

# Improving the Learning and Teaching of Mathematical Logic Elements Using Maple

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*The study of the elements of mathematical logic and its sections, especially in the process of teaching in high school in general and specialized courses of mathematics or higher educational institutions, is a rather tricky task. The paper demonstrates how one can design and develop a package of interactive applications to help learn, understand, and apply the essential elements of mathematical logic. The research aims to develop and describe an interactive application for studying the order of constructing truth tables and studying and demonstrating the production of principal normal disjunctive and conjunctive forms. The development is based on the methods of the Maple computer algebra system of the Maplet library. In addition, other methods are used to solve the research issues – theoretical and accurate analytical solutions to mathematical logic issues. All constructed material (Maplet) will serve as a good simulator for students, allowing one to perform an infinite number of attempts to analyze, synthesize, and build truth tables.*

*Keywords: Maple, interactive application, mathematical logic, teaching mathematical logic, Maplet, mathematical education, learning mathematical logic, truth table, principal normal forms*

## INTRODUCTION

Maple software is a computer algebra system (CAS, or Symbolic Algebra System [SAS]), in other words, a program that can represent information in symbolic or algebraic form. In addition, Maple CAS allows one to produce accurate analytical solutions to many mathematical problems (Kumar & Kumaresan, 2008; Olenev, Kirichek, Potekhina & Pelikh, 2020; Olenev, Shuvaev, Migacheva, Kulevskaya & Nazarenko, 2020; Shingareva & Lizárraga-Celaya, 2009). Currently, Maple is a robust system that can be used to solve almost any mathematical problem in a university course, and even more so in a school mathematics course. The system has powerful mathematical libraries that cover almost the entire university course of mathematical disciplines. However, they do not explain the procedures and stages of problem solving, and students have to learn a new and not always familiar language, which takes up their time intended for education.

Maple has a package called Maplet (Kamerich, 1999; Lopez, 1994; Monagan et. al., 2009), which allows one to create ergonomic tools in the form of interactive applications. This package can create windows, buttons, dialog boxes, and other visual interface elements that create user interaction. The Maplet package is one of the most effective ways to develop a Graphic User Interface (GUI).

The primary goal of the created Maplets is to ensure the integration of the Maple mathematical libraries for use in user procedures equipped with interactive windows and dialogs. The authors use Maple as a support tool for creating several Maplets, which are an effective didactic tool to help students and teachers learn the order of construction and analysis and synthesis of Boolean algebra truth tables, as well as to obtain principal conjunctive and disjunctive normal forms (PCNF, PDNF) when studying a school or university program of discrete mathematics.

Maplets solve user-entered problems and allow students to learn the process of solving them. Students can find hints, get immediate feedback, and check the answers repeatedly. The authors believe that this is no less effective than working with a teacher or an instructor. More than that, the developed maplets can be used to demonstrate specific examples of problem solving.

Thus, the developed maplets will allow students to find their own mistakes, correct them, and better understand the issues of the topic under study. They serve as a powerful motivating force for encouraging students to do homework and also provide an opportunity for cooperation between students when completing tasks and projects. Therefore, the developed software product can balance the development of an understanding of the studied material and the acquisition of technical programming skills. The primary research goal is to analyze and present the possibilities of using the Maple computer algebra system to study the order of constructing truth tables and obtaining principal disjunctive and conjunctive forms using the Maplet GUI. Moreover, the paper aims to develop and show the use case of the Maplets complex in studying these concepts.

## **MATERIALS AND METHODS**

Obtaining truth tables and principal normal forms of a logical function in solving mathematical logic problems and their analysis is carried out by computer algebra methods using the Maple CAS (Rosen, 2012). To demonstrate symbolic solutions to problems of constructing truth tables and obtaining principal normal forms of a logical function, one uses the capabilities of the Maple CAS (Maplets and Logic libraries). The following basic algorithms of symbolic calculations are implemented – demons – tration of performing automated obtaining of a truth table, the solution of which is brought to a convenient form for perception, use and analysis (the results can be obtained in various types of representation), and obtaining principal normal forms of a logical function from the truth table.

The basis for constructing the truth table is additionally based on the tabular organization of the data representation, for which one uses the Maple-ShowTable function. In the process of using this function, the software sets the necessary form of presentation of the result and displays it in a tabular form as the most informative form for the presentation, understanding, and analysis of information. Besides, the construction of principal normal conjunctive and disjunctive forms is additionally based on the mathematical representation of information using the function of the Maplets library – MathMLViewer. The software sets the required form of representation of the result and outputs it in mathematical form.

Problems solved by Maplet:

- Performing task entry;
- Task description in tabular form;
- Checking the answer correctness;
- Availability of help and assistance.

One should use the following sections and buttons in order to create a GUI:

A. To build a truth table:

- There are two fields for entering the information necessary for building a truth table. In one field, one should enter logical (Boolean) function under study, and in the other – variables, the behavior of which needs to be studied;
  - There are two fields for displaying information from the constructed truth table. The internal software implementation allows one to represent information in two ways – using Boolean variables “true” and “false,” or “0” and “1.” The corresponding button is responsible for issuing the relevant information;
  - The corresponding buttons are also provided to give access to information in a tabular form;
  - The information section consists of three buttons that provide actions with the entered information and “help and info” button.
- B. To construct principal normal disjunctive and conjunctive forms:
- Information block (4 buttons), which introduces the procedure of Maplet usage, the help block, and general actions with Maplet;
  - Two information input fields allow one to enter the initial information on the truth table under study;
  - There are two fields with corresponding buttons for displaying information on the obtained principal normal conjunctive and disjunctive forms, the information in which is presented in the format of the Maple computer algebra system;
  - In addition, there is a separate block for the presentation and analysis of information on principal normal conjunctive and disjunctive forms, represented in mathematical form.

## RESULTS

The authors demonstrate the practicality of the developed Maplets. Maplet “Truth table” solves the problem of constructing a truth table and presenting the study results in various forms. Figure 1 shows the results of the study of the function  $a \vee b$ . Using the representation of the results of the Boolean variables “true” and “false,” as well as “0” and “1” (Fig.1).

**FIGURE 1**  
**RESULTS OF CONSTRUCTING A TRUTH TABLE FOR THE FUNCTION  $a \vee b$  IN THE**  
**PROCESS OF USING THE BOOLEAN VARIABLES “TRUE” AND “FALSE” AND**  
**“0” AND “1”**

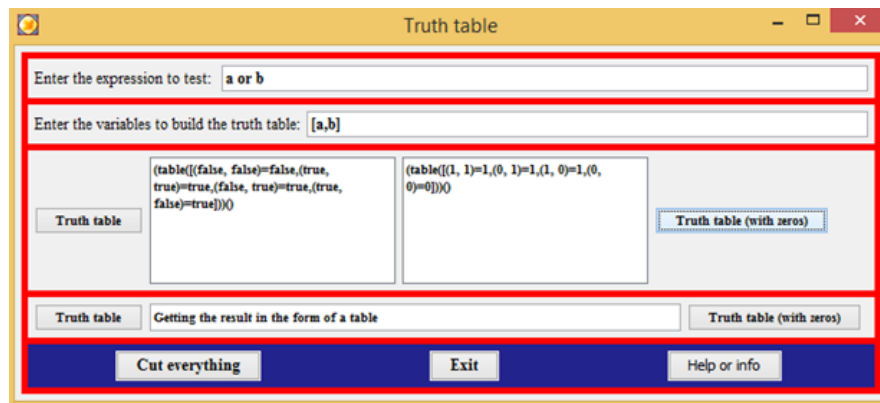


Figure 2 depicts the study results of a more complex function  $a \wedge b \vee c$  (Fig. 2).

**FIGURE 2**  
**RESULTS OF CONSTRUCTING A TRUTH TABLE FOR THE FUNCTION  $a \wedge b \vee c$  IN THE**  
**PROCESS OF USING THE BOOLEAN VARIABLES “TRUE” AND “FALSE” AND**  
**“0” AND “1”**

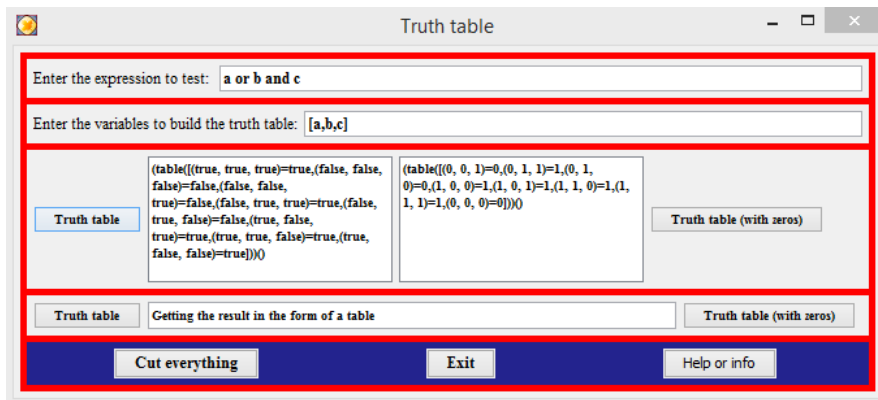


Figure 3 demonstrates the results and the study process of the same function  $a \wedge b \vee c$ , but the results are presented in tabular form (Fig. 3).

**FIGURE 3**  
**RESULTS OF CONSTRUCTING A TRUTH TABLE FOR THE FUNCTION  $a \wedge b \vee c$  IN**  
**TABULAR FORM – IN THE PROCESS OF USING THE BOOLEAN VARIABLES A)**  
**“TRUE” AND “FALSE” AND B) “0” AND “1”**

a	b	c	Function o...
true	true	true	true
false	false	false	false
false	false	true	false
false	true	true	true
false	true	false	false
true	false	true	true
true	true	false	true
true	false	false	true

**A)**

a	b	c	Function o...
0	0	1	0
0	1	1	1
0	1	0	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	0	0

**B)**

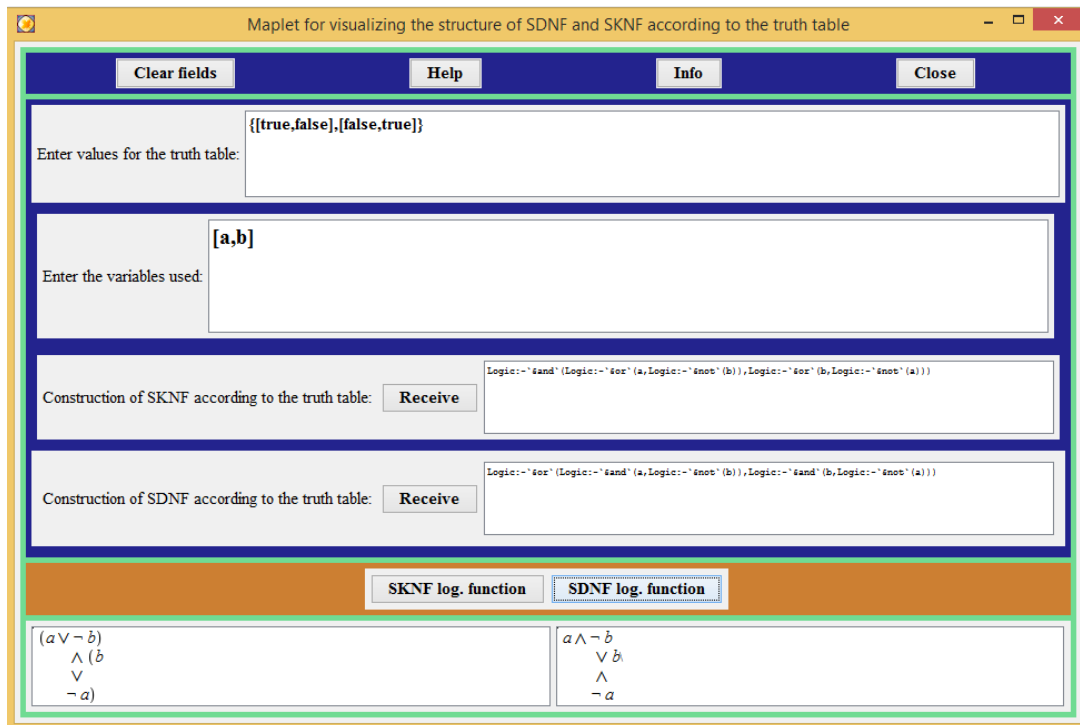
The following “Maplet for visualizing the structure of PNDP and PNCF according to the truth table” is used to solve the problem of constructing principal normal disjunctive and conjunctive forms from the truth table and presenting the results of the study in various forms. For example, the following truth table is used (Table 1).

**TABLE 1**  
**THE TRUTH TABLE**

a	b	F(a,b)
true	true	false
true	false	true
false	true	true
false	false	false

Figure 4 shows the results of the study of the mentioned truth table. The obtained results are presented in two forms – using the format of the Maple CAS and mathematical form (Fig. 4).

**FIGURE 4**  
**THE RESULTS OF CONSTRUCTING THE PNDP AND PNCF FOR THE TRUTH TABLE**



## DISCUSSION

Based on the presented material, the authors can draw the following conclusion – the created Maplet allows one to study the order of constructing truth tables of various Boolean functions and makes it possible to construct principal normal forms from the truth table. The results are correct, and they can be obtained quickly and easily. Since the proposed maplets allow one to practically explore and demonstrate the results of constructing truth tables and principal normal forms and not just use the Maple library routines, the developed Maplet contributes to the acquisition of in-depth mathematical knowledge and skills. Students can create their own projects under proper guidance, as the used Maplet elements are intuitive and easy to understand and use.

The authors' results correlate with the results obtained by colleagues from Mexico in that it is necessary to support the study of mathematics, including the elements of mathematical logic as a branch of discrete mathematics, using various types of software (Jiménez-Hernández, Oktaba, Díaz-Barriga & Piattini, 2020). from Turkey (Güyer, 2008), including the use of computer algebra systems, which are more often used in science and engineering fields (Klima & Sigmon, 2012). The Maple CAS can be used in teaching mathematics (Røyrvik, 2002). The computational capabilities of Maple can also be used to formulate, prove, or refute statements, to solve various applied problems in the logic of statements, as confirmed by researchers from Bulgaria (Durcheva & Nikolova, 2018) and from the United States (Mlsna & Liszewski, 2005) in the aspect of the effectiveness of using Maplet in simplifying logical expressions.

The limitations of using the developed Maplet for solving the problems of constructing truth tables and principal normal forms are related only to the number of programmed capabilities based on the program requirements for studying this section. A potential area of research in the context of this paper is the

development of Maplet for the study of other branches of discrete mathematics – graph theory, propositional logic, logic, and predicates, etc.

## CONCLUSION

The developed Maplets greatly simplify the teaching management of a large number of students. They also provide students with relevant practice to prepare for and accompany the study of mathematical logic sections. It can be used both to demonstrate the construction of truth tables and normal forms in lectures to bring new educational material, and in practical classes as a research tool, as a simulator to check the correctness of individual tasks, or as a modeling environment (Bailey, Borwein, Calkin, Luke, Girgensohn & Moll, 2007; Borwein, Bailey & Girgensohn, 2004; Borwein & Bailey, 2008; Tyncherov, Olenev, Selivanova & Kirichek, 2020).

Maplets can support both randomly generated problems and problems generated by the user. It allows students to solve the problem of constructing a truth table and obtain principal normal forms taken from other sources. More than that, algorithmically generated tasks allow learners to work on different tasks (both technical and mathematical) and use their experience to derive a common method for solving similar problems.

For example, the maplet “Maplet for visualizing the structure of PPDF and PPDF according to the truth table” gives a general idea of the order of finding both principal conjunctive and disjunctive normal forms depending on the necessity. In contrast, the Maplet “Truth Table” uses a combination of symbolic and verbal techniques to study the solution to the problem of constructing a truth table. The features of almost instant feedback and the possibility of repeated use of a Maplet will be beneficial when students solve various problems in a step-by-step mode. The mentioned features allow students to use both developed Maplets for learning without a teacher’s physical presence at any time and in a comfortable environment.

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