Exploring the Meaning-Making Process in Conceptual Mathematics: The Case of Linguistic Markers in Algebraic Equations

Myla L. Santos
Central Luzon State University

As research established, linguistic features are essential to understanding mathematical word problems, and these features are seen as a significant factor of students' difficulty in word problems. Students are unable to translate the words into numbers or formula to work on the needed mathematical operations. This present study's aim is two-fold. First, the analysis of the syntactic features of mathematical language is hoped to contribute to the understanding of mathematical sentence by making its lexis, syntax and semantics explicit. Second, by focusing on the linguistic markers in algebraic equation, the meaning-making process in the mathematical language was described by determining how these linguistic markers prompted the mathematical conditions in the expressions and statements. Results show that algebraic sentences of equalities were characterized by inflectional phrase (InflP) projection, and 'if…then' syntax, which are considered in lexico-grammatical precision. Predication in algebraic sentence of equalities was signified essentially by the linking verb 'is' in definition, description and illustration frames.

Keywords: conceptual mathematics, linguistic markers, meaning-making process, assumption-conclusion syntax, lexico-grammatical

INTRODUCTION

Mathematical word problems can be described and examined linguistically on three levels: (a) verbal formulation, (b) underlying mathematical relations, and (c) symbolic or formulaic mathematical expression. The mathematical properties of these levels include constants or the values given in the problem and variables, and the unknown values or quantities. The relationships between the two values are identified by algebraic operations. The arrangement of these values and the algebraic operations show the mathematical process illustrating the make-up of the phrase structures of mathematical word problems.

Also called verbal problems, mathematical word problems use both common and specialized language. This originates from the nature of the mathematical language in which they are functionally situated. Mathematical language (ML) is quite different from the natural language (NL) as the former somehow deviates from the norms of the latter. As a specialized language, ML has its own lexical proceedings more often than not observed in the NL. The expanse of meaning these words in the ML expressed in the mathematical word problems is considered context-free. This makes the meaning of each mathematical term independent of its situation. Hence, ML as a context-free language derives its meaning within the bounds of its grammatical construction and compositionality. Nevertheless, Mathematical word problems use a grammatical system that is specific of its intention and meaning.

Verbal formulation is the style of expressing meaning through words. According to Pierce, et al. (2009), verbal formulation in mathematics is the manner in which the algebraic statements are linguistically expressed to convey meaning. Hence, algebraic verbal formulation in algebraic equation involves the phrase structures of the statements and the use of these structures to perform the mathematical property of the expressions. Using algebraic expressions and sentences as baseline forms, phrase structures of mathematical word problems follow generally the bottom up direction.

Like algebraic phrases, algebraic sentences are composed of numbers and variables with signs of operations like addition, subtraction, multiplication, division, brackets, rational power and root extraction. Unlike algebraic phrases however, algebraic sentences are statements of equality of two expressions formulated by applying a set of variables in algebraic operations. They are often characterized by the linking verb 'is' and the verb 'equals' or their variations. Hence, algebraic sentences are usually not just a mathematical notation but are completed by linguistic markers in applying arithmetic operations or solutions of finding a number or set of numbers that if substituted for the variables in the equation reduces them to an identity which is often referred as solution.

Based on construction and compositionality, algebraic equations also called verbal sentences consist of a complete subject and a complete predicate. They are equations with equal sign between two algebraic expressions or algebraic phrases which is a linguistic marker bearing specific algebraic meaning. Mathematically, the logic of algebraic sentences calls for the subject to be algebraically equal to the predicate. They employ linguistic markers or cue words instead of the phrasal verb 'equal to' such as 'is', 'is the same as', gives', 'was', and 'will be'.

Research Questions

The syntax of mathematical language (ML) is highly structured and complex. However, it requires simplicity and clarity, at the same time should also be precise to follow its algorithm. The elements of the algebraic equation in ML are constructed in a way such requisites are achieved. In general, this study's aim is two-fold. First, the analysis of the syntactic features of mathematical language was hoped to contribute to the understanding of mathematical sentence by making its lexis, syntax and semantics explicit. Second, by focusing on the linguistic markers in algebraic equation, the meaning-making process in the mathematical language was described by determining how these linguistic markers prompted the mathematical conditions in the expressions and statements. Specifically, this paper hoped to answer the following;

1. What are the linguistic markers in the syntax of algebraic sentence of equalities?
2. What are the linguistic markers that prompt the predicate arguments in algebraic sentence of equalities?
3. How is the lexico-grammatical precision algebraic sentence of equalities versus the natural language achieved through linguistic markers?

METHODOLOGY

In this study, formal language analysis framework in phrase structure grammar was used as a reference model for presenting the grammar of mathematical word problems (MWPs). In particular, the constructional and compositional analysis illustrate the MWPs crossing over the natural language (NL) to highlight the specialized mathematical language (ML). Although there are varieties of grammar system for describing the structure of a language, constructional and compositional analyses were used in an attempt to describe and explain mathematical language syntax in terms of more general principles and operations that are typical of ML but somehow deviations from the idiosyncrasies of the NL system which allows context-free analysis of the ML system; hence, the phrase structure grammar.

Generally, the phrase structure grammar (PSG) consists of a finite set of terminals such as leaf nodes in a parse tree or minimal projections, the actual lexical units that made up a phrase or a sentence including its morpho-syntactic features such as parts of speech, case, type of complement, agreement and the like. PSG also outlines finite set of non-terminals (NT) such as phrase type, or syntactic categories of the terminals as noun phrase (NP), verb phrase (VP), prepositional phrase (PP), quantifier phrase (QP) among others. Complement phrase structure in the PSG allows the projection of TP to further describe the minimal projection within the sentence. Lastly, PSG includes symbols of syntactical function such as the null constituents (ø), showing the deep structure of the sentence and the parenthesis for grammatical relationship;

Thus, the derivation of a syntactic structure is a bottom-up process, wherein the merging functions check the features of a terminal or lexical unit and their features with un-attributed values called lexical variables. These lexical variables initiate the introduction of another unit whose feature-values are unified with the said variables. Finally, this merging creates binary structures and is applied recursively until all features of the sentences are interpreted or are given values in the phrase structure.

Review of Related Literature

Ashlock (2001) classifies mathematics in terms of its focus; conceptual and procedural. According to him, the conceptual mathematics is largely dependent on the linguistic structure of the mathematical concepts. He says that conceptual mathematics is understanding and conceptualizing mathematics knowledge referring to the understanding of ideas and generalizations that connect to mathematical construct. Similarly, Jitendra and Star (2012) state that conceptual understanding in Mathematics is similar to comprehension in reading. Procedural mathematics, on the other hand, is understanding and identifying knowledge on a step-wise semantics. Conceptual Mathematics uses linguistic features while procedural mathematics uses arithmetic conventions.

Mathematics has its own language (Guthrie, 1977; Muth,1982 in Tonio et al., 2019). According to them it is composed of numerical and literal constants and variables. These constants and variable are arranged logically into mathematical sentences known as operations to express calculations and measurements. Blachowicz & Ogle (1982) say there are many symbols in Mathematics and they are part of our everyday lives. Commonly, we consider them as the language of mathematics because the expression of this language enables us to understand its meaning and use them in return to create meaning. Hence, mathematical language is viewed as a communication tool and is devised conventionally (Martiniello, 2008). The conventions of Mathematical Language (ML) are decided upon by its users to equate mathematical symbols to linguistic meaning. For instance, Ward (2005) illustrates that the positioning or ordering of number and symbols in relation to each other gives meaning as in the ordering of words and sentences in utterances. He further exemplifies that different positions and orders create different contexts and result in different meanings. This means that to appropriate meanings in mathematical language is also similar to how context functions in linguistic operations. Also, Brown, Cady & Taylor (2009) emphasize that ML syntax and semantics are a precise form of short cut in representing meanings in Mathematics, and to make them sensible is to understand contexts and conventions.

For Cummins (1981), Mathematics is a language of symbols, grammar and logic. He says that all languages are ways of conveying information and meaning. Mathematical language is not an exception. And to be able to express logical information and meaning, Cummins explains that Mathematical Language follows a grammar, a rule for arranging its elements into meaningful statements.

However, language, according to Winsor (2007) can be a barrier to understanding mathematics. In fact in his paper about teaching mathematics, he illustrates that teaching the specific language of mathematics in an explicit way includes using terminology correctly, and explaining and modeling technical terms and concepts in ways which connect meaningfully with the children’s existing knowledge, language and experiences. Similarly, Langeness (2011) concludes that to develop the language of mathematics with their students is complex and teachers must make it an integral but deliberate component of day to day teaching. Manzo & Sherk (1975) in Martin and Mulls (2012) and Anudin (2019), clarify that syntax has two meanings; the first deals with the observable and underlying structures of a sentence and the other is the scientific study of grammatical system. When taken as a whole, they say these two common definitions of syntax are similar with Mathematics.

Making links between the specialized mathematical language of instruction and a different home language demands particular care. Variations of meaning and the nature of contexts must be considered carefully.

In their paper in learning the language of mathematics, Hughes & Fries (2015) discuss the use of language as a tool for teaching mathematical concepts. In it, they show how making the syntactical and rhetorical structure of mathematical language clear and explicit so students can increase their understanding of fundamental mathematical concepts. Also, in order to understand the linguistic difficulties in mathematics context, O'Keeffe and O'Donoghue (2015) say it is important to understand the linguistic features constructing mathematics word problems. They explicate that mathematical word problem is constructed by its own language system. This language system organizes choices of language function, mathematical symbol, and visual display. Failure in understanding this system will lead to failure in understanding the mathematical word problems due to its linguistic features.

Furthermore, in many mathematical tasks and activities learner encounter language and literacy barriers as they read and interpret written words and symbols. Hence it must also ensure that mathematics is accessible despite literacy challenges shaping mathematics language as a priority (Vizconde, 2006).

RESULTS AND DISCUSSION

Algebraic Equation and Its Linguistic Markers

Mathematical word problems use both sophisticated and specialized language. This originates from the nature of the mathematical language (ML) in which they are functionally situated. ML is quite different from the natural language (NL) as the former somehow deviates from the norms of the latter. As a specialized language, algebraic equations of ML have their own lexical proceedings more often than not observed in the NL. The expanse of meaning these words in the algebraic sentence of equalities expressed in the mathematical word problems is considered context-free. This makes the meaning of each mathematical term independent of its situation. Hence, algebraic equations in ML as a context-free language derives its meaning within the bounds of its grammatical construction and compositionality. Nevertheless, algebraic sentence of equalities uses a grammatical system that is specific and precise.

The syntax of algebraic equations is highly structured and complex. However, it requires simplicity and clarity, at the same time should also be exact. The elements of the word problems of ML are constructed in a way such requisites are achieved. Also, its compositionality enables the interpretation to be accounted based on the target algebraic process. Based on the phrase structure analysis, the algebraic equations are characterized by the following linguistic features as cued by the linguistic markers.

If… Then Syntax

In general, the phrase structure of the algebraic equations shows that the syntax is of algebraic equation in tense phrase (TP) which has two sides as indicated by the NP at the left side and the T' (T bar) at the right – hand side. Their relationship is connected by an equal sign if interpreted algebraically. Algebraic equations (1) 'Seven more than twice an unknown number X is twenty three', if divided by the left – hand side and the right – hand side will be using the linking verb 'is' as a reference. The left – hand side is the NP 'Seven more than twice an unknown number X; while the right – hand side starts with a verb phrase including the tense verb or the T' (T bar) 'is twenty three'.

Seven More Than Twice an Unknown Number X Is Twenty Three

According to the syntax and semantics of the two elements of the TP projection of algebraic equation, the logic between the two shows the assumption – conclusion relationship as indicated by the linking verb 'is' as the linguistic marker. The NP projection and the T' or VP projection are both parts of the assumption, and the solution which involve the algebraic process of finding the unknown or the variable is the conclusion. The TP 'Seven more than twice an unknown number X is twenty three' as an assumption – conclusion construction can be in 'If …. then compositionality or a statement of condition and inference.

That is saying ***If*** 'Seven more than twice an unknown number X is twenty three', ***then*** 'unknown number X' must be…

To arrive at the condition of the **If….then**, TP 'Seven more than twice an unknown number X is twenty three' is interpreted as the following based on phrase structure analysis.

NP 'Seven more than twice an unknown number X'

2X + 7; as the quantifier 'more than' is interpreted as algebraic addition, and the adverb 'twice' as algebraic multiplication.

T' – VP 'is twenty three'

= 23; with 'is' interpreted as sign for equalities.

TP 'Seven more than twice an unknown number X is twenty three'

$$2X + 7 = \mathbf{23}$$

As stated above, the algebraic equation illustrated as **If…… then….** condition, algebraic equation (1) is interpreted as **If** 2X + 7 = 23, **then** X is ______. That is saying the assumption that if 2X + 7 is 23, the conclusion is that X is a number that will make the left – hand side of the equation equal with the right – hand side of the equation as the LV 'is' means 'equals' or 'is equal to'. To check if the condition is met, mathematical process will be done following the algebraic operations involved in the equation. As such, using the involved algebraic operations, 2X + 7 = **23** will be simplified as

$$2X + 7 = \mathbf{23}$$

2X = 23 – 7 (performing the inverse operation when constants are simplified and are transposed)

$$\mathbf{2X = 16}$$

X = 16/2 (inverse operation when transposing values to the other side, of equation)

$$\mathbf{X = 8}$$

To check whether the conceptual mathematical relationship that **if** 2X + 7 = **23, then X = 8** is met based on the interpretation made from its construction and compositionality, the value of the unknown number X is substituted to the equation;

$$2X + 7 = \mathbf{23}$$

$$\mathbf{2(8) + 7 = 23}$$

$$\mathbf{7 + 16 =23}$$

$$\mathbf{23 = 23}$$

Indeed, algebraic equation follows a specific syntax so as to achieve its specified function. Algebraic equations are of assumptions and conclusions. Hence, the groups of words in the statements behave as a unit as far as reference is concern as shown in the conceptual analysis of the composition.

Predicate Arguments in Algebraic Equation

Following a linear order in phrase structure analysis, algebraic equations rely on predication to identify the elements of their mathematical meaning. The predicate is represented by the linking verb 'is' as it connects linguistically the left side to the right side of the equation. The use of 'is' in algebraic equation is one characteristic of ML that sets it apart from NL. Algebraic sentences are non-temporal – there is no past, present or future in the context of algebraic expressions (Santos, 2016). Based on the construction and compositionality, there are three semantic frames of the linking verb 'is' as used for coordination in ASE.

Definition – *'Is' as 'Equals' or 'Equal to'*. Generally, 'is' is used to mean 'equals' or 'is equal to' in algebraic equations. The expression 'is' literally expresses that the left- side expressions are collectively and mathematically equal to the right side expressions. This is determined by performing the identified algebraic operations in the sentence.

Twelve Is Sixteen Less Than Four Times a Number

The algebraic equations above with its phrase structure is constructed in a way that the subject of the sentence or the left – hand side of the equation is defined by the circumstances of the number at the right – hand side of the equation. In mathematical language, defining a number is more or less to designate a condition to count, to measure and to label (O'Hallora, 2015). This is done by following the operations involved in counting, measuring and labeling a number. By counting, measuring or labeling a number, the number is placed in a mathematical test on the different algebraic manifestation of the said number. Manifestation, according to Sumarwati (2014) is expressing in a different manner the property of a number but is more or less defining the same quantity or quality. Hence, the manifestation of the number being defined is the operationalization of the values at the right side of the algebraic equation.

In the algebraic equation (2) 'Twelve is sixteen less than four times a number', the NP 'Twelve' is defined by expressing its manifestation in the operations involved in NP 'sixteen less than four times a number'. Collectively, the conditions set by the manifestation of the NP at the right side are expected to result in exactly the same value of the NP at the left – hand side. Nonetheless, defining in algebraic equations is done through the use of 'is'. As such, 'Twelve is sixteen less than four times a number' defined mathematically is 'Twelve equals sixteen less than four times a number, or 'Twelve is equal to sixteen less than four times a number', which is translated algebraically as;

$$\mathbf{12 = 4X - 16}$$

To illustrate algebraically that the manifestation is exactly the same as the defined value, the 'if…… then' syntax is applied; that is computing for the value of the variable: 'If 12 is 4X – 16, then X is'.

$$\mathbf{4X - 16 = 12}$$
$$\mathbf{4X = 12 + 16}$$
$$\mathbf{4X = 28}$$
$$\mathbf{X = 28/4}$$
$$\mathbf{X = 7}$$

Substituting the value of X;

$$\mathbf{4\,(7) - 16 = 12}$$
$$\mathbf{28 - 16 = 12}$$
$$\mathbf{12 = 12}$$

Description – *'Is' and the NP as Adjectival Phrase*. Based on the principle of phrase structure, syntactical elements follow a specific order to express the target meaning. Similarly, numerical and literal values in algebraic equations follow a distinct order to form an expression and relationship to get the correct solution. This involves the mathematical order of operation (Hughes et al. 2015)

The Product of Three More Than Six and Twice the Sum of a Number and Three Is 20

According to Ashlock (2011), mathematical language deviates from the NL as far as lexical properties are concerned. In the present. study, one deviation that was observed is the NP class used as a modifier instead of a subject or an object. In the algebraic equation (2) 'The product of three more than six and twice the sum of a number and three is 20', the NP at the side which is linguistically identified as a subject is an example. The description of the number '20' is expressed in the DP 'The product of three more than six and twice the sum of a number and three. This means that if the mathematical condition of the DP projection at the left – hand side of the phrase structure is operationalized, it will yield the count, measure or label of what the number 20 expresses. The algebraic conditions of the left – hand side which are taken as a unit have the mathematical property of that at the right – hand side. Conversely, the expression 'any number less than or equal to 19 is 20' is a general description of the value 20. Hence, any equation that fits the said condition is a description of the number 20. The DP ‘The product of three more than six and twice the sum of a number and three’ expresses an attribute of the value ‘20’.

The verb ’is’ therefore is not only a linker but it is also a mathematical limitation which describes the identified value. It implies that the conditions set by the operations in the left side of the algebraic equation are the specified construction that leads to the given value at the right side of the algebraic equation. Hence, the ‘is’ in an algebraic equations indicates that a symbol is needed to ensure the proper operationalization of the elements at the left side of the equation. This symbol which is usually a set of square brackets [ ] ensures the proper order of operation required to arrive at the specified condition of the right side element. A set of brackets in a natural language is a punctuation mark used to set a word or phrase aside from the rest of a sentence. In mathematical language, brackets set elements apart from the rest to ensure proper grouping in terms of algebraic operation.

The expression ‘The product of three more than six and twice the sum of a number and three’ is translated algebraically as;

$$\mathbf{(6+3)\ [\ 2\ (X+3]}$$

Following the principle of mathematical order of operation, algebraic operation inside the parenthesis should be worked on first than those outside the parenthesis. In c there are brackets, working on those in the brackets follows a specific construction. Brackets are very important in the construction of the algebraic expressions.

Apparently, the algebraic equation (2) is simplified as;

$$\mathbf{(6+3)\ [\ 2\ (X+3)] = 20}$$
$$\mathbf{(9)\ (2X+6) = 20}$$
$$\mathbf{18X + 54 = 20}$$

If the brackets will be removed, the algebraic expression will look like this;

$$\mathbf{(3+6)\ (2)\ X + 3}$$

That if the expression will be simplified, it will follow a possible order of operation below;

$$\mathbf{(3+6)\ (2)\ X + 3 = 20}$$
$$\mathbf{(9)\ 2X + 3 = 20}$$
$$\mathbf{18X + 3 = 20}$$

Looking at the order of operation, it seems that (9) should only operate with **2X** and not with **3** because of the separation indicated by the (+) sign. Indeed, **18X + 27** is algebraically different from **18X + 3**.

Noting the difference on construction between the two semantic frames, definition and description, the first starts with the key word, while the latter starts with the manifestation or the conditions. Like in NL, a typical definition follows the construction which leads to a statement expressing its essential nature (Martiniello, 2008). It may give the exact meaning or the degree of distinctness. In the mathematical language, the latter is often the case of defining algebraic expressions. Definition is done by stating the keyword first followed by the distinct condition of the value of the keyword as in ‘9 is the square root of 81’. Description on the other hand usually starts with the explanation first before the keyword. Unlike in natural language, mathematical language describes a term by giving first the conditions that lead to the identification of the keyword. Description of algebraic term or value follows the logic of mathematical operation or the manner of computation for the unknown. The progressive operations are to be followed first arriving at the target value. For instance, ‘the difference of 12 and 7 is 5. Five is described as the answer to the algebraic subtraction stated in the DP ‘the difference’.

Illustration - 'Is' as 'an Example of'

According to Hughes, et al. (2015), there is a nearly universally accepted logical and rhetorical structure to mathematical exposition. One of the most common structures is illustration. Dominguez (2005) explains that illustration in mathematical language is citing sample applications of the algebraic values at hand. Hence, to illustrate is to give instances where the stated value can be operationalized. This includes several mathematical applications. Hence, the use of 'is' in algebraic equations influence the solution strategies for the algebraic equations. As the 'is' introduces the description of the constants and variables, it also arranges their order and relationships toward the correct operations. Correct operation strongly depends on the order and relationship described in either the left – hand or the right – hand side of the equation. The marked difference between description and illustration in algebraic sentences is the implication/s of their mathematical structure. Description gives a situation where the value of the mathematical concept is realized and operationalized. However, illustration provides one or more conditions which are mathematically equivalent, hence, 'is' in this case introduces the semantics of 'an example of.

Three More Than Five times the Smaller Number, the Sum of Four Times the Larger Number and Three Times the Smaller Number Is 71

Based on the phrase structure of algebraic equation 3, it is noted that the left side is projected into two TP constructions. Originally in the algebraic sentence, the two elements in the left side are separated by a punctuation mark that marks a compositionality of a series pertaining to a single object at the right – hand side of the maximal projection. The punctuation mark is an important element in presenting a series in mathematical language. Also, the complementary characteristic of the elements introduced by the linking verb 'is' emphasizes the illustrative stance of the construction. Unlike in a natural language, punctuation mark used to introduce a series in algebraic sentences like the comma is not accompanied by a conjunct. The TP series of projections are different manifestations because they are composed of different constants and variables as well as order of operations. In fact, although the three are of algebraic transitivity, one element is mathematically different from the two.

Both elements are of the same mathematical operation, the constants and variables involved in the operation differ making the algebraic equation different form each other in terms of contents. The first minimal TP projection 'Three more than five times the smaller number X' is composed of variables different form the two, and is translated algebraically;

$$\mathbf{(5 + 3)\,(X) \text{ or } X\,(5 + 3)}$$

The second minimal TP is 'the sum of four times the larger number and three times the smaller number'. Both algebraic equations use the same algebraic operation as elicited by the DP 'the sum', as 'sum' is the mathematical result of addition, and the quantifier 'more than' a process involving algebraic addition, their results differ significantly. The second minimal TP projection 'the sum of four times the larger number Y and three times the smaller number X is translated as;

$$\mathbf{4(Y) + 3\,(X)}$$

The use of 'and' in the prior TP is somehow confusing. It is tricky. As cited above, there are two elements at the left side of the maximal projection, that which is separated by a comma. In natural language, two elements in a series are separated by a conjunct, 'and' for that matter even with the employment of a comma. Hence, seeing the 'and' in the construction with a comma, may be misinterpreted as there are three elements in the construction. However, analyzing the compositionality, key words that express the algebraic operation involved identify only two TP elements; hence, the construction below;

(a) [Three more than five times the smaller number], [the sum of four times the larger number and three times the smaller number]
and not;

(b) [Three more than five times the smaller number], [the sum of four times the larger number] and [three times the smaller number]

What makes the first compositionality valid and correct is the key word that contains the mathematical property, the word that expresses which algebraic operation to be used. The first uses the quantifier ‘more than’ as a mathematical property of the NP ‘three more than five times the smaller number’ for it implies algebraic addition. The second element seemed to be the confusing one because of the comma (,) and the word ‘and’ (see (a) and (b)). However, checking the compositionality, the second element DP ‘the sum of four times the larger number’ seems incomplete in terms of mathematical property. The ‘sum’ as an algebraic operation is referred to as the answer to algebraic addition, and therefore requires the constituents of addition known as addends, at least two. Translating the DP ‘the sum of four times the larger number’ is translated as;

4Y, with the variable Y representing the larger number

It can be noticed that there is only one constituent, and the operation cannot be completed. In this sense, the projection should be extended. In doing so, the ‘and’ in the next DP projection will not be considered as a conjunct of the supposed to be two elements, but just a connector of words in the DP the sum of four times the larger number and three times the smaller number’. Based on the construction, the ‘and’ expression connects the compositionality of &P projection in the PP projection ‘of four times the larger number and three times the smaller number’. Consequently, the two objects in the PP projection are also the constituents of the DP ‘the sum’. Hence, it is possible to complete the algebraic operation implied in the mathematical property ‘the sum’ of the second element in the right side of the algebraic equation. The TP ’Three more than five times the smaller number, the sum of four times the larger number and three times the smaller number is 71’ is translated algebraically as;

(5X + 3) = 71 as (4Y + 3X) = 71

The two construction at the left side of algebraic equation have different compositionality but are pertaining to the same value at the right side of the algebraic equation. **(5X + 3)** and **(4Y + 3X)** are the illustration or examples of the possible conditions of the value **71**. The comma between the first algebraic equation and the second algebraic equation is interpreted as a conjunct since there is no lexical entry showing a relationship between the first algebraic equation and the second algebraic equation. Since both equations are put on the same implication of the right side, they are both related mathematically to a construction and composition leading to the value at the right – hand side which is the 71. This multiple implications that is equal to the given value use the illustration stance of the subject – complement construction.

Lexico-Grammatical Precision

Mathematical language like a natural language is using a grammar that is precise. In fact, the main reason for using mathematical grammar is that the mathematical statements are supposed to be accurate. This can be achieved through the use of a language that is devoid of vagueness and ambiguities. Algebraic sentences, however are highly complex lexically and syntactically, and quite prone to ambiguities. It is important, therefore, to point out lexical and syntactical precision based on form and function.

To illustrate the level of precision that is preferred in mathematical discourse, as the algebraic sentence below.

(1) *The ratio of seven increased by the product of two numbers XY and one half of the sum of a number M and six times n is the sum of M and N and eight*

As explained earlier, no space between a constant (number) and a variable (letter) indicates an algebraic multiplication as expressed by the verb ‘times’. This TP is considered embedded TP and not a distinct TP of the statement because it is linked to a prior construction by the conjunct ‘and’. Looking at the &P construction, the embedded TP is part of the &P together with the DP ‘a number M’. The & P, apparently is the next linear order. However, a mathematical statement should include an expression of algebraic operation. Hence, the next projection is the DP 'the sum of a number M and six times N' which is translated as

$$\mathbf{M + 6N}$$

The DP 'the sum' as a noun-based lexical category is translated as a mathematical phrasal verb 'is added to'. This illustrates the limitation of the given definition of lexical categories, in this case, the noun.

Going further the next linear order, the DP 'the sum of a number M and six times N' is a compositionality of PP with the P 'of' as its head. In natural language, a PP construction is generally a complement of the prior NP or its variations. In mathematical language however, it is the other way around. The noun 'one half' rather than functioning as an object or focus becomes an algebraic modifier expressed as 'one half of' describing the condition of the object of the preposition 'the sum of M and six times N'. Also, it carries the algebraic operation in which the PP functions. So to at least complete mathematically the meaning of the PP construction, the NP or its variations prior to the PP should also be accorded as a compositional phrase, hence, the QP 'one half of the sum of a number M and six times N, which is translated algebraically as;

$$\mathbf{\tfrac{1}{2}\,(M + 6N)}$$

'One half of' is translated as algebraic multiplication if it is attached as a fraction to the entity it modifies.

$$\text{or } \frac{(\mathbf{M+6N})}{|\mathbf{2}}$$

'One half of' is translated as algebraic division if it is a simplified operation of dividing the entity to halves, with which fraction is translated as algebraic division. These transformations from a noun category to mathematical adjective and verb are illustration of how a lexical category can be transformed to another in the mathematical language.

Further, the QP 'one half of the sum of a number M and six times N' is preceded by the conjunct 'and' which makes it a part of coordination in &P construction. Therefore, the next linear order of the phrase structure is the &P projection. Since the first element of the &P following the bottom – top projection is the QP 'one half of the sum of a number M and six times N344', coordination rule requires that the other element in the &P should be parallel compositionally to the prior one. It is expected therefore that the other element of &P is also a QP or an equal variation such as DP or surface NP. Consequently, the adjacent parallel compositionality to the prior QP is another QP 'two numbers'. However, the QP 'two numbers' is preceded by a preposition, therefore is a completer of a PP construction. The PP with the embedded QP is not grammatically parallel with the other element in the &P which is a distinct QP. Also, the QP 'two numbers' cannot be translated within an algebraic sentence without a mathematical property presented linguistically through lexical positioning and semantic cues. Extending the phrase structure, the QP 'two numbers' as a completer in the PP projection is the object of the DP 'the product'. This DP has a mathematical property translated as algebraic multiplication which can be attached to the PP 'of two numbers' adjacent to it to warrant its representation as algebraic statement. The DP 'the product of two numbers is translated algebraically as;

$\mathbf{X \cdot Y}$, small dot between two variables indicates algebraic multiplication
or $\mathbf{XY}$, no space between two variables indicates algebraic multiplication

The NP 'seven increased by the product of two numbers' is preceded by the preposition 'of' resulting in a PP 'of seven increased by the product of two numbers' construction. As discussed earlier, in

mathematical language specifically in algebraic sentences, head noun projection is a nominative accusative type non-terminating constructional form where noun phrases are projected from the noun to its modifier. The PP construction as a modifier is preceded by the noun 'ratio'. The noun 'ratio' and the modifier PP 'of seven increased by the product of two numbers' compose the nominative accusative type. Finally, the maximal projection DP 'the ratio of seven increased by the product of two numbers' construction also shows how precise words contribute to the clarity of the algebraic equation. Precision in this case means the specific function of the words and not just mere categories. The DP 'the ratio' is interpreted as algebraic division expressed in mathematical verb 'divided by'. Hence, DP 'the ratio' accounts its transitivity on the argument structure of a phrase that is head verb. In this case, the argument structure is not determined by the lexical category, rather is determined by the syntactic construction. Since the DP 'the ratio' has at least two arguments, the subject and the patient, its transitivity establishes the relationship between the two possible nouns in its valence. The valency of DP 'the ratio' is in the PP construction, thus the object/s of the preposition 'of'. Based on the linguistic feature, the preposition 'of' is completed by the &P 'seven increased by the product of two numbers, and one half of the sum of a number M and six times N'. The elements of &P (1) 'seven increased by the product of two numbers', and 'one half of the sum of a number M and six times N' are the arguments of the mathematical verb 'ratio' interpreted as 'divided by'. As an algebraic division, 'ratio' requires a dividend (agent/reactor) and a divisor (patient/receiver). Unlike in natural language where the noun after the verb is the patient or the receiver/completer of the action, in algebraic sentence construction, the first element is usually the dividend and the latter is the divisor. Hence, in the algebraic sentence, the first element is the patient/receiver of the action. In the &P construction, NP 'seven increased by the product of two numbers' is the receiver/completer of the phrasal verb 'divided by' and QP 'one half of the sum of a number M and six times N' is the agent/reactor of the action.
Finally, based on the transitivity, DP 'the ratio of seven increased by the product of two numbers XY' is the patient/receiver of the action 'divided by'; and, 'one half of the sum of a number M and six times N' is the agent/reactor. The DP is interpreted algebraically as;

$$\mathbf{7 + XY}$$
$$½(\mathbf{M + 6N})$$

The above construction is the compositionality of the left side of the equalities further categorized as the subject of the algebraic sentence. The right –hand side of the equalities is the predicate headed by the linking verb 'is', and is indicated as the T'. Since a TP projection is a tense projection (Radford, 2006), its verb phrase projection is tense – headed. The linking verb is the tense indicator of the projection. The predicate of the equalities is T' 'is the sum of M and N and eight.

According to Langliness (2011), the linking verb in mathematical language is used to show equalities between the left – hand side expressions and the right – hand side expressions. The linking verb therefore means 'equals' or 'is equal to'. Hence, the T' 'is the sum of M and N and eight' mathematically means 'equals the sum of M and N and eight'. To algebraically translate the T', the mathematical condition of all the expressions should be identified. The DP 'the sum' is the mathematical condition as it is translated as algebraic addition. Since, it is addition, there should be at least two elements to be added. Two elements in a phrase structure construction is introduced in &P projection. However, looking at the compositionality of the DP projection, there are two conjuncts 'and'; (1) 'M and N', and (2) 'and eight'. The two 'ands' function differently. Since it would be mathematically redundant to say "sum of the sum of M and N" (versus 'sum of M, N and eight), the first 'and' is an algebraic addition joining two variables (literal values). M and N is translated as M + N. The second 'and' is also notably algebraic addition as it attributes its meaning with the linguistic character of the conjunction 'and' expressing addition. Consequently, T' 'is the sum of M and N and eight' is translated algebraically as;

$$\mathbf{= M + N + 8}$$

To complete the elements of the equalities, the left – hand side e expressions are placed side-by-side with the right – hand side expression using the sign for equalities (=). Hence, the algebraic equation 'The ratio of seven increased by the product of two numbers and one half of the sum of a number M and six times N is the sum of M and N and eight' is algebraically translated as;

$$\frac{\mathbf{7 + XY}}{\frac{1}{2}(\boldsymbol{M + 6N})} = (\boldsymbol{M + N + 8})$$

CONCLUSIONS AND PEDAGOGICAL IMPLICATIONS

Having established that the language of mathematics is based on a natural language such as English, but the behaviors of its lexis, syntax and semantics prove the uniqueness of its expressions and thought. Understanding linguistic aspects of algebraic sentence of equalities leads to the understanding of how the system of the language of mathematics operates. Hence, in order to address linguistic difficulties in understanding the algebraic sentence of equalities, it is appropriate to explore the construction and compositionality operating in the language they are premised, specifically those linguistic markers that prompts meaning.

Teachers' and students' awareness of the complexities must then be explicitly taught to students. Process of deconstructing a problem text is essential. For instance, the lexical properties of algebraic sentence of equalities must be focused such as identifying linguistic markers in the if-then syntax, predicate arguments in algebraic sentence of equalities which include the definition, description and illustration, and in lexico-grammatical precision of the sentences. This can assist in locating nominal expressions and structures which often indicate the process or operations, object complementation in linking verb 'is' that determine mathematical relationships in definition, description and illustration, and the lexical shifts that happen in the semantics of algebraic sentence of equalities. Usual words in everyday English but are differently meant in the context of mathematics language should also be pointed out. Nevertheless, it should be a pedagogical priority that in mathematics, students should be taught or trained to use close-reading strategy, that is, to focus on every word in the text. Also, In terms of syntactical features, algebraic sentence of equalities generally follow lexico-grammatical precision. Hence, to remind students of steps and process involved in the MWPs, construction text that involved modified operations should be modeled. This would create impression to the students that they need not always follow the same pattern but have to learn to derive formula that fit the condition specified by the given problem text.

Finally, conceptual understanding should be maximized in dealing with algebraic equations. It means that in comprehending mathematical word problems, interpreting algebraic equations should be done linguistically and presentation of information into comprehensible concepts should be a focus in teaching. As demonstrated in the examples, relevant concepts are seldom explicitly presented in the texts linguistically, but signaled by a variety of linguistic markers. Attention to these key linguistic features may raise awareness about the ways mathematical word problems specifically the algebraic equations are constructed to achieve understanding of the mathematical conditions and algebraic operations required to solve them. Based on the analysis of texts, close attention to linguistic markers of the algebraic equations requires explicit teaching of the linguistic markers or cues. While existing mathematical knowledge is crucial in understanding the conditions and operations embedded in text, consistent use of the linguistic markers as cue is essential. In this manner, verbalization of the discourse of problem solving can be achieved gradually but comprehensively.

REFERENCES

Anudin, A. (2019). The use and overuse of transitional conjuncts in written Asian Englishes. *The Asian EFL Journal*, *25*(5.1), 453–477.

Ashlock, R.B. (2001). *Error patterns in computation: Using error patterns to improve instruction*. Prentice Hall.

Blachowicz, S., & Ogle, G. (1982). "Why Are There So Many Words in Math?": Planning for Content-Area Vocabulary Instruction. *Voices from the Middle*, *20*(1), 43.

Brown, C.L., Cady, J.A., & Taylor, P.M. (2009). Problem Solving and the English Language Learner. *Mathematics Teaching in the Middle School*, *14*(9), 532–539.

Capraro, R.M., Capraro, M.M., & Rupley, W.H. (2012). Reading-enhanced word problem solving: A theoretical model. *European Journal of Psychology of Education*, *27*(1), 91–114.

Dominguez, H. (2005). Bilingual students' articulation and gesticulation of mathematical knowledge during problem solving. *Bilingual Research Journal*, *29*(2), 269–295

Hughes, E.M., & Fries, K.M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly*, *31*(3), 235–252.

Jitendra, A.H., & Star, J.R. (2012). An exploratory study contrasting high– and low achieving students' percent word problem solving. Learn. *Individ. Differ*, *22*, 151–158. DOI:10.1016/j.lindif.2011.11.003

Langeness, J. (2011). *Methods to Improve Student Ability in Solving Math Word Problems*. Hamline University, St. Paul, MN.

Martin, M.O., & Mullis, I.V. (2013). *TIMSS and PIRLS 2011: Relationships among reading, mathematics, and science achievement at the fourth grade - Implications for early learning*. Boston: IEA.

Martiniello, M. (2008). Language and the performance of English-language learners in math word problems. *Harv. Educ. Rev*., *78*, 333–368.

Muth, K.D. (1982). Solving word problems: Middle school students and extraneous information. *School Science and Mathematics*, *86*(2), 108–111.

Namirah Fatmanissa & Kusnandi (2017). The linguistic challenges of Mathematics word problems: A research and literature review. *Malaysian Journal of Learning and Instruction (MJLI)*, Special issue on Graduate Students Research on Education, pp. 73–92.

O'Keeffe, L., & O'Donoghue, J. (2015). A role for language analysis in mathematics textbook analysis. *International Journal of Science and Mathematics Education*, *13*(3), 605–630.

Pierce, M.E., & Fontaine, L.M. (2009). Designing vocabulary instruction in mathematics. *The Reading Teacher*, *63*(3), 239–243.

Santos, M.L. (2019). Analysis of the Grammatical Construction and Compositionality of Verbal Problems: Linguistics and Logic Interface in the Language Mathematics. *Asian EFL Journal*, *24*(4.2), 313–349.

Tonio, J.Z., Anudin, A.G., & Lucas, R.I. (2019). Lexical Features and Motivations of Product Reviews on Selected Philippine Online Shops. *Asian EFL Journal*, *24*(4.2), 161–190.

Vizconde, C. (2006). *Attitudes of Student Teachers Towards the Use of English as Language of Instruction for Science and Mathematics in the Philippines*.

Ward, R.A. (2005). Using Children's Literature to Inspire K–8 Preservice teachers' future mathematics pedagogy. *The Reading Teacher*, *59*(2), 132–143.

Winsor, M. (2007). Bridging the Language Barrier in Mathematics. *Mathematics Teacher*, *101*, 372–378.