Prospective Teacher's Visual Geometric Perceptions of Problem Solving

Agung Deddiliawan Ismail
Universitas Negeri Malang
Universitas Muhammadiyah Malang

Cholis Sa'dijah
Universitas Negeri Malang

Susiswo
Universitas Negeri Malang

Sisworo
Universitas Negeri Malang

This study was intended to describe the geometric visual perception of prospective teachers in problem-solving. The research subjects consisted of three people who were selected from 33 prospective teachers based on the differences in the answers given. The results showed that in visual perception, prospective teachers with middle mathematics skill followed the continuous perceptual theory. However, prospective teachers with high and low mathematics skill had different visual perceptions with constant perceptual theory. In solving these two questions, prospective teacher with middle mathematics skill tended to have the same perception. But prospective teachers with high and low mathematics skills had different visual perceptions. Projected in the visualization of the answer to question number 2, subject with high mathematics skills added a new point to build a new rectangle that was used to obtain supporting information in finding the value asked by the question. Likewise, the prospective teacher with low mathematics skills showed that the visual perception was line length being sought, resulting in addition and subtraction operations in line segments.

Keywords: *visual perception, problem-solving, geometry problems*

INTRODUCTION

Recent research regarding visual perception is only associated with data and spatial structures. For example, Gal & Linchevski (2010) recently researched the difficulty of visual geometric perception, and Krukar et al. (2021) researched 2D and 3D visual perception. Based on previous research, no research has been found on visual perception associated with solving geometric problems. The human thought process is complex, from capturing objects from the eye, then information processing in the brain, and ending with information stored in the brain. In the process of capturing objects, an activity called perception occurs.

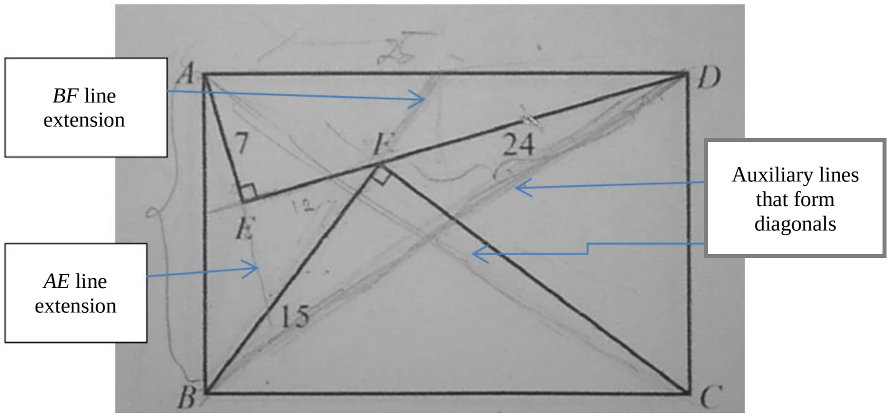
Perception is related to cognitive processes (Tacca, 2011; Wang et al., 2019). Sternberg & Sternberg (2012) explain that perception is a series of processes by which a person can recognize, organize, and understand the sensations received from stimuli. Perception includes many psychological phenomena. The visual system will provide information about the properties of the outside world (Wade, 2013). The visual process is the initial stage for recognizing geometric problems (Baiduri, Ismail, & Sulfiyah, 2020). Gibson (1978) creates a framework for visual perception, including the concept of distal objects, informational mediums, proximal stimulation, and perceptual objects. Distal objects are objects that exist in the outside world. An event that produces a pattern is called the informational medium. Reflected light, waves, and touch are also informational mediums; when these come into contact with sensory receptors, proximal stimulation occurs. Meanwhile, a perceptual object occurs when a reflection of the properties of the outside world is formed. The objective of visual perceptions is to estimate an object by distracting eyesight while observing object (Wang et al., 2019).

In the 21st century, competent teachers in community life are needed. They are the prospective teachers who are prepared to have skills. Rahman (2019) emphasizes that, knowledge is not enough for a student to succeed. Students need to acquire skills for the 21st century, such as problem-solving, being creative and innovative, metacognitive ability, communication, and others. Problem-solving is one of the foundations of cognitive processing. Meaningful learning is based on a constructivist approach and is developed per cognitive learning theory (Krawec, 2014; Jonassen, 2000; Sadijah et al., 2021). Problem-solving is a process that involves systematic observation and critical thinking to find the right solution or way to achieve the desired goal. Kim et al. (2018) explain that there is a positive influence between problem-solving and innovative behavior and the perception of opportunities that are needed in the 21st-century generation. So that by having problem-solving abilities, humans as individuals can develop innovative abilities and can read opportunities in social life. Aydoğdu & Ayaz (2015) also argue that problem-solving is essential because it has a functional role for individuals and society. Problem-solving is also a strategy that can be used to solve problems, learn new concepts and abilities, and strengthen existing concepts (Ismail, 2016; Walida et al., 2022).

Geometry problem solving is one of the fields that exist in mathematical problem-solving. Geometry is an interesting field because it not only contains numbers but also symbols in the form of images as visualization (Henderson, 1982). Lin & Lin (2016) pinpoint that problem solving in geometry is challenging for students who understand problems and concepts. However, it becomes problematic when students do not have the knowledge or information on their working memory, schemas on their long-term memory, and visual understanding of geometry drawing. Geometry is an interesting mathematical field, but many students have difficulty solving geometric problems. Geometry is a branch of mathematics that studies two and three-dimensional shapes. The results of a study conducted by Chen et al. (2020) show that in the 2011 TIMSS data, students have the lowest score in geometry. Sulistiowati et al., (2019) argue that the main difficulty experienced by students is the visualization stage in interpreting the problem into a mathematical model and conducting the analysis stage. The previous research state that students who have moderate and low abilities tend to only be in the visualization and analysis stages (Nurani et al., 2016; Suwito et al., 2017). Visualization and analysis are the initial stages in the geometric process (Wahidah et al., 2017).

Based on the results of preliminary research conducted on three prospective teachers of Class A in the second semester of the Mathematics Education Study Program, who had taken the Geometry course, it was found that two of them answered incorrectly on the given geometry problem-solving test questions. The test questions were adopted from SIMAK UI 2018. The analysis uncovered that the errors occurred during the visualization and image analysis stages. The following is a picture of a prospective teacher's answer.

FIGURE 1
ONE OF PROSPECTIVE TEACHER'S ANSWER

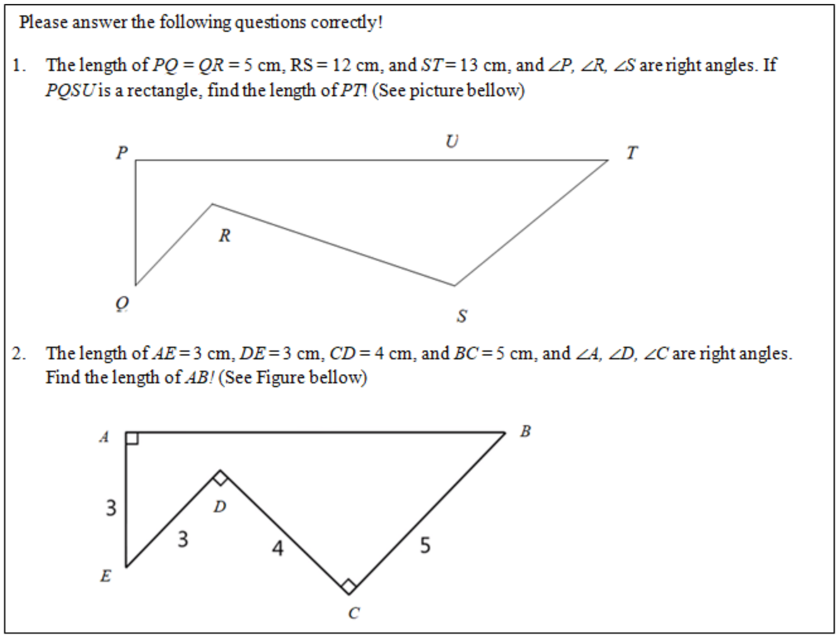


In Figure 1, it is known that there is an *ABCD* rectangle and on the inside, there are two right triangles *AED* and *BFC*. Note that the lengths are $AE = 7$, $DE = 24$, and $BF = 15$. To solve the question, the prospective teacher made auxiliary lines, namely *AC*, *BD*, and the extension of the *BF* and *AE* lines, to solve the question. If the prospective teacher experienced difficulties in this process, he would not be able to continue at the next stage in solving geometric problems. Gal & Linchevski (2010) conclude in the results of their study that there is a need for research on perceptual analysis in solving geometric problems. So based on this explanation, it is necessary to study and analyze more deeply geometric visual perceptions in problem-solving. This study aims to describe the visual perception of geometry that occurs in prospective teachers' problem-solving skills.

METHOD

The present study employed an in-depth analysis to obtain clear and structured data. To achieve this objective, this study implemented a qualitative descriptive approach. Qualitative descriptive research seeks to describe and interpret objects according to what they are. The research subjects were 4th semester of mathematics education prospective teachers in Universitas Muhammadiyah Malang. Criteria for the subject in this study were prospective teachers who had taken the geometry course and, in the answer, the visual perception was found in solving geometric problems. Another criterion was that the informant had communicative skills, both written and oral. To collect data, researchers exemplified several data collection techniques. First is the test technique. This technique was carried out twice at the time of the initial research study and at the time of the research. Secondly, the observation technique. This technique was done when the research process was running by observing the research subjects. Researchers were the main instrument in this research. The secondary instrument was in the form of test questions, observation sheets, and an interview guide. The test questions were given in the form of a problem-solving question sheet with one item with an essay answer to the description. The following is a test question adapted from Gercekboss (2019).

FIGURE 2
THE TEST QUESTION


Please answer the following questions correctly!

1. The length of $PQ = QR = 5$ cm, RS = 12 cm, and $ST = 13$ cm, and $\angle P$, $\angle R$, $\angle S$ are right angles. If $PQSU$ is a rectangle, find the length of PT! (See picture bellow)

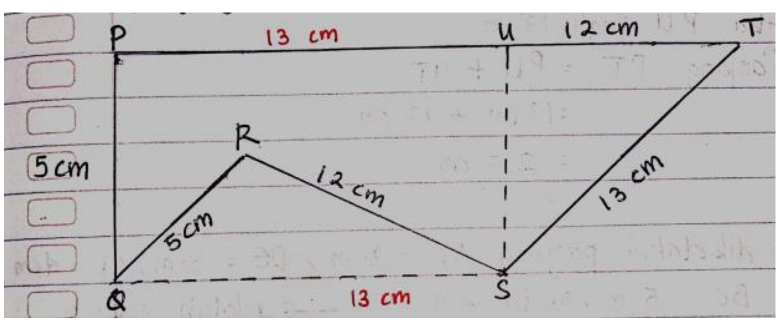
2. The length of $AE = 3$ cm, $DE = 3$ cm, $CD = 4$ cm, and $BC = 5$ cm, and $\angle A$, $\angle D$, $\angle C$ are right angles. Find the length of AB! (See Figure bellow)

RESULTS

Based on the test results from 33 respondents, 29 prospective teachers answered the questions, and 4 prospective teachers did not answer the questions. Based on the 29 respondents who answered, it was found that 27 prospective teachers had the same visual perceptions in working on questions 1 and 2. From the answers, it was also found that 2 answers had different pictures or visualizations. There are three subjects in this research. Subject 1 is prospective teacher with high mathematics skill, subject 2 is prospective teacher with middle mathematics skill, and subject 3 is prospective teacher with low mathematics skill.

Subject 1

FIGURE 3
VISUALIZATION OF SUBJECT 1 FOR QUESTION NUMBER 1



Based on subject 1's answer, it was obtained in Figure 3. From Figure 3, it could be see that to determine the length of the PT, subject 1 described the geometric shape in question number 1. Then subject 1 provided information in the image, namely the length of $PT = 5cm$, $QR = 5cm$, $RS = 12cm$, and $ST = 13cm$.

Figure 3 also shows the QS and US auxiliary lines. The following is a snippet of the interview with subject 1.

Researcher : How did you determine the length of PT
Subject 1 : We had to find the length of the PU and UT, Sir. To find the length of PU, we first looked for the QS length with the help of QR and RS. For UT, we used US and ST.
Researcher : The QR, RS, US, and ST lines were used to what extent?
Subject 1 : To find the lengths of the QS and UT Sir, using the Pythagorean formula.
Researcher : What was the relation between PU and QS lines?
Subject 1 : $PUSQ$ is a rectangle, Sir. So PU and QS were parallel and opposite, and therefore they had the same length.

From the description of the image and the interview, it can be concluded that to solve problem number 1, subject 1 redraws the geometry image in the problem. Next, subject 1 gave the information on the picture. To find the length being asked, subject 1 used auxiliary lines. The created auxiliary lines formed a rectangular shape and a right triangle. Using the Pythagorean formula and the properties of the rectangle, subject 1 could find the length in question.

FIGURE 4
VISUALIZATION OF SUBJECT 1 FOR QUESTION NUMBER 2

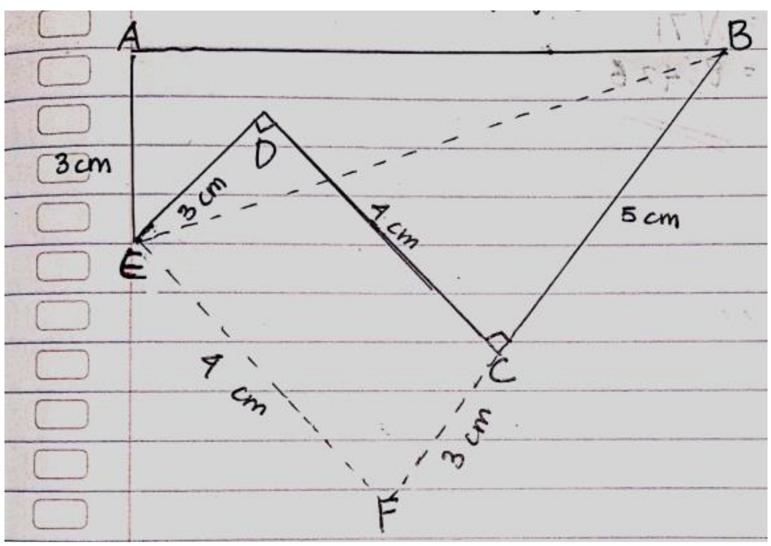


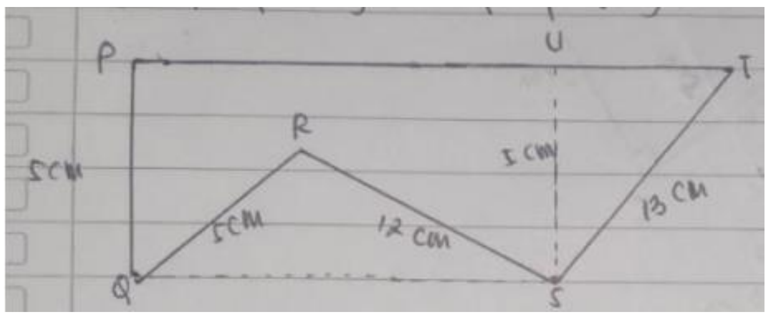
Figure 4 indicated that to work on problem number 2, subject 1 found the length of AB by drawing geometric shapes and providing information in the image. Among the lines' lengths were the length of $AE = 3cm, ED = 3cm, DC = 4cm$, and $CB = 5cm$. Subject 1 used the *BE* auxiliary line and made *F* point, which would form the FE and FC auxiliary lines. The connected $EDCF$ points formed a rectangular shape so that the parallel and opposite lines had the same length. The following is a quote from an interview with subject 1.

Researcher : How did you find the length of AB?
Subject 1 : I have to find the length of AB by first finding the length of BE. To find the length of BE, I made point F, so that I could find the length of EF and FC. When I found it, I could measure the length of BE by using the EF and FB lines.

From the results of the interview quotation with subject 1 and Figure 4, it can be concluded that to find the length in question, namely the length of AB, subject 1 drew an auxiliary line connecting points B and E. From points $A, B,$ and E, a right triangle ABE was formed. To find the length of $BE,$ subject 1 created a new point F. The point was then connected to points C and E so that the FC and FE auxiliary lines were able to be drawn. After looking for the length, a right triangle BFE was formed. The length of BE could be calculated using Pythagoras in the BFE triangle. Because the lengths of BE and AE were known by using Pythagoras, thus the length of AB could be found.

Subject 2

FIGURE 5
VISUALIZATION OF SUBJECT 2 FOR QUESTION NO. 1



Based on Figure 5, it could be seen that to find the length of PT, subject 2 added the length of PU and UT. PU length was obtained from $PU = QS$. Meanwhile, the length of UT was derived from operating the US and ST using Pythagoras. The following is an excerpt from an interview with subject 2.

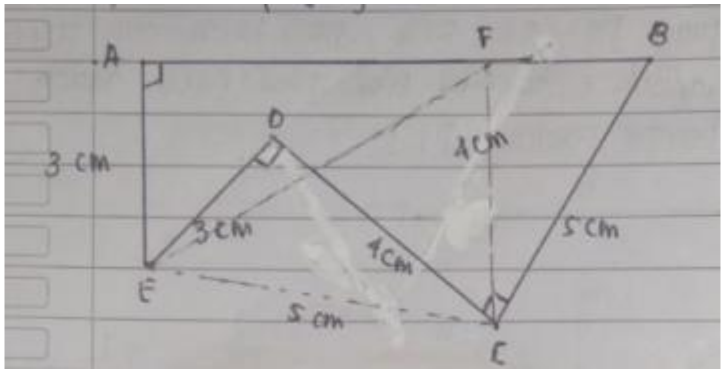
Researcher : Pay attention to the picture you drew. Can you tell me how you can determine the length of the PT line?
Subject 2 : Mhmm. The length of $PT = PU + UT$. Then, $PU = QS$. $PQSU$ shape then was a rectangle, right? The length of UT can be found by using $PQ = SU$ and ST

From the results of the interview excerpt and Figure 5, it could be said that subject 2 used the Pythagorean formula to find the side lengths of a right triangle and used the properties of the rectangle. To find the length in question, subject 2 uses auxiliary lines. There were two auxiliary lines used. The US auxiliary line was used to form the right triangle of SUT. Since $PUSQ$ was a rectangle, and the length of $PQ = US$. Then, the UT length could be calculated using the Pythagorean formula.

To solve problem 2, subject 2 formed the geometry from the question. Subject 2 redrew the existing picture in the problem, which was shown in Figure 6 by writing the length measurements on each side. Subject 2 seemed to know what the question meant. The following is an excerpt from an interview with subject 2.

Researcher : Could you please explain your picture?
Subject 2 : This picture was drawn from the question. In the question, I got the information that the length of $AE = 3cm, DE = 3cm, CD = 4cm$, and $BC = 5cm$, whereas $\angle A, \angle D, \angle C$ were right angles. Then I was asked to find the length of AB.
Researcher : What was to find from the question?
Subject 2 : the length of AB.

FIGURE 6
VISUALIZATION OF SUBJECT 2 FOR QUESTION NO. 2



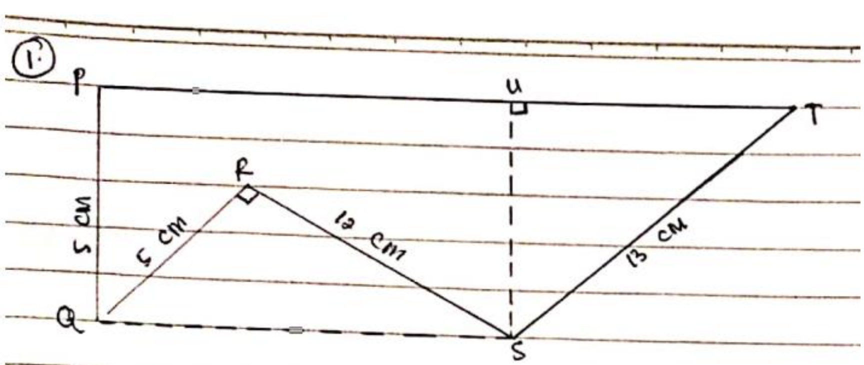
Based on figure 6, to determine the length of AB, subject 2 added up the AF and FB lengths. AF was obtained from the EAF triangle, and BC was obtained from the BFC triangle. The following is an excerpt of an interview with subject 2.

Researcher : Please tell me how you got the length of the AB line from your picture!
Subject 2 : It was similar to number 1 Sir, the length of AB equal to $AF + FB$. The length of AF was derived from the EAF triangle, and the length of FB was acquired from the BFC triangle.
Researcher : The EF in EAF triangle, how can you get that?
Subject 2 : The EF was found from the ECF triangle.

From Figure 6 and the interview with subject 2 it could be explained that to find the length of AB, there were two stages, namely finding the length of AF and BF, used to find the length of AB. To find the AF length, subject 2 used EF's side assist. To find the length of EF, you have to find the extension CE and DF. Since DE and CD were known, CE could be searched using Pythagoras. Subject 2 assumes that the CDE and CFB segments of triangles are congruent, so the lengths of CD and CF were the same. Subject 2 also thought that the FCE triangle was a right triangle. So, to find the length of EF, the Pythagorean formula was exemplified.

Subject 3

FIGURE 7
VISUALIZATION OF SUBJECT 3 FOR QUESTION NO. 1



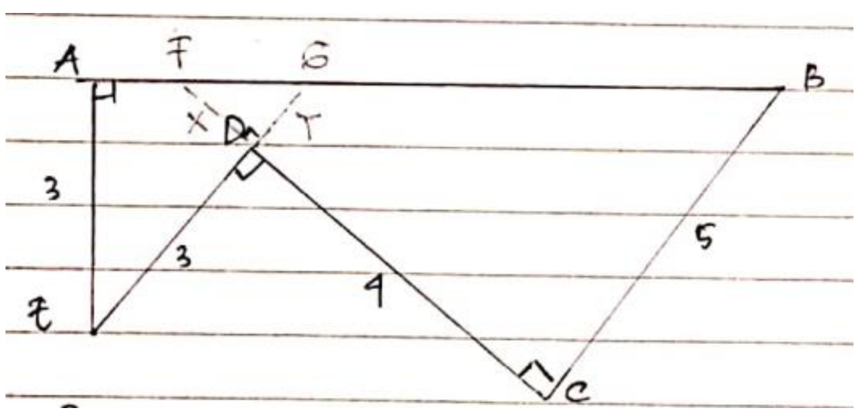
Based on Figure 7, in finding the length of PT, subject 3 implemented the concept of a rectangle and a right triangle, these two concepts were used to find the length of PU. Besides, to find the length of UT, the subject also implemented a right triangle. The following is an excerpt of the interview with subject 3.

Researcher : *Please have a look at your picture. Could you please explain to me how to find the length of* PT*?*

Subject 3 : *Well, the* PT*'s length could be derived from adding* PU *and* UT*.* $PU = QS$ *as* $PU//QS$ *because they were in a rectangular shape of* $PQSU$*. For* UT*'s length, I could calculate from the* SUT *right triangle.*

From the interview excerpt with subject 3 and Figure 7, it could be explained that to find the length of PT, what should be done was to find the length of PU and UT. The PU length was obtained by finding the QS length with the help of QR and RS using the Pythagorean formula. Since $PUSQ$ was a rectangle, then the length of $PU = QS$. To find the UT length, subject 3 uses the rectangular property and got the length $US = PQ$. Using the Pythagorean formula, the UT length could be found using the US and ST line lengths.

FIGURE 8
VISUALIZATION OF SUBJECT 3 FOR QUESTION NO. 2



Subject 3 also redrew what was known in problem number 2. Interestingly, from Figure 8, it could be seen that subject 3 only described 2 auxiliary lines. Among them were the DF line symbolized by *x* and the DG line symbolized by *y*. Based on this figure, subject 3 searched for the length of AB by adding AG and FB, and the result was reduced by FG. AG was obtained from the EAG right triangle, FB was obtained from the BCF right triangle, and FG was obtained from the FDG right triangle, respectively. The following is an excerpt from the interview.

Researcher : *Based on the picture you drew for question number 2, could you please elaborate on how to get the length of the* AB *line?*

Subject 3 : *I think, in order to find the length of the* AB *line, I could use three right triangles* EAG*,* FDG*, and* FCB*. From the three right triangles, I could find the length of* AG*,* FG*, and* FB *lines. Afterward, I used the following formula* $AB = AG + FB - FG$*.*

Based on Figure 8 and the interview with subject 3, it could be explained that to find the length of *AB*, subject 3 made an auxiliary line by extending the ED and CD lines that cut the AB line. The point of intersection was named G and F. The auxiliary line DF was named x, and DG was called y. Subject 3 assumed that the length $AB = AG + FB - FG$. *AG* was obtained using the Pythagorean formula for the right triangle EAG. Line FG was obtained from the right triangle FDG, while FB is obtained from triangle BCF.

DISCUSSION

This study aimed to describe the visual perception that occurred in research subjects when working on geometric solving problems. Geometry and visual perception are closely related or interrelated (Öğmen & Herzog, 2010). Previous research results indicate that there is a correlation between visualization skills and performance in geometry (Cui et al., 2017; Weckbacher & Okamoto, 2018; Ilhan et al., 2019). If someone can visualize geometry problems, it will surely have an impact on their performance in solving geometric mathematical problems. Visual perception can describe a problem to be solved depending on properties such as the shape, color, or surface of an object.

The questions tested on the research subjects were similar but not the same. Sternberg & Sternberg (2012) explain that in a visual session, there is a constant perception where the perception of an object will be used to create perceptions on objects of different sizes. When viewed from the answers or pictures made by the research subjects, prospective teachers were proven to have similar visual perceptions. Based on the results of the study, this opinion was observed under subject 2. The results showed constant perceptuality when working on questions 1 and 2 (Roberts & Suppes, 1967). Subject 2 implemented almost the same method in determining the length of the line in question. The auxiliary lines used were almost the same. Auxiliary lines and symbols were very important in solving geometric problems, handwriting can help develop visual perception (Vinci-Booher & James, 2020). Making auxiliary lines is an observation activity on parts of the image that stand out and are then linked to one another (Wang et al., 2019). It can be seen that the CF line in Figure 5 has the same shape and position as the SU line in Figure 4. While the CE line in Figure 5 was the same as the QS line in Figure 4 (Palatnik & Sigler, 2019). Based on the quotes of the interview with subject 2, it could be seen that subject 2 thought that to find the length of the AB side in problem 2 was the same as finding the length of the PT side in problem 1. The shapes made by subject 2 by adding auxiliary lines were mentioned as visual perceptions that were made. Visual perception can describe problems to be solved depending on properties such as the shape, color, or surface of an object (Murray & Adams, 2019).

In contrast to subjects 1 and 3, the results of the study showed that the perceptual constant described by (Sternberg & Sternberg, 2012) did not apply to subjects 1 and 3. In question number 1, subject 1 had the perception that there were two shapes there, namely a rectangle and a right triangle, by using the auxiliary lines QS and SU. Whereas in question number 2, in finding the length of AB, subject 1 used the rectangular property and the Pythagorean formula on a right triangle. Even though the pictures in questions 1 and 2 were almost the same, subject 1 had a different perception of the image. On the other hand, subject 2 made a new point which was point F in figure 3 so that it could form a $CDEF$ rectangle. To determine each side length of the $CDEF$ rectangle, subject 2 made a comparison. Visual perception contributes to the comparison of numbers (Cui et al., 2017). This stage was one of the procedures for making auxiliary lines (Suwa & Motoda, 1989). Subject 2 used a combination of recalling and anticipating reasoning (Palatnik & Sigler, 2019). The rectangle was intended to find the side lengths of BF and EF, which would later be used to find the length of BE. If line BE was known, then AB length could be found. Subject 1 recognized visual patterns from the shapes made (Gal & Linchevski, 2010). Another perception also occurred in subject 3. To find the length of PT in question number 1, it could be seen that subject 3 implemented the concept of rectangles and right triangles. The results showed that to find the length of PT, subject 3 added PU and UT. Line PU was obtained by operating the QRS triangle in Figure 6 and then using the rectangular properties. To find the UT length, subject 3 used a right triangle principle. For the second problem, subject 3 did not use the perception that had been done in problem 1. In this problem, subject 3 used a right triangle by extending the CD and ED lines to cut the AB line. The provision of the auxiliary lines affects solving geometric problems (Fan et al., 2017). The use of line definitions and carrying out the process of extending is anticipating reasoning conducted by subject 3 (Palatnik & Sigler, 2019), so that the AG, FG, and FB lines were obtained, which were the sides of the $EAG, FDG,$ and FCB triangles. From these lines, we could find the length of $AB = AG + FB - FG$.

CONCLUSION

The study concluded that each person's visual perception is different. Even though the objects that are seen have similarities, perceptual consistency is also sometimes different for some people, as occurred to is prospective teachers with high and middle skill mathematics. Prospective teacher with low skill mathematics carried out visual perceptions in accordance with the theory where the initial perceptions he had would be used to create perceptions of other objects that had almost the same shape. In contrast to prospective teachers with high and low skill mathematics, the visual perception created in question number 1 was not used to create visual perceptions in question 2, which had almost the same shape. In solving the two questions, prospective teachers with middle skill mathematics tends to have the same perception. It can be seen from the auxiliary lines used that there is a similarity in location or position. Prospective teachers with high and low skill mathematics have different visual perceptions when working on these two questions. Projected in the visualization of the answer to question number 2, prospective teacher with high sill mathematics added a new point to build a new rectangle that is used to obtain supporting information in finding the value or length asked by the question. Likewise, prospective teachers with low skill mathematics created the visual perception that the line length sought was the result of addition and subtraction operations on the line segments.

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