

Determination and Comparison Pedagogical Content Knowledge of Pre-Service and In-Service Elementary School Teachers in Mathematics

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The research investigates and compares pedagogical content knowledge of pre-service and in-service elementary school teachers in mathematics. The study used a quantitative method design, and the participants were 96 pre-service teachers and 80 in-service elementary school teachers. The data collected by questionnaires using 5-point Likers scala consists of content knowledge, pedagogical knowledge, and pedagogical content knowledge. The data is analyzed using a one-way ANOVA and MANOVA. The results showed that pre-service and in-service teachers showed higher levels of pedagogical content knowledge. Teachers in services scored higher in pedagogical content knowledge than pre-service teachers significantly. In-service teachers have higher scores in each of the components. Pre-service teachers have higher scores in pedagogical knowledge and pedagogical content knowledge except for content knowledge. Teacher educators need to pay attention to increased pre-service content knowledge and activating pre-service teachers in elementary school as co-teacher. In-service teachers can improve their skills through educational seminars and webinars online that do not interfere with teaching time.

Keywords: pedagogical content knowledge, pre-service teacher, in-service teacher, mathematics

INTRODUCTION

Nowadays, knowledge is evolving very fast, and every country is racing to produce new findings in all disciplines. These advances were triggered by computerized technology that accelerates performance in designing, processing information, evaluating, and decision making. Besides, teachers are challenged to carry out education following the demands of 21century education. The characteristics of 21Century education are globalization and collaboration facilitated by information communication and technology (ICT). ICT makes global information easily accessible and enables students to obtain more information than in school quickly. ICT allows students to collaborate with friends around the world. This situation makes educators adapt to very rapid changes and seek to use the power of students as digital natives to support their learning outcomes.

To meet the educational needs of the 21st century, the government conduct training or workshop for professional teacher development—likewise, the educational institution’s reforms in curriculum and teaching materials. Also, lecturers are monitored periodically by quality insurance at the university level or national level. After various efforts in improving pre-service and in-service competencies, it is necessary

to know how these efforts have impacted pedagogical content knowledge levels. Pedagogical content knowledge (PCK) is one of the mathematical knowledge for teaching that positively affects the learning outcomes of mathematics students (Charalambous et al., 2020; Valiandes & Neophytou, 2018; Yu & Singh, 2018). In fact, over the past two decades, international research on professional development has resulted in the understanding that teachers who have pedagogical abilities can improve the quality of teaching and student learning outcomes (Charalambous et al., 2020; Valiandes & Neophytou, 2018; Yu & Singh, 2018). Some researchers found that professional development that prioritizes PCK development improves teacher performance and student learning(Charalambous et al., 2020).

This research aims to determine the level of PCK ability of pre-service and in-service teachers and elementary school teachers. This information will be a consideration in the future design and implementation of PCK development efforts. Best practice PCK development in in-service teachers able to implement in pre-service teachers or vice versa. The problem the study identified was "what are the level of PCK preservice and in-service elementary school teachers in math and the difference between them?". In this context, its sub-problems were selected as follows:

- Are there any significant differences in teachers' PCK among pre-service and in-service teachers?
- What are the pre-service and in-service mathematics teachers' strengths and weaknesses in their PCK levels?
- What are the differences between pre-service and in-service teachers' levels in each component of PCK?

LITERATURE REVIEW

Professional teachers need to create quality and innovative instruction. The role of the teacher is crucial because it develops essential knowledge, skills, and dispositions that are very influential on the development of mathematics students at the education level afterward. Teaching ability is known as pedagogical content knowledge (PCK). PCK is the knowledge that distinguishes an educator from an expert, such as determining a math teacher from a mathematician (Kwong et al., 2007). Also, PCK focus on the learning strategies of specific disciplines, unlike general knowledge implemented to all (Darling-Hammond, 2006). PCK depends on the characteristics of the subject matter, so PCK for mathematics differs from another subject. Therefore, PCK ability is essential for teachers to teach effectively.

PCK is a knowledge base concerned with the representation and formulation of concepts, pedagogical techniques, and knowledge of what makes concepts difficult or easy to learn (Mishra & Koehler, 2006). This is in line with Ball et al. (2009) that PCK is knowledge of content related to students, knowledge of content related to teaching, and knowledge of content related to the curriculum(Ball et al., 2009). PCK is a combination of understanding, knowledge, skills, and dispositions effectively applied by teachers in the teaching (Grieser & Hendricks, 2018).

Meanwhile, PCK in mathematics is knowledge of instructional strategies and representations of a mathematical concept and how to teach mathematics to students (Depaepe et al., 2015). Also, PCK is related to how and why concepts are related to each other and how a procedure solves a problem (Johnson et al., 2020; Leavy et al., 2015). Moreover, PCK also combines knowledge about teaching, mathematics content, and knowledge of instructional design, such as strategy to introduce a content (Scheiner et al., 2019).

One of the central aspects of mathematics pedagogical knowledge is knowledge of learning strategy and representation of mathematical concepts and how to teach mathematics (Tröbst et al., 2019a). The familiar theory about teaching mathematics, argued by (Bruner, n.d.), students learn mathematics through enactive, iconic, and symbolic models. Similar to Tall (2004), mathematics understanding constructs from real-world perceptions and activities, enforced, and iconic representations in the real world will be formed fundamentally through language to support abstract concepts, including symbols. According to Lee (2017) the components of PCK in mathematics are: 1) Pay attention to the environment and situations to engage students in learning mathematics, 2) Consider the students' math activities, 3) Improve students' understanding of mathematics and thinking.

According to Tröbst et al. (2019b), PCK consists of pedagogical knowledge and content knowledge. Pedagogical knowledge (PK) combines understanding, knowledge, skill, and disposition that teachers effectively teach (Tröbst et al., 2019b). Chai et al. (2015) define PK as the knowledge of teaching students, instructional methods, educational theory, and assessment applicable to all subjects. Some factors influence the effective teaching process, including knowledge of the subject matter, student knowledge, knowledge of student cognition, and knowledge of teaching and decision making (Turnuklu & Yesildere, 2007). Lee (2017) defines PK as knowledge of how to teach or transfer that knowledge to the learners concerned. Good PK helps teachers choose proper teaching tools, skills, and techniques to teach specific content to make topics understandable to students. It also allows teachers to manage classroom activities and time (Akhtar et al., 2016). Knowing different strategies is not enough because PK relates to students; classroom management, planning, and assessment of students are included in pedagogical knowledge.

Content knowledge (CK) of mathematics consists of conceptual and procedural knowledge. Conceptual knowledge refers to the abstraction of concepts, including understanding concepts and their relevance (Rittle-Johnson et al., 2015). Van de Walle et al. (2019) extend the definition of conceptual knowledge as understanding or structure of concepts and the relationships between concepts. A person with conceptual knowledge can explain concepts, understand the relationships between concepts, and find concepts (Zuya, 2017).

Conceptual knowledge is information related to each other, a network where interrelationships are as crucial as different information (Österman & Bråting, 2019). Some studies have found that conceptual knowledge plays a role in the success of learning mathematics. Zulnadi & Zamri (2017) report that conceptual knowledge positively influences learning achievement and conceptual considerations for obtaining procedural knowledge. Good conceptual understanding allows children to develop strategies when solving problems, for example, making connections between concepts. Therefore children with good conceptual understanding usually have good procedural skills (Schneider et al., 2011).

Meanwhile, procedural knowledge as a rule or step to solving problems through mathematical presentations (Van de Walle et al., 2019). The mathematical representation includes symbols, notation, mathematics equations, graphics, tables, words, etc. Procedural knowledge is an answer to the question "how." Hence procedural knowledge build on conceptual understanding; it's essential to develop students' conceptual knowledge involved in physical and mental activities using concrete material.

A previous study proves that CK provides a significant contribution to PCK. Depaepé et al. (2015) found conceptual and procedural knowledge related to PCK. Gess-Newsome et al. (2017) study the impact of teacher content knowledge, pedagogical knowledge, and PCK on instructional quality and student achievement. They conclude a positive correlation between content knowledge and pedagogical content knowledge. Norton (2019) concluded that mathematical content knowledge is highly predictive of mathematical pedagogical content knowledge performance and suggests merit in developing the two aspects of teacher knowledge in tandem rather than in different courses. Therefore, mathematical knowledge for teaching combines content knowledge and pedagogical content knowledge. It is impossible to give instructions on pedagogical content knowledge without implicitly supplying some instruction on the knowledge of appropriate content (Tröbst et al., 2019b).

METHODS

This study is based on survey research to get much information from the population to learn about their opinions. There were 80 participants (52 women, 28 men) in services teachers from public and private elementary schools. Their teaching experience is 26.25% less than five years, 21.25% between 6-10 years, 23, 75% 11-15 years, and 28.65% over 16 years. Also, 96 pre-service teachers (74 women and 22 men) enrolled in Arithmetic instruction courses.

The data was collected by questionnaire referring to Aksu et al. (2014). PCK instruments consist of three-part: CK (15 items), PK (15 items), and PCK (20 items). We used the 5-point Likert scale, each: 1 (strongly agree), 2 (agree), 3 (unidentified), 4 (disagree), 5 (strongly disagree). The data was analyzed using JASP statistical tools. The average scores of pre-service and in-service teachers in each group (CK, PK,

and PCK) are calculated. One-way ANOVA is used to find group differences, followed by the Tuckey test or Scheffe tests are applied to analyze the appropriate data. MANOVA is used to compare data groups (CK, PK, PCK) between pre-service and in-service teachers.

RESULTS

Descriptive Analysis

Table 1 presents descriptive statistics (number of participants, average, and standard deviation) for pre-service and in-service on three sub-scale instruments (CK, PK, and PCK). PCK of pre-service teachers falls into the moderate category ($\bar{x} = 3.755$) and in-service teachers in the high category ($\bar{x} = 4.095$). The average score of CK in pre-service teachers ($\bar{x} = 3.617$) was lower than the average score of PK ($\bar{x} = 3.851$) and PCK ($\bar{x} = 3.755$). The average score of in-service teachers in CK ($\bar{x} = 4.504$) is lower than the average score of PK ($\bar{x} = 4.109$) and PCK ($\bar{x} = 4.121$).

TABLE 1
DESCRIPTIVE ANALYSIS

| | Pre-service | | | In-service | | |
|----------------|-------------|-------|-------|------------|-------|-------|
| | CK | PK | PCK | CK | PK | PCK |
| N | 96 | 96 | 96 | 80 | 80 | 80 |
| Mean | 3.617 | 3.851 | 3.797 | 4.054 | 4.109 | 4.121 |
| Std. Deviation | 0.477 | 0.408 | 0.454 | 0.388 | 0.371 | 0.402 |

Based on the participant's responses, pre-service teachers have higher scores in connecting topics with daily life ($\bar{x} = 4.01$), knowing students' misconceptions when learning new topics ($\bar{x} = 3,814$), understanding mathematical concepts ($\bar{x} = 3,804$). But it has a low average score on the knowledge of national and international mathematical scientists ($\bar{x} = 2,835$). At the same time, in-service teachers have a high category in CK ($\bar{x} = 4,054$). They have the highest score on knowledge of mathematics ($\bar{x} = 4,313$), desire to participate in seminars, symposiums, and workshops related to mathematics ($\bar{x} = 4,363$). They can relate topics to everyday life ($\bar{x} = 4,275$). The lowest average score was mathematicians' knowledge at the national or international level ($\bar{x} = 3,325$).

In pedagogical knowledge, the pre-service teacher obtained the highest average score on the ability to control my emotions during the lesson ($\bar{x} = 4,123$), set up a democratic environment that allowed students to express their opinions ($\bar{x} = 4,041$) and use time effectively in lessons ($\bar{x} = 3,958$). In contrast, pre-service teachers had the lowest scores on knowledge of instructional design ($\bar{x} = 3,395$). In-service teachers have the highest average score in using learning tools in teaching ($\bar{x} = 4,225$), while the lowest is knowledge of instructional design ($\bar{x} = 3,838$).

On PCK, pre-service teachers have a higher score on understanding concepts that exemplify everyday life for students in lessons ($\bar{x} = 4,061$), doing a question and answer during the learning process ($\bar{x} = 4,144$), and using appropriate assessment ($\bar{x} = 3,969$). But they have low scores on explaining the material with strategies that can be understood by all students even though they have different abilities ($\bar{x} = 3,556$). On the other hand, teachers in the department have high scores on all questionnaires. The lowest score is creating a conversation about a mathematical idea with students ($\bar{x} = 3,963$), which is already relatively good.

Research Question 1: *Are there any significant differences in teachers' PCK among pre-service and in-service teachers?*

One-Way ANOVA is used to determine if there are significant differences between pre-service and in-service teachers to PCK. As a result, it is well known that there is a substantial difference in the ability of

PCK ($F = 101.000$; $p < 0.001$) between pre-service and in-service teachers. Previously obtained normally distributed data (Leverne's, $p = 0.066$). Tuckey tests showed the average PCK in-service teacher score was significantly higher than the pre-service teacher ($p < 0.001$).

Research Question 2: *What are the pre-service and in-service mathematics teachers' strengths and weaknesses in their PCK levels?*

One-Way ANOVA was performed to determine whether there were significant differences in PCK and each of its components (CK, PK, PCK). Pre-service teachers' data is homogenous (Leverne's, $p = 0.568$). ANOVA's one-way test results found a significant difference between CK, PK and PCK ($F = 14.600$; $p < 0.001$). Furthermore, the Tukey test is conducted to determine the direction of the difference and which has a significant difference between CK, PK, and PCK.

TABLE 2
POST HOC COMPARISONS

| | | Mean Difference | SE | t | p Tukey |
|----|-----|-----------------|-------|--------|---------|
| CK | PK | -0.303 | 0.060 | -5.066 | < .001 |
| | PCK | -0.249 | 0.060 | -4.162 | < .001 |
| PK | PCK | 0.054 | 0.060 | 0.904 | 0.638 |

Table 2 shows that there are significant differences between CK and PK, and PCK. However, there is no difference in the ability of PK and PCK pre-service teachers. At the same time, in-services teachers' data is also homogeneous (Leverne's, $p = 0.798$). One-way ANOVA test results found no significant difference between CK, PK, and PCK ($F = 0.675$; $p = 0.510$).

Research Question 3: *What are the differences between pre-service and in-service teachers' levels in each component of PCK?*

MANOVA was performed to compare PCK teachers pre-service and in-service. Assumption Checks by Box's M-test for homogeneity of covariance matrices is $\chi^2 = 80.29$ and $p < 0.001$. We use the Shapiro-Wilk test for multivariate normality ($F = 0.953$ and $p < 0.001$). No significant violation of linearity, normality, or homogeneity was found.

The MANOVA Pillai test (Table. 3) showed significant differences between CK, PK, and PCK. CK, PK, and PCK in-service teachers are better than pre-service teachers. :

TABLE 3
MANOVA

| Cases | df | Approx. F | Trace _{Pillai} | Num df | Den df | p |
|-----------|-----|-----------|-------------------------|--------|--------|--------|
| Intercept | 1 | 8387.12 | 0.993 | 3 | 172.0 | < .001 |
| teachers | 1 | 26.93 | 0.320 | 3 | 172.0 | < .001 |
| Residuals | 174 | | | | | |

DISCUSSION

The study investigates and compares teachers' confidence in services and pre-services in pedagogical content knowledge. The results showed that in-service teachers attain higher confidence in pedagogical content knowledge than pre-service teachers. These findings align with the findings (Şahin et al., 2016) that PCK in-service teachers are not sufficient to identify student mis conceptualizations. The average score of

each component of PCK(CK, PK, PCK) of in-service teachers is high, and there is no significant difference between components. In addition, the average score of CK, PK, and PCK teachers in the position is significantly higher than pre-service teachers. The experience of teaching affects PCK of in-service teachers. In this case, pre-service teachers develop PCK through lectures and observations to school or observe instructional videos.

Pre-service teachers have the lowest scores on knowledge of mathematics for teaching. As a math teacher, mastery of math content is a crucial requirement. Many studies show that content knowledge affects PCK (Depaepe et al., 2015; Gess-Newsome et al., 2019; Norton, 2019). Kleickmann et al. (2013) concluded that CK is essential in developing PCK. They added that a high CK would increase the opportunity for PCK acquisition.

Content knowledge depends on conceptual and procedural knowledge. Both knowledge is indispensable when students devise strategies to solve problems. Pre-service teachers' weaknesses in conceptual knowledge impact their ability to teach mathematics. In addition, teaching strategies consider what, why, and how concepts are interrelated (Grieser & Hendricks, 2018; Johnson et al., 2020). Therefore, the PCK ability of pre-service teachers who are still moderate is likely due to conceptual capabilities that have not been maximized. In addition, having high procedural knowledge also has an impact on PCK. Teachers who have high procedural knowledge tend to focus on teaching procedures. Therefore, it needs to get attention from lecturers and do CK development through curriculum and teaching materials improvement. In addition, lecturers need to take advantage of the power of pre-service teachers, who are the 21st century generation and have many opportunities to explore mathematical concepts from various resources. As a digital generation, they can enrich insights through an online environment that provides information in video and image illustrations that can be accessed easily. Lecturers can reform the curriculum by activating pre-service teachers in elementary school as co-teachers who get involved in; lesson plans, guide students in small groups; recognize students' needs, and offer support as required. They are modeling the established limits, expectations, and classroom routines.

CONCLUSION

The study found that the PCK of in-service was better than pre-service teachers. The weakness of PCK of pre-service is the ability of mathematical content, which is also low, including conceptual and procedural knowledge. This becomes a challenge for educators to improve the quality of education of prospective teachers, especially in mathematics. The opportunity to develop the competence of PCK pre-service teachers is to take advantage of their computer technology abilities. Lecturers can reform the curriculum by adding individual and group learning activities outside the classroom to inform online learning organized by LMS.

In-service teachers can improve their skills through educational seminars and webinars online that do not interfere with teaching time. Also, in the teacher community, they can make learning media such as animations presentations using technology that will lead to the development of TPACK (Technological Pedagogical Content Knowledge). Educational technology is growing rapidly, and in-service teachers should not be left behind.

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