$

$g(x) = 3x-1$

$p(x) = f(x) \cdot g(x)$

$$= x(2x+5) \cdot (3x-1)$$

$$= (2x^2 + 5x)(3x-1)$$

$$= 6x^3 - 2x^2 + 15x^2 - 5x$$

$$= 6x^3 + 13x^2 - 5x \quad \checkmark \rightarrow \text{blm}$$

II.  $f(x) = x(2x+5)$

$g(x) = 3x-1$

$u = x(2x+5) = 2x^2 + 5x$        $v = 3x-1$

$u' = 4x+5$        $v' = 3$

$p'(x) = u'v + uv'$

$$(4x+5)(3x-1) + (2x^2+5x)3$$

$$12x^2 - 4x + 15x - 5 + 6x^2 + 15x$$

$$18x^2 + 26x - 5 \quad \checkmark$$

*Saran: ditunjukkan*

Student Work Results AR (medium ability)

**FIGURE 3**  
**AR'S STUDENT WORK**

a. Tentukan  $P'(x)$   
 Cara 1 : kalikan  $f(x)$  dengan  $g(x)$  untuk mendapatkan  $P(x)$   
 $P(x) = f(x) \cdot g(x)$   
 $= x(2x+5) \cdot (3x-1)$   
 $= (2x^2+5x)(3x-1)$   
 $= 6x^3 - 2x^2 + 15x^2 - 5x$   
 $= 6x^3 + 13x^2 - 5x$   
 Setelah mendapatkan nilai  $P(x)$  maka kita cari nilai  $P'(x)$  dengan rumus  $P'(x) = n \cdot a \cdot x^{n-1}$   
 $P'(x) = 6x^3 + 13x^2 - 5x$   
 $P'(x) = 3 \cdot 6x^{3-1} + 2 \cdot 13x^{2-1} - 1 \cdot 5x^{1-1}$   
 $= 18x^2 + 26x - 5$

Cara II :  
 Untuk mencari nilai  $P'(x)$  kita bisa langsung cari turunan pertama dari  $f(x)$  dan  $g(x)$  dahulu.  
 $P'(x) = f'(x) \cdot g(x)$   
 $P'(x) = x(2x+5)(3x-1)$   
 $P'(x) = (2x^2+5x)(3x-1)$   
 $P'(x) = f'(x) \cdot g(x)$   
 $= (2 \cdot 2x^{2-1} + 1 \cdot 5x^{1-1})(3x-1)$   
 $= (4x+5)(3x-1)$   
 $= 12x + 15$   
 Ternyata itu beda dan cara yang mudah itu menggunakan cara yang pertama karena mudah untuk dipalangi.

Student Work Results IJ (low ability)

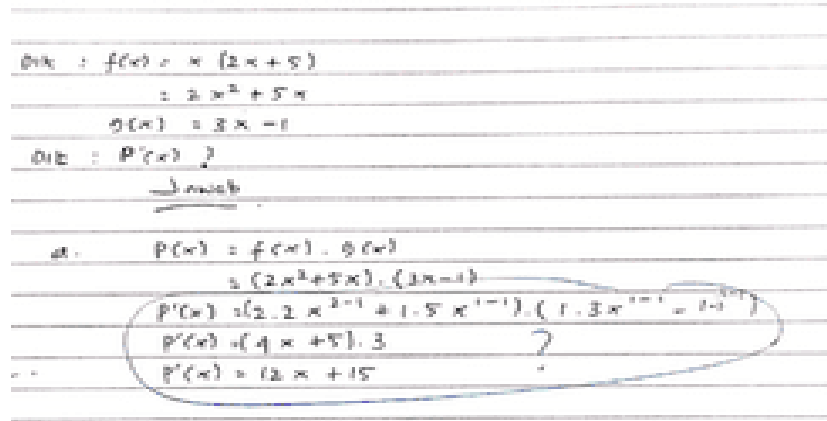
**FIGURE 4**  
**IJ'S STUDENT WORK**

1)  $f(x) = x(2x+5)$  dan  $g(x) = 3x-1$  dan  $P(x) = f(x) \cdot g(x)$   
 $\therefore f(x) = x(2x+5) \quad \cdot g(x) = 3x-1$   
 $\therefore f(x) = 2x^2 + 5x \quad \cdot g(x) = 3x-1$   
 $x = 4 \text{ dan } \quad x = 2$   
 $x = 9 \quad \quad \quad x = 3$

a.  $P'(x) = f'(x) \cdot g(x)$   
 $\therefore f'(x) = 2 \cdot g(x)$   
 $f(9) = g(2)^2 \quad \quad \quad f(9) = 2 \cdot g(2)^2$   
 $f(9) = g(4) \quad \quad \quad f(9) = 2 \cdot g(4)$   
 $f_9 = g \quad \quad \quad f = 4$   
 $\frac{f_9}{4} \quad \quad \quad \frac{f}{3}$

Student Work Results FT (low ability)

**FIGURE 5**  
**FT'S STUDENT WORK**



Aspects/Component Measured	High Ability Students	Students with Moderates Abilities	Low Ability Students	Analysis
Fluency	This student with the initials MP did not write down what was known and what would be completed. However, he could write down the equation $P(x) = f(x) \cdot g(x)$ well. MP has also been able to write down the derivative of the equation function $P(x) = f(x) \cdot g(x)$ .	Students with the initials MU and AR also do not write down what is known and what will be completed. However, they can write function equations and their derivatives well.	IJ and FT did not write down what was known and what was asked from the questions. They also cannot write equations well.	Based on student work and confirmation of the students studied, it was found that: Students with high, medium, and low abilities have can think creatively mathematically, but the levels are different. In low ability students, the results obtained are still wrong. IJ gave an answer irrelevant to the problem. This obstacle can be caused by a lack of initial knowledge and a lack of



				student concepts in solving problems.
Flexibility	Students with this ability have been able to solve problems using two ways of solving. It can be seen in the students' answers, that MP has done 2 ways at once in solving the problem, namely by multiplying the two factors first and then deriving them and by using the product rule.	Students with the initials MU and AR in this ability are only able to solve problems in one way, namely using the product rule formula. When using the second method, namely multiplying two factors, and then lowering them, they make an error. MU's mistake is not doing derivatives on the results obtained from the multiplication of two factors. Meanwhile, AR made an error when it derived the multiplication of two factors. $P'(x) = f'(x) \cdot g'(x)$ $= (2x^2+5x) (3x-1)$ $= (2 \cdot 2x^{2-1}+5x^{1-1})$ $(3 \cdot x^{1-1} - 0)$ $= (4x + 5) (3)$ $= 12x + 15$ (Not relevant to the product rule)	Students with low abilities, namely IJ and FT, cannot solve questions properly and correctly. They only do one way of solving and the solution is also not correct.	According to students with high and medium abilities, solving problems using the formula for the derivative of product is easier to understand and use because they only use formulas. If you are careful in entering the formula, students can do it. Meanwhile, low students experience barriers to thinking when solving questions at the beginning. They look confused what to do and how. So, you cannot solve the problem in any way.

**Problems 2**

The weight in grams of a malignant tumor at time  $t$  is  $w(t) = 0,2t^2 - 0,09t$ , where  $t$  is measured in weeks. Determine the tumor growth rate when  $t = 10$ .

Student Work Results MP (high ability)

**FIGURE 6**  
**MP'S STUDENT WORK**

Handwritten work for Figure 6:

Left side (Direct Derivative):

$$w(t) = 0,2t^2 - 0,09t$$

$$t = 10$$

Dit.  $\rightarrow$  Menc. Sesuai

$$w'(t) = 0,4t - 0,09$$

$$4 - 0,09$$

$$= 3,91 \text{ gram/minggu.}$$

Right side (Limit Definition):

Definis:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{0,2(2t^2) - 0,09(2t) - (0,2t^2 - 0,09t)}{h}$$

$$\lim_{h \rightarrow 0} \frac{0,2(2t^2 + 2t(2h) + 2h^2) - (0,09(2t + 0,09h)) - (0,2t^2 - 0,09t)}{h}$$

$$\lim_{h \rightarrow 0} \frac{0,2(2t^2 + 0,42h + 0,2h^2) - 0,09(2t + 0,09h) - 0,2t^2 + 0,09t}{h}$$

$$\lim_{h \rightarrow 0} \frac{0,42t + 0,04h - 0,09}{h} \rightarrow 0,42t - 0,09$$

Student Work Results MU (medium ability)

**FIGURE 7**  
**MU'S STUDENT WORK**

Handwritten work for Figure 7:

Left side (Direct Derivative):

Diket:  $w(t) = 0,2t^2 - 0,09t$   
 $t = 10$

Diket:  $w(t) = 0,2t^2 - 0,09t$   
 $w'(t) = 0,4t - 0,09$   
 $= 0,4(10) - 0,09$   
 $= 4 - 0,09$   
 $= 3,91$

Jadi laju pertumbuhan pertumbuhan tumor saat  $t=10$  adalah  $3,91$  g/minggu.

$$f(x) = 0,2x^2 - 0,09x$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{0,2(x+h)^2 - 0,09(x+h) - (0,2x^2 - 0,09x)}{h}$$

Right side (Limit Definition):

$$= \lim_{h \rightarrow 0} \frac{0,2(x^2 + 2xh + h^2) - 0,09x + 0,09h - (0,2x^2 - 0,09x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0,4xh + 0,2h^2 + 0,09h}{h}$$

$$= \lim_{h \rightarrow 0} (0,4x + 0,2h + 0,09)$$

$$= 0,4x + 0,09$$

Student Work Results AR (medium ability)

**FIGURE 8**  
**AR'S STUDENT WORK**

<p>Dik: <math>w(t) = 0,2t^2 - 0,09t</math>                  Dit: laju pertumbuhan saat <math>t = 10</math>                  Jawab:  <math>w'(t) = 2(0,2t) - 0,09</math>  <math>= 0,4t - 0,09</math>  <math>w'(10) = 0,4(10) - 0,09</math>  <math>= 4 - 0,09</math>  <math>= 3,91 \text{ gr/minggu}</math>                  Jadi, laju pertumbuhan jamur saat <math>t = 10</math> adalah <math>3,91 \text{ gr/minggu}</math>.</p>	<p><math>f(x) = 0,2x^2 - 0,09x</math>  <math>f'(x) = \lim_{h \rightarrow 0} \frac{0,2(x+h)^2 - 0,09(x+h) - [0,2x^2 - 0,09x]}{h}</math>  <math>\lim_{h \rightarrow 0} \frac{0,2(x^2 + 2xh + h^2) - 0,09x - 0,09h - 0,2x^2 + 0,09x}{h}</math>  <math>\lim_{h \rightarrow 0} \frac{0,2x^2 + 0,4xh + 0,2h^2 - 0,09x - 0,09h - 0,2x^2 + 0,09x}{h}</math>  <math>\lim_{h \rightarrow 0} \frac{0,4xh + 0,2h^2 - 0,09h}{h}</math>  <math>\lim_{h \rightarrow 0} \frac{h(0,4x + 0,2h - 0,09)}{h}</math>  <math>= 0,4x + 0,2h - 0,09</math>  <math>= 0,4x - 0,09</math></p>
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Student Work Results IJ (low ability)

**FIGURE 9**  
**IJ'S STUDENT WORK**

<p>Dik:  <math>w(t) = 0,2t^2 - 0,09t</math>  <math>w'(t) = 2 \cdot 0,2t - 0,09</math>  <math>= 0,4t - 0,09</math>  <math>w'(10) = 0,4(10) - 0,09</math>  <math>= 4 - 0,09</math>  <math>= 3,91 \text{ gr/minggu}</math></p>	<p>Definisir  <math>w'(t) = \lim_{h \rightarrow 0} \frac{0,2(t+h)^2 - f(t)}{h}</math>  <math>= \lim_{h \rightarrow 0} \frac{0,2(t^2 + 2ht + h^2) - (0,2t^2 - 0,09t)}{h}</math>  <math>= \lim_{h \rightarrow 0} \frac{0,2t^2 + 4ht + 2h^2 - 0,2t^2 + 0,09t}{h}</math>  <math>= \lim_{h \rightarrow 0} \frac{4t + 2h + 0,09t}{h}</math>  <math>\lim_{h \rightarrow 0} 4t + 2h + 0,09t</math>  <math>w'(t) = 4t + 0,09</math>  <math>w'(10) = 4(10) + 0,09</math></p>
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Student Work Results FT (low ability)

**FIGURE 10**  
**FT'S STUDENT WORK**

$$\begin{aligned}
 w'(t) &= 0,4t - 0,09 \\
 w'(10) &= 0,4(10) - 0,09 \\
 &= 4 - 0,09 \\
 &= 3,91
 \end{aligned}$$
  

$$\begin{aligned}
 f(x) &= \lim_{h \rightarrow 0} \frac{0,2(t+h)^2 - 0,09(t+h) - [0,2t^2 - 0,09t]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{0,2(t^2 + 2th + h^2) - 0,09t - 0,09h - 0,2t^2 + 0,09t}{h} \\
 &= \lim_{h \rightarrow 0} \frac{0,2t^2 + 0,4th + 0,2h^2 - 0,09t - 0,09h - 0,2t^2 + 0,09t}{h} \\
 &= \lim_{h \rightarrow 0} \frac{0,4th + 0,2h^2 - 0,09h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(0,4t + 0,2h - 0,09)}{h} \\
 &= 0,4t + 0,2(0) - 0,09 \\
 &= 0,4t + 0,09
 \end{aligned}$$

Aspects/Component Measured	High Ability Students	Students with Moderates Abilities	Low Ability Students	Analysis
Fluency	<p>MP can write down what is known and what is asked in the problem. MP can also write derivatives of functions well  <math>w(t) = 0.2t^2 - 0.09t</math>                      MP does not explain in detail the substitution of the variable value <math>t = 10</math> into the derivative function.</p> $  \begin{aligned}  w'(t) &= 0,4t - 0,09 \\  &= 4 - 0,09 \\  &= 3,91 \text{ gram/menagya.}  \end{aligned}  $ <p>MP suddenly writes <math>4 - 0.09</math> without explaining in advance how to get it.</p>	<p>MU and AR can write down what is known and what will be solved, make final conclusions, and write derivatives of functions <math>w(t) = 0.2t^2 - 0.09t</math> well. In addition, MU and AR can also substitute the value of the variable <math>t = 10</math> into the derivative function.</p>	<p>1. IJ has not been able to write down what is being asked in the problem. However, he can write the derivative of the function  <math>w(t) = 0.2t^2 - 0.09t</math>                      and can also substitute the variable value <math>t = 10</math> into the derivative function.</p> <p>2. FT can write down what is known and what is asked and can write down the derivative of the function.</p>	<p>Students with high, medium, and low abilities can think smoothly in solving problems.</p>

Flexibility	MP can solve problems using 2 ways, namely using derivative rules and derivative definitions. When using the definition of the derivative, MP made the mistake of using parentheses. In mathematics, it is important to pay attention to the meaning of the use of brackets in addition and subtraction operations. At the end of the conclusion, MP cannot interpret the results. MP just writes functions $0,04x - 0,09$ .	MU can solve the problem using 2 ways. But in the second way, namely using the definition of a derivative, MU makes a mistake. Both errors in the process and the result. The resulting functional equation should be $f'(x) = 0,04x - 0,09$ , but MU wrote $f'(x) = 0,04x + 0,09$ . This error occurs because there is a mistake and inaccuracy MU in using parentheses for addition and subtraction operations. Meanwhile, AR is also not much different from MU, making the mistake of using brackets for addition and subtraction operations. The difference is, AR can write down the result correctly.	In solving question number 2, IJ and FT use 2 ways. However, the results are different. The second method used by IJ and FT is wrong and wrong.	Students with high, medium, and low abilities still cannot think flexibly. They can use 2 ways of solving but still have errors in the process and produce wrong results.
Originality	Students work on it in their own way, but the steps used are still found errors.	Students work on it in their own way, but the steps used are still found errors.	Students work on it in their own way, but the steps used are still found errors.	Students with high, medium, and low ability levels are already able to work on the questions in their own way, but the steps used are still

				found to be errors. However, in this indicator, new ideas have not been found from each student's abilities.
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Quantitatively, students' scores in solving problems with mathematical creative thinking skills can be shown in Table 3 below.

**TABLE 3**  
**MATHEMATICAL CREATIVE THINKING ABILITY SCORING**

Question Number 1					
Subject	MP	MU	AR	IJ	FT
Aspect	Fluency				
Indicator	Give more than one idea, produce the right and correct answer	Delivers more than one idea smoothly, but results in incorrect answers	Delivers more than one idea smoothly, but results in incorrect answers	Gives answers that are irrelevant to the problem	Gives an idea, but the calculation process is wrong resulting in the wrong answer
Scoring	4	3	3	0	1
Aspect	Flexibility				
Indicator	Doing the problem in more than one way, the calculation process and the results are correct	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Do the problem one way but produce the wrong answer	Do the problem one way but produce the wrong answer
Scoring	4	3	3	1	1

Question Number 2					
Subject	MP	MU	AR	IJ	FT
Aspect	Fluency				
Indicator	Give more than one idea, produce the right and correct answer	Delivers more than one idea smoothly, but results in incorrect answers	Give more than one idea, produce the right and correct answer	Delivers more than one idea smoothly, but results in incorrect answers	Delivers more than one idea smoothly, but results in incorrect answers
Scoring	4	3	4	3	3
Aspect	Flexibility				
Indicator	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong	Doing the problem in more than one way, but there is an error in the calculation process so that the results are wrong
Scoring	3	3	3	3	3
Aspect	Originality				
Indicator	Doing it in its own way, the calculation process is directed but there are steps that have not been completed so the result has not been found or done in a way that is often used and the results are correct	Doing it in its own way, the calculation process is directed but there are steps that have not been completed so the result has not been found or done in a way that is often used and the results are correct	Doing it in his own way, the calculation process is directed, the steps taken are complete but the results are not yet	Doing it in his own way, the calculation process is directed, the steps taken are complete but the results are not yet	Doing it in his own way, the calculation process is directed, the steps taken are complete but the results are not yet
Scoring	2	2	3	3	3

Based on what has been obtained in this study, it can be said that students in the subject of this research can think creatively mathematically on the indicators of flexibility, fluency, and originality, but the levels are different. The level of mathematical creative thinking ability can depend on the questions or problems and the material being studied. Derivatives are a material that is quite difficult to develop mathematical creative abilities because they have many functions and symbols and seem stiff to solve them. Barriers to the ability to think creatively mathematically in this study can occur due to several factors, including 1) due to a lack of prior knowledge of students; 2) the lack of strong concepts possessed by students, for example,

the concept of using parentheses in addition and subtraction operations and the concept of using product rules. In addition, students often experience difficulties in entering variable values to conclude the results, as they do not seem to understand what is desired in the problem. Students understand more about the use of derivative rules than using other methods; this indicates that the originality of students' thinking skills is still quite weak; that is, it still depends on what is conveyed by the lecturer and what is contained in the book. They have not been able to develop the ideas or novelty they have. Therefore, it is necessary to help students in the form of scaffolding so they can develop their creative thinking skills. The scaffolding used in this study refers to research (Anghileri, 2006), namely, scaffolding level 1 (environmental provisions), scaffolding level 2 (explaining, reviewing, and restructuring), and scaffolding level 3 (developing conceptual thinking). This scaffolding is given when implementing Treffinger learning by using analogical reasoning. To increase the initial knowledge and concepts of students who often experience difficulties in solving problems, analogies are also needed. The importance of analogy in learning is not widely known by many people and not many people use it. Even though theoretically, it has been shown that analogical reasoning has an influence on the development of creative thinking skills. Analogical reasoning is crucial in forming perceptions and finding solutions to problems (Kristayulita, Nusantara, As'ari, & Sa'dijah, 2019). Through this analogical reasoning, it is not only able to develop students' mathematical creative thinking skills in answering questions but also able to solve problems in everyday life quickly, cleverly, and smartly (in a broad sense).

The following is the scaffolding design given in this study as a form of effort to overcome obstacles to students' mathematical creative thinking.

**TABLE 4**  
**FORM OF SCAFFOLDING**

Scaffolding Components	Activities Performed
Level 1: Environmental Provisions	<ul style="list-style-type: none"> <li>Recall about the definition of derivative  <math display="block">f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}</math> </li> <li>Remind students again about adjusting the use of symbols such as <math>x</math>, <math>c</math>, or <math>t</math> in accordance with known questions. For example, the problem is known  <math display="block">w(t) = 0,2t^2 - 0,09t</math> then use  <math display="block">\text{laju perubahan} = \lim_{h \rightarrow 0} \frac{w(t+h) - w(t)}{h}</math> </li> <li>Remind students again about the concept of the product rule, namely by using the example <math>u</math> and <math>v</math>, or by using <math>(f \cdot g)'(x) = f'(x)g(x) + g'(x)f(x)</math></li> </ul>
Level 2: Explaining	<ul style="list-style-type: none"> <li>Provide direction in the form of explanations and emphasis on questions that are not yet understood so that they can be easily understood.</li> <li>Give directions to students, that the problem has a way of solving that is like the following problem  <p><b>Source Problem</b>  <i>If it is known that <math>x</math> = your month of birth, then determine the rate of change of the function at</i></p> <math display="block">f(x) = (x^2 - 3)^2!</math> </li> </ul>



	<p><b>Targets Problem</b>  <i>The weight in grams of a malignant tumor at time <math>t</math> is <math>w(t) = 0,2t^2 - 0,09t</math>, where <math>t</math> is measured in weeks. Determine the tumor growth rate when <math>t = 10</math>.</i></p> <ul style="list-style-type: none"> <li>• Give emphasis to students about multiplication of algebraic functions using powers of two such as <math>0,2(t+h)^2</math>. This form can be solved by squaring first and then multiplying by 0.2, so it becomes <math>0,2t^2 + 0,2h^2 + 0,4th</math>.</li> <li>• Give emphasis to students on the use of decimal numbers, for example the subtraction operation <math>4 - 0.09 = 3.91</math>.</li> <li>• Give emphasis to students about the use of the product rule.</li> </ul>
Reviewing	<ul style="list-style-type: none"> <li>• When determining the result, students are asked to be more careful in calculating the operations of multiplication and subtraction of decimal numbers, for example  <math display="block">0,4(10) - 0,09 = 4 - 0,09 = 3,91.</math></li> <li>• Remind students to write down units in the final answer, for example grams/week, meters/second, and others.</li> </ul>
Restructuring	<ul style="list-style-type: none"> <li>• Ask students to re-check the process of solving problems that are solved using the definition of derivatives, for example in finding the rate of change.</li> <li>• Directing students to be able to write correct final conclusions, for example, so the rate of tumor change when <math>t = 10</math> is 3.91 grams/week.</li> </ul>
Level 3: Developing Thinking	<p>Conceptual</p> <ul style="list-style-type: none"> <li>• Ask students to find other alternatives in solving problems.</li> <li>• Provide guiding questions that make students discover other concepts related to the problem.</li> <li>• Provide direction to students in compiling problems related to everyday contexts. Students can use book sources as references and objects around them.</li> </ul>

Barriers to thinking due to lack of prior knowledge and student concepts in this study occurred at the stages of practice with the process and working with real problems contained in Treffinger learning. This Treffinger learning can be implemented for all students of various backgrounds and ability levels to be applied to this study. This learning can also help students learn and develop new ideas by using creative thinking, having a flexible nature, and seeing complex thinking. In the learning process, this shows the link between creative and critical thinking so that maximum results will be obtained in developing divergent thinking skills (Ariani Wirahayu, Purwito, & Juarti, 2018). This learning has three stages: basic tools, practice with process, and working with real problems (Shoimin, 2014). The Basic Tools stage is the foundation or foundation where creative learning develops. At this stage, focus more on how children can think divergently or openly without thinking that the opinions conveyed are right or wrong. At this stage what is carried out by the lecturer is that the lecturer conveys the learning achievements, materials, and stimuli that can dig up student information about the material to be delivered; the lecturer divides students into several groups based on their ability to think creatively mathematically; the lecturer gives open problems to students; and the lecturer guides students in conveying their ideas. In the practice with process stage, students solve problems by using analogical reasoning to develop their mathematical creative thinking abilities. This is in line with Hayes' opinion (in Fasko, 2000) that creativity can be increased in several ways: (1) developing basic knowledge; (2) creating the right atmosphere for creativity; (3) looking for analogies. Analogical reasoning aims to apply similar relationships in helping to understand problems or new mathematical concepts by going through previous mathematical material abilities so that if students or students use analogical reasoning in solving problems, it is possible to be able to find possible answers

to the same questions. In the process of learning mathematics students or students are often required to think or reason in looking for similarities or similarities or relatedness of the nature of a particular concept to other concepts through comparisons. In fact, it is not only required in learning mathematics but also in everyday life.

This stage provides an opportunity to be able to apply the skills that have been acquired and learned at the Basic Tools stage. The stages of analogical reasoning used to consist of the stages of recognition, representation, structuring, mapping, applying, and verifying. The findings in this study are that there are two new stages added to the early stages of analogical reasoning, namely the stages of recognition and representation. These two stages are important to add because students need knowledge at the beginning, namely recognition and students also need to represent what they know and what they will solve.

## **CONCLUSION**

Based on the research results, it was found that students in the subject of this study each could think creatively mathematically on the indicators of flexibility, fluency, and originality, but the levels were different. The level of mathematical creative thinking ability can depend on the questions or problems and the material being studied. However, one of the indicators of the ability to think creatively mathematically, which is very weak for students, is originality because students seem rigid with what they have obtained from lecturers and books. Students find it difficult to come up with ideas in solving problems. Barriers to the ability to think creatively mathematically can occur due to several factors, including 1) due to a lack of prior knowledge of students; 2) a lack of strong concepts possessed by students. Barriers to thinking in solving derivative problems are caused by the lack of initial knowledge experienced by students in understanding problems and determining ideas in planning solutions. One way for lecturers to overcome these obstacles is to provide scaffolding starting from the level of environmental provisions or preparing other descriptions when students do not understand the initial problems. Explaining or giving an explanation. Reviewing or reflecting on answers and improving work results. Restructuring, namely questions or directions to find answers and answer back with a better design. While the developing conceptual thinking stage or looking for other alternatives to solve the problem and provide direction to find other related concepts. The provision of scaffolding is carried out using Treffinger learning with several stages, namely basic tools, practice with process, and working with real problems. This scaffolding activity is given at the stages of practice with the process and working with real problems. In the practice with process stage, students solve problems by using analogical reasoning as an effort to develop their mathematical creative thinking abilities. This stage provides an opportunity to be able to apply the skills that have been acquired and learned at the Basic Tools stage. The stages of analogical reasoning used to consist of the stages of recognition, representation, structuring, mapping, applying, and verifying. The findings in this study are that there are two new stages added to the early stages of analogical reasoning, namely the stages of recognition and representation.

This research was only conducted at the Curup State Islamic Institute, where the number of students was still low in quantity, so it can be suggested that the more subjects are taken, the more likely the obstacles to thinking will be obtained.

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