A Portrait of Controversial Mathematics Problems and Students' Metacognitive Awareness: A Case of Indonesia

Sikky El Walida
Universitas Negeri Malang
Universitas Islam Malang

Cholis Sa'dijah
Universitas Negeri Malang

Subanji
Universitas Negeri Malang

Sisworo
Universitas Negeri Malang

This study sheds light on the students' use of metacognitive strategies in solving controversial mathematical problems. This study involved 80 students. Data on metacognitive strategies were obtained through interviews and the results of students' work on test items. Data analysis and interpretation of the findings were carried out through text analysis on metacognitive strategy data obtained at the planning, monitoring, and evaluating stages. The results of data analysis depict that at the planning stage, students identify the aspect of the problems they have known and recognize the contradictory things. At the monitoring stage, students explore the elements that cause controversy. In the evaluating stage, students clarify, strengthen, improve, and conclude solutions to controversial mathematical problems. The results of this study indicate that it is necessary to apply metacognitive strategies to facilitate students to trace back the aspects inducing the controversy, eradicating the controversies among students. To this end, lecturers and students can develop mathematics learning models that involve metacognitive strategies for the success of learning mathematics in the future.

Keywords: metacognitive strategies, problem-solving, controversial mathematical problems

INTRODUCTION

In the last five years, studies on controversial mathematical problems have been carried out. Simic-Muller et al. (2015), for example, conducted research on controversial reasoning in mathematics education. Mueller and Yankelewitz (2014) revealed that controversial issues lead to many different reasons. Meanwhile, Subanji et al. (2021) suggested the level of reasoning to solve mathematical problems. In addition, Rosyadi et al. (2022) revealed the link between controversial problem solving and critical thinking using HOTS (higher-order thinking skills). Based on those previous studies, there has been limited research linking controversial mathematical problems with metacognitive strategies.

The students' procedures to solve controversial problems are essential to metacognitive strategies. Controversial problems refer to situations that cause debate due to different points of view (Simic-Muller et al., 2015). They are problems that are contrary to the existing schema. These problems arise as a result of understanding an unresolved problem, causing conflicts in one's thinking. Mueller and Yankelewitz (2014) reported that the existence of controversial issues could lead to diverse reasoning from students' understanding.

Controversial problems are also experienced by university students in the Mathematics Education Study Program in Malang. The students face many failures in solving math problems because of their inability to apply their metacognitive skills. It is reflected in the students‘ law average score (< 70) in Linear Algebra courses in the last three years. Metacognitive strategies facilitate students to monitor and control their thinking processes (Ku & Ho, 2010).

Learning mathematics is a process of developing a beneficial mindset to solve problems, one of the five standards of the mathematics learning process (NCTM, 2014). In the context of learning mathematics, controversy can occur when a student finds an uncommon problem that is different from the prevalent problem. Controversial problems also often occur in mathematics learning, placing problem-solving skills as the fundamental elements of mathematics learning.

The importance of problem-solving in learning mathematics has resulted in extensive studies on problem-solving (Gurat, 2018; Intaros et al., 2014; Sa'dijah et al., 2020; Schoenfeld, 2016). Gurat (2018) explored problem-solving strategies in teacher-student interaction. Furthermore, Sa'dijah et al. (2020) pointed out that problem-solving is a core activity in learning mathematics. Therefore, improving problem-solving skills is the focus of learning mathematics. Schoenfeld (2016) stated that problem-solving is a learning process to complete new and unfamiliar tasks when the appropriate solution-finding method is partially or not known. Problem-solving skills is required in solving controversial mathematical problems.

The frequent emergence of unrealized controversial issues has encouraged many experts to study the controversial problems (Dewhurst, 1992; Goldberg & Savenije, 2018; Kello, 2016; Oulton et al., 2004; Simic-Muller et al., 2015). However, there is still limited research examining controversial mathematics problem-solving. Dewhurst (1992), for example, unveiled that students need to be familiarized with controversial knowledge and equipped with skills to deal with controversial issues. Oulton et al. (2004) reported that students must develop a realistic understanding to resolve controversial problems. Meanwhile, Simic-Muller et al. (2015) found that teacher candidates were open to teaching mathematics in a real-world context but conflicted over using controversial problems.

The process of solving controversial mathematical problems involves not only cognitive conflict but also metacognitive strategies. These two factors are critical in improving the mathematics learning process. In dealing with controversial problems, students will experience cognitive conflicts due to the contrary between the issues they face and the existing thinking schemes. Therefore, students will revisit the problems they face and apply metacognitive strategies to find the factors that cause controversy. Several researchers have studied cognitive conflict (Kang et al., 2004; Watson, 2002; Watson, 2007). In general, cognitive conflict can be considered one of the crucial factors in the concept learning process because the conflict may transform the students' thinking structure.

Purnomo et al. (2017) stated that failure in solving mathematical problems is caused by a lack of understanding of metacognitive aspects, primarily related to the problem-solving steps. Multiple studies have shown that metacognition may improve students' problem-solving skills because it enhances problem-solving attempts. Consequently, metacognition is essential for students in solving mathematical problems (Kusumaningtyas et al., 2018). Likewise, metacognitive strategies have also been widely studied by several researchers (Desoete & De Craene, 2019; Goh, 2008; Ku & Ho, 2010).

The previous studies on cognitive conflict and metacognitive strategies suggest that the application of metacognitive strategies in solving controversial mathematical problems can be traced. The controversial math problem contains a conflict between the completion process and the obtained final result, producing cognitive conflicts. Thus, a metacognitive strategy is required for the solvency of that problem.

Several studies have highlighted issues on controversial problems (Goldberg & Savenije, 2018; Mueller & Yankelewitz, 2014) and metacognition (Kusumaningtyas et al., 2018; Purnomo et al., 2017). However, no research investigates metacognitive strategies for solving controversial mathematical problems. Our preliminary studies have produced metacognitive strategies for solving controversial problems. Anchored by the previous reviews and preliminary study, this research was carried out to reveal the students' metacognitive strategies in solving controversial mathematical problems. The results of this study are expected to provide awareness to the lecturers that students may receive a different understanding of the materials delivered by the lecturers. Therefore, it is necessary to retrace the causes of controversy to remove controversy among students. At this point, the results of the study can provide input for lecturers as well as students in developing models of mathematics learning that involve metacognitive strategies for the success of mathematics learning in the future.

METHOD

Participants

This research applied a qualitative approach with a case study design. A qualitative approach was used to describe the students' procedures of using metacognitive strategies in solving controversial mathematical problems. This study involved 80 students consisting of 19 male and 61 female students from the mathematics education study program at two private universities in Malang. The students had taken Linear Algebra courses. The research instruments were in the form of a test about controversial mathematical problems and an interview guide. The controversial mathematical test was used to describe the students' analysis process when finding a problem requiring an unusual problem-solving process. Meanwhile, the interview guide was used to clarify the obtained data and identify the suitability of the student's written answers and spoken explanations in solving controversial problems. The controversial mathematical problems given to the research subjects are as follows.

FIGURE 1
THE CONTROVERSIAL MATHEMATICAL PROBLEMS

Consider a student's solution to the following equation.

$$\frac{x-4}{x+2}=\frac{x-4}{x+3}$$
$$(x-4)(x+3)=(x-4)(x+2)$$
$$x+3=x+2$$
$$3=2$$

1. Are the completion steps made by the student logical? Explain!
2. How is the correct completion? Explain!

Data Collection

The data were obtained from students' written answers and think-aloud in solving controversial mathematical problems. Data collection was carried out by recording or documenting the students' process of solving controversial mathematical problems and the interview process. From these data, several types of students' answers were found in solving controversial mathematical problems. First, some students worked on problems using cross multiplication, while other students worked on the problem by multiplying both sides with $(x+3)$ $(x+2)$. Also, some students eliminated the element $(x-4)$. Further, three student answers which represented logical and illogical answers, were selected. The selection was carried out based on the student's ability to write down the reasons for their answers.

Collecting data, reducing data, showing data in the form of pictures or tables, and drawing conclusions from tests and interviews were all parts of data analysis. Students were asked to work on a controversial math problem that was given to them. By using data reduction, three of the 80 students who had conflicts were chosen. People were talked to about the three people being studied. This study also used task-based interviews and the think-aloud method to find out the metacognitive strategies that students used to solve controversial mathematical problems. Trocki et al. (2015) stated that the think-aloud method could encourage students to share their thoughts. In addition, both the interview findings and the students' work were analyzed. The analytical findings were provided as narrative text. The last phase was to draw conclusions from the study findings in response to the research problem formulation.

RESULTS

Based on the data analysis, students have applied some metacognitive strategy activities in solving controversial mathematical problems. Their metacognitive strategies are presented in Table 2.

TABLE 1
METACOGNITIVE STRATEGY ACTIVITIES

Metacognitive Strategy	Description	Metacognitive Strategy Activities
Planning	▪ Identifying the elements in the problems ▪ Recognizing the existence of contradictory aspect	1. Redetermining the purpose of the problem to be solved 2. Rementioning the elements in the problem 3. Restating that the provided answer is logical or illogical 4. Restating the presence of a conflict between the process and the resulting answer
Monitoring	Tracing the things that cause conflict	1. Reunderstanding the concept of equations 2. Reunderstanding the concept of division 3. Reunderstanding the concept of undefined division conditions 4. Retracing contradictory elements 5. Rementioning the elements causing conflict
Evaluating	Clarifying, strengthening, correcting, and concluding solutions	1. Reexplaining the elements that cause conflict 2. Concluding the causes of conflict 3. Reimproving solutions based on the correct concepts and theories, then producing solutions

These metacognitive strategies were used as a guide in solving controversial mathematical problems. Thus, based on the metacognitive strategy activities in Table 3, the answers from the research subjects are presented in Figure 2.

Based on the analysis of students' works, 24 students experienced conflicts between their answers and the steps that should be taken. Some presented logical processes even though the final results were contradictory, while some had illogical processes even though they generated the expected final results. Of the 24 students, 18 students stated that they worked on problems using cross multiplication, as shown in Figure 3, and six students stated that they worked on problems by multiplying both sides $(x + 3)$ $(x + 2)$, as shown in Figure 5. In the next step, there were similarities in their answers as they eliminated or crossed out $(x - 4)$ and then removed the x.

The following describes students' metacognitive strategy activities in solving controversial mathematical problems.

Planning

FIGURE 2
ANSWER FROM SUBJECT 1

Translated Version:

1. No, because what is being asked in the question is the value of x. While in the student's answer the value of x is not known.
2. $\frac{x-4}{x+2} = \frac{x-4}{x+3}$

$(x-4)(x+3) = (x+2)(x-4)$

$x^2 + 3x - 4x - 12 = x^2 - 4x + 2x - 8$

$x^2 - x - 12 = x^2 - 2x - 8$

$x - 4 = 0$

$x = 4$

So, $x = 4$

The following are the interview excerpts of Subject 1.

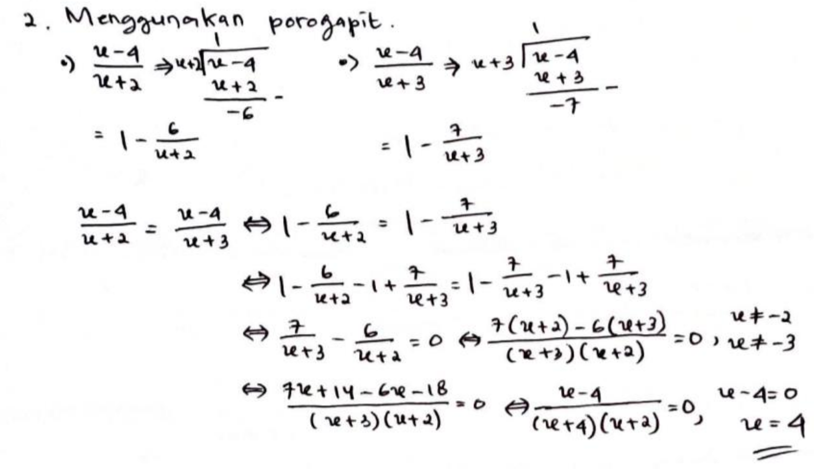
P : Why is the step said to be illogical?
S1 : Because the question asks to find the value of x that meets the equation, but the value of x is not found. ***(fulfilling activities P1 and P3)***
P : What do you know about the questions?
S1 : There is an equation in which both sides are fractions. But at the time of completing the process, the wrong answer appears at the end of the answer, namely $3 = 2$. ***(fulfilling activities P2 and P4)***

Based on the interview, Subject 1 completed the planning stage because she carried out activities to redetermine the objectives of the problem (P1) and rementioned the elements in the problem (P2). Besides, she stated that the description of the given answers is illogical (P3). It means that Subject 1 experienced cognitive conflict because the problem she faced was contrary to her thinking scheme. Also, she stated that there was a conflict between the process described in the problem and the obtained final answer, namely $3 = 2$ (P4).

FIGURE 3
THE ANSWER FROM SUBJECT 2

Translated Version:

1. Logically this step is correct so that $(x-4)$ on the left and right sides has the same value. But, it does not apply if $x=4$. So, the student's work step is not correct because it does not apply to all $x \in \mathbb{R}$.
2. Using "*porogapit*" (tiered division)



2. Menggunakan porogapit.

•) $\frac{x-4}{x+2} \Rightarrow x+2\overline{\big)\,x-4}$ with quotient 1, subtracting $x+2$ gives -6

$= 1-\frac{6}{x+2}$

•) $\frac{x-4}{x+3} \Rightarrow x+3\overline{\big)\,x-4}$ with quotient 1, subtracting $x+3$ gives -7

$= 1-\frac{7}{x+3}$

$$\frac{x-4}{x+2}=\frac{x-4}{x+3} \Leftrightarrow 1-\frac{6}{x+2}=1-\frac{7}{x+3}$$

$$\Leftrightarrow 1-\frac{6}{x+2}-1+\frac{7}{x+3}=1-\frac{7}{x+3}-1+\frac{7}{x+3}$$

$$\Leftrightarrow \frac{7}{x+3}-\frac{6}{x+2}=0 \Leftrightarrow \frac{7(x+2)-6(x+3)}{(x+3)(x+2)}=0,\ \begin{matrix} x\neq -2 \\ x\neq -3 \end{matrix}$$

$$\Leftrightarrow \frac{7x+14-6x-18}{(x+3)(x+2)}=0 \Leftrightarrow \frac{x-4}{(x+4)(x+2)}=0,\ \begin{matrix} x-4=0 \\ x=4 \end{matrix}$$

The following are interview results with Subject 2.

P : Why is the student's step said to be logical?

S2 : I think it is logical because the value of $(x-4)$ on both sides is the same, but this will not apply if $x=4$. ***(fulfilling activity P3)***

P : What do you know about the questions?

S2 : The step taken by students is by eliminating $(x-4)$ not justified because if $x=4$, the 0 can't be shared with 0. ***(fulfilling activities P1 and P2)***

P : Then how should it be?

*S2 : To solve the equation, other conditions must be added, namely $x \neq -2$ and $x \neq -3$. Then I used "**porogapit**"*(tiered division) *to minimize possible errors in the answers presented by students so that the obtained final result is $x=4$.* ***(fulfilling activity P4)***

The interview results show that Subject 2 has fulfilled all the planning stages (P1-P4).

FIGURE 4
THE ANSWER FROM SUBJECT 3

Translated version:

1. The steps used by students are logical, because students use cross multiplication, but the final answer 3 = 2 is a wrong statement.
2. $\frac{x-4}{x+2} = \frac{x-4}{x+3}$

   $\frac{x-4}{x+2}(x+2)(x+3) = \frac{x-4}{x+3}(x+2)(x+3)$ both sides is multiplied by $(x+2)(x+3)$

   $(x-4)(x+3) = (x-4)(x+2)$

   $x^2 + 3x - 4x - 12 = x^2 - 4x + 2x - 8$

   $x^2 - x^2 - x + 2x - 12 + 8 = 0$

   $x - 4 = 0$

   $x = 4$

   So, $x = 4$

The results of the interviews with Subject 3 depict that Subject 3 had the same completion steps as Subject 1. Subject 3 restated that her answers were logical because she had done cross multiplication, but a wrong thing appeared at the end of her answer, namely $3 = 2$. A different thing emerged in the first step of solving which should be done to solve the problem. Subject 3 gave an argument that because the problem is a fraction, the fraction must first be converted into a number that is not a fraction by multiplying both sides with the denominator of both, namely $(x + 2)(x + 3)$. Based on the results of the interview, it can be concluded that Subject 3 has also fulfilled all the planning stages (P1-P4).

Monitoring

Based on the answer of Subject 1 in Figure 1, a follow-up interview was done as follows.

P : Then how should you solve the problem?

S1 : I think I should use the following steps.

1. *Students cross multiply between the left and right sides to obtain the equation* $(x - 4)(x + 3) = (x - 4)(x + 2)$ ***(fulfilling activities M1, M4, M5)***
2. *After that, I do multiplication on each side so that I get* $x^2 - x + 12 = x^2 - 2x - 8$ ***(fulfilling activities M4, M5)***
3. *Next, I move the right side to the left so that the equation is equal to zero* ***(fulfilling activities M4, M5)***
4. *Then I calculated and obtained* $x - 4 = 0$ *as the result* ***(fulfilling activity E3)***
5. *The obtained final answer is* $x = 4$ ***(fulfilling activity E3)***

Subject 1 has not performed the monitoring stage optimally because she has not well understood the concept of division (M2) and the concept of undefined division (M3). She has already understood the concept of equation (M1). Because Subject 1 experienced cognitive conflict in the planning stage, she can then retrace the contradictory things (M4) and mention the causes of conflict (M5). Subject 1 well understands that what should be asked in the question was to find a value for x that meets the criteria, but the final result was wrong. Subject 2 also experienced cognitive conflict. Furthermore, he applied a monitoring stage with the fulfilment of all monitoring activities (M1-M4). Similarly, Subject 3 experienced cognitive conflict. She restated that the answer description is logical, but the final result was wrong. Thus, in the monitoring activities, Subject 3 was able to retrace back the causes of the conflict. It means that Subject 3 has not performed the monitoring stage optimally because she did not understand well the concept of division (M2) and the concept of undefined division (M3). However, she understands the concept of equality (M1). Subject 3 could retrace the contradictory things (M4) and remention the causes of the conflict (M5), namely the final result obtained is wrong.

Evaluating

In the evaluating stage, Subject 1 has not reexplained the things that cause conflict (E1) and concluded the cause of the conflict (E2). However, in the end, Subject 1 was able to reimprove the solution based on the correct concepts and theories and produce a solution (E3) by finding a solution from the equation that, that is $x = 4$. The same thing also happened to Subject 3. Meanwhile, Subject 2 has fulfilled all evaluating activities (E1-E3) meaning that she has performed all stages of metacognitive strategies well.

DISCUSSION

A learning environment which has an exchange of arguments will lead to the involvement of students in generating ideas to develop arguments in the form of reasoning and the teacher as a facilitator (Mueller and Yankelewitz, 2014). In this case, when facing a controversial problem, students will be confronted with a description of different answers that require a logical argument from the problem at hand.

The problem used in this study is the controversial one (Goldberg & Savenije, 2018; Mueller & Yankelewitz, 2014; Simic-Muller et al., 2015). A controversial issue is a situation that causes debate because of different points of view (Simic-Muller et al., 2015). Controversial problems arise as a result of understanding an unresolved problem, causing conflicts in one's thinking. Mueller and Yankelewitz (2014) uncovered that the existence of controversial issues can lead to diverse reasoning from students' understanding. In the context of mathematics, controversy can occur when students find problems that are different from problems that are usually considered commonplace. In addition, controversial mathematical problems arise due to an unfinished understanding of a problem, which causes conflicts in students' thinking. In mathematics, controversial mathematical problems can also be found when there is an incomplete solution to a problem resulting in a lot of debate to produce the correct answer. This can be seen in the description of the given questions where the conflicting final answer appears, namely $3 \neq 2$. Thus, students were asked to analyze whether the obtained final answer has a wrong completion step.

In dealing with controversial problems, students will experience cognitive conflict because the problems they face are contrary to the thinking schemes they already have. When students are given a controversial mathematical problem, the problem will provide students with new evidence that contradicts the existing conceptions. This will lead to cognitive conflict leading students to consider or find alternative concepts that can explain inappropriate events (Kang et al., 2004). The cognitive conflict generated by dissatisfaction is the first step toward conceptual change (Lee & Yi, 2013) with the finding echoes Limón's (2001) research connecting the new knowledge with the prior knowledge to bring up meaningful learning is essential. Therefore, the process of solving mathematical controversial problems will also involve cognitive conflict.

In mathematics education, McLeod and Schoenfeld (1987) presented a theory of the interaction between cognitive and metacognitive procedures that occurs when students solve mathematical problems. The failure of students to solve problems seems to arise because of the failure of their metacognitive function. This means that students have the necessary mathematical knowledge, but they fail to use it because they cannot control and monitor it (Schoenfeld, 2016a).

The existence of metacognition is paramount of importance for students in solving mathematical problems (Kusumaningtyas et al., 2018). According to Schoenfeld (2016a), metacognition helps students to become more effective problem solvers because they can define their targets, monitor their thoughts and assess whether their actions lead to targets. Research on metacognition associated with problem-solving has been carried out extensively (see, for example, (Fortunato, I., Hecht, D., Tittle, C.K. & Alvarez, 1991; Kapa, 2001; Karlen, 2016; Kramarski & Mevarech, 1997; Mevarech & Fridkin, 2006; Mokos & Kafoussi, 2013). From these studies, several important things can be found. First, students' metacognitive investigations are required during mathematical problem-solving activities. Second, knowledge of the actions that students must take to successfully solve problems is based on knowledge of metacognitive strategies. Metacognition can strengthen students' ability to become better problem solvers because metacognitive strategies support efforts during problem-solving. Third, students will gain a better ability to solve problems if they control and monitor the used strategies a lot.

The concept of the metacognitive strategies used in this study refers to knowledge of what actions can be taken to successfully solve problems (Karlen, 2016; Vula et al, 2017). Metacognitive strategies are used when solving problems (Ariyati & Royanto, 2018; Vula et al., 2017). The metacognitive strategies described in this study include the stages of planning, monitoring, and evaluating.

Planning is a stage that reflects the tendency of students to set goals or think about tasks before starting to solve problems, determining procedures that direct thinking, and selecting the appropriate strategy (Karlen, 2016; Ku & Ho, 2010). In the planning stage, there are activities to reidentify things that are in problem and rerealize that there are contradictory things. The planning stage in solving mathematical controversial problems can be seen in the answers of Subjects 1, 2, and 3 as shown in Figures 2, 3, and 4. The three subjects managed to fulfil all stages of planning. This finding corresponds to the previous research by Santrock (2011) that a known problem is an important part of the thinking process. Likewise, Boyle et al. (2016) argue that planning involves the identification and selection of appropriate strategies and allocation of resources, such as attention. This stage also involves goal setting, background knowledge activation, and time awareness.

Monitoring is a stage of retracing the causes of conflict. Subject 2 has fulfilled all stages of monitoring, while Subjects 1 and 3 only fulfilled several stages of monitoring, namely in activities M1, M3, and M4 as shown in the results of the interview. Because students experience cognitive conflicts in which the problems faced are contrary to the thinking schemes they already have, it is necessary to retrace the things or concepts that cause conflict. At the monitoring stage, students should be able to reexplain the prerequisite concepts used in solving controversial mathematical problems such as equations, divisions, and undefined division conditions. Within a similar vein, Fyfe et al. (2012) revealed that prerequisite knowledge can be used in solving controversial problems. Furthermore, Grant (2014) unveiled that rather than a lack of mathematical knowledge, the inability of students to carry out the monitoring process in learning is a factor behind low math performance.

Evaluating is a stage of reclarifying, restrengthening, reimproving, and concluding solutions. Based on the answers given by Subjects 1, 2, and 3, the study showsaed that only Subject 2 has fulfilled all stages of evaluating metacognitive strategies, while Subjects 1 and 3 only fulfilled the E3 activity, namely reimproving the solution based on the correct concept and theory and producing a solution. This can be seen in the results of the work of Subjects 1, 2, and 3 in Figure 2 which is also confirmed by the results of the interview. These current findings reflect that the subjects are not yet fully aware of their metacognitive learning strategies. They have very limited awareness of their own metacognitive (Yilmaz & Baydas, 2017). This might be because there was no previous metacognitive-related training, and it becomes an obstacle in the application of metacognitive strategies (Alzahrani, 2017).

CONCLUSION

The stages of metacognitive strategies have not yet emerged completely when students are asked to solve controversial mathematical problems. The planning stage has been carried out well because all the activities have been fulfiled. Meanwhile, the stages of monitoring and evaluating have not been implemented optimally because several activities are not yet fulfiled. Students are expected to be able to provide logical arguments in the problem-solving process when encountering controversial mathematical problems. Therefore, they can understand the given problems well based on the concepts used in solving mathematical controversial problems. At this point, it is necessary to habituate metacognitive learning. The use of the introduced metacognitive strategies must be engaged directly to the improvement of both monitoring and evaluating stages when dealing with controversial mathematical problems.

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