Private Equity Valuation and IRR Algorithm

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An algorithm is developed that calculates the IRR for various private equity entities within a private equity leveraged transaction. The algorithm calculates the total anticipated value for a transaction and then produces the associated IRRs based on the exit EBITDA, the EBITDA multiple, and the available cash. The benefits of the algorithm are that multiple programming formats become available, insights emerge that are difficult to perceive using an equivalent spreadsheet pro forma analysis, and other types of analyses become possible for examining individual parameters.

Keywords: private equity, valuation, carried interest, IRR, EBITDA multiple

INTRODUCTION

Firm valuation algorithms (see Arnold and James (2000, 2003) and Arnold and North (2009, 2011)) have been applied to the pro forma analysis of a firm with ongoing operations. A benefit of an algorithm is that it is programmable in multiple settings and more easily allows for the application of scenario and sensitivity analyses, as well as, Monte Carlo simulations. The resulting analyses can provide rich insights into the main drivers of the financial outcome being modeled and where to focus efforts to provide the highest impact on those outcomes.

Although there is extensive research regarding investment in private equity (see Ang, Chen, Goetzmann, and Phalippou (2018) and Ivashina and Lerner (2019)), venture capital (see Block, Fisch, Vismara, and Andres (2019)), angel investing (see Harrison, Bock, and Gregson (2021)) and more recently, carried interest (Phallippou (2020)), this research generally centers around investment return or on the criteria for investing and not how the investments (i.e. the firms that receive the funding) generate the return through the growth of the firm. Essentially, the means by which a private firm is expected to generate the return for these various funding sources has not been extensively explored systematically.

In order to address how a private firm is expected to generate return to its investors by growing its operations, generally, a financial statement model (usually propriety) is developed based on the intended growth projections of a firm with private equity funding and significant financing from senior and mezzanine debt. This type of model is an essential part of the analysis performed when an investment is considered and can provide greater insight into the specific source of the generated returns. Projections of IRRs for debt and equity holders, as well as the valuation of warrants and carried interest, are generated by the model, usually with some scenario analyses to determine the vulnerability of the investment.

The contribution of this presentation is the development of an algorithm that captures the financial statement model/analysis systematically. The algorithm can be applied to a single investment deal and is readily adaptable to various simulation analyses. Further, the algorithm has the potential to provide implicit values for inputs, such as revenue growth, when investment return levels are set to specific targets and other model inputs are set to historical industry levels. The benefit of the algorithm beyond a spreadsheet pro forma analysis, is that the algorithm can be more readily incorporated into the examination of multiple investments under multiple scenarios and is easily programable in various platforms (e.g. Python or other coding languages) beyond a spreadsheet environment.

For example, suppose the algorithm is set to an aggregated form of financing within an industry. In that case, the implied revenue growth necessary to achieve a certain level of return performance provides an anticipated revenue growth rate for the industry. This is similar to how an option pricing model is set at an actual option price to produce an implied volatility of the future underlying security spot prices. Taking this further, future empirical analyses of private equity fund returns can use the algorithm to determine anticipated ex-ante fund returns (by setting revenue growth goals, or other inputs, to match previously realized levels) or the algorithm can fix ex-post fund returns (by setting returns at historical industry levels) and back out what the revenue growth goals or goals of other inputs would need to be to achieve the fixed return outcome. The preconceived input or output goals can be decomposed while considering different combinations of available funding sources.

Using implied values for revenue growth or considering funding effects relative to implied revenue growth is a foreign idea for publicly traded firms with regular financial reporting requirements. Because private firms do not have such reporting requirements, the ability to imply model inputs from an algorithm is very beneficial given what data resources are available. Although this discussion has focused on implying revenue growth, any model parameter (i.e., cost of goods sold, fixed asset growth, etc.) can be implied with other parameters set to fixed values based on historical industry-specific assumptions. Private Equity fund's often have commonly established terms in their partnership agreements across the industry. The well-known "2-and-20" determination of management fees and carried interest, specified waterfall provisions, and range of acceptable preferred interest rates are just a few examples. In leveraged deals, interest rates would be set to match market conditions. The algorithm can utilize these existing boundaries, allowing the model to focus further on other more dynamic key drivers and return outcomes. Further, if return performance is taken as a given within a private equity transaction, an optimal mix between mezzanine debt (with warrants) and equity can be implied from the algorithm and then compared to the actual mix of financing used if studying multiple transactions or as a means of creating an optimal design for a current transaction.

In the first section, a generic leveraged buyout (LBO) model is produced that generates IRRs, warrant valuation, and carried interest, i.e., the "waterfall." In the following section, the algorithm for the LBO model is generated and programmed in Excel (note: other programming platforms are possible). The third section demonstrates how the sequence of revenue growth affects return even when EBITDA, a common metric for valuation, is unchanged. This is an insight not easily demonstrated using a traditional pro forma analysis. This section also includes a sensitivity analysis for investment exit and an example of using the model to solve for an input parameter given a set investment return. The fourth section concludes the presentation.

LBO MODEL

A firm will be part of a private equity transaction with the anticipation of an exit from the transaction (i.e. a sale of the firm to another entity) five years in the future. The initial transaction will set the "new" firm's capital structure. The exit transaction will pay off all debt and distribute the remaining funds depending on warrants, equity claims, and performance incentives. FIGURE 1 illustrates how the initial transaction works. The private equity fund purchases the firm for $9,000,000 or 10.0 times the EBITDA (EBITDAX) from the firm's last year of operations (Year 0).

FIGURE 1
PRIVATE EQUITY TRANSACTION EFFECT ON BALANCE SHEET

Transaction:	The Private Equity Fund raises $5,000,000 in senior debt, $1,000,000 in mezzanine debt, and $3,000,000 in stock (20% supplied by the existing managers and 80% supplied by the Fund). The total purchase price ignoring transaction fees is $9,000,000 or 10.0 times EBITDA (10.0 = $9,000,000 transaction/$900,000 EBITDA in Year 0). Senior debt: due in five years with an interest rate of 8.00% APR Mezzanine debt: due in five years with an interest rate of 10.50% APR and 3.00% warrants due when the firm is sold at some point in the future Stock: 20% is supplied by the existing managers and 80% is supplied by the Fund		
Income Statement:	**Year 0:**	**Transaction:**	**New Firm Year 0:**
Revenue:	2,000,000	Used as a model input	2,000,000
Cost of goods sold (COGS):	900,000	Used as a model input	900,000
SGA:	200,000	Used as a model input	200,000
EBITDA[a]:	900,000		
Depreciation:	400,000		
Amortization[b]:	0		
EBIT[c]:	500,000		
Senior debt interest:	280,000		
Mezzanine debt interest[d]:	0		
EBT[e]:	220,000		
Tax:	66,000		
EAT[f]:	154,000		
Balance Sheet (Assets):			
Cash:	1,526,450	Only 200,000 transfers	200,000
Accounts receivable:	305,556	Full amount transfers	305,556
Inventory:	100,000	Full amount transfers	100,000
Total current assets:	1,932,006		605,556
Fixed assets:	12,000,000	Reset to new value based on market value	8,000,000
Accumulated depreciation:	6,000,000	Depreciation restarts	0
Goodwill[b]:	0	Set to balance the balance sheet	456,944
Total fixed assets:	6,000,000		8,456,944

Total assets:	7,932,006		9,062,500
Balance Sheet (Liabilities):			
Accounts payable:	62,500	Full amount transfers	62,500
Senior debt:	4,000,000	Reset based on transaction	5,000,000
Mezzanine debt[d]:	0	Reset based on transaction	1,000,000
Total liabilities:	4,062,500		6,062,500
Balance Sheet (Equity):			
Stock[g]:	100	Reset based on transaction	3,000,000
Retained earnings:	3,869,406	Resets to zero	0
Total equity:	3,869,506		3,000,000
Total liabilities and equity:	7,932,006		9,062,500

[a] EBITDA (Earnings before interest, taxes, depreciation, and amortization) = Revenue – COGS – SGA
[b] The firm does not have a any goodwill to amortize prior to the sale
[c] EBIT (Earnings before interest and taxes) = EBITDA – Depreciation – Amortization
[d] The firm does not have any mezzanine debt prior to the sale
[e] EBT (Earnings before interest and taxes) = EBIT – Senior debt interest – Mezzanine debt interest
[f] EAT (Earnings after taxes) = EBT – Taxes
[g] Stock is set at $1 and for 100 shares, generally retained earnings accounts for most of the equity

FIGURE 2 illustrates the associated pro forma model for the transaction. To add context to the example, the target firm is small and in a high-growth industry. The private equity fund is purchasing the firm to add growth capital and expertise to achieve high projected revenue growth levels. The existing management will be retained and supply some of the equity. Note: if existing management is to be retained without a significant equity stake, they would likely be issued warrants to incentivize performance. Warrants of X% become X% of the firm's equity when a performance goal is met, or a particular event occurs in the future. In this case, warrants are exercised upon the sale of the firm in the future and are given to mezzanine lenders.

FIGURE 2
PRO FORMA ANALYSIS OF A PRIVATE EQUITY TRANSACTION

Revenue:	Initially $2,000,000, will grow annually by: 20%, 30%, 40%, 30%, and 20%
Gross profit margin:	55%, cost of goods sold = revenue × (1 – 55%)
Selling, general, administrative (SGA) costs:	Initially, $200,000, will grow 3% annually
Senior debt:	$5,000,000 with 8.00% annual interest, debt with interest is due in 5 years
Mezzanine debt:	$1,000,000 with 10.50% annual interest, debt with interest is due in 5 years, has 3% in warrants
Tax rate:	30%
Days in receivables:	55 days in a 360-day year, (0.15278 = 55 / 360)
Days in inventory:	40 days in a 360-day year, (0.11111 = 40 / 360)
Days in payables:	25 days in a 360-day year, (0.06944 = 25 / 360)
Initial cash:	$200,000
Initial accounts receivable:	$305,556, (55 = $305,556 / [$2,000,000 / 360])
Initial inventory:	$100,000, (40 = $100,000 / [$2,000,000 × (1 – 55%) / 360]

Initial accounts payable:	$62,500, (25 = $62,500 / [$2,000,000 × (1 – 55%) / 360]					
Fixed assets:	Initially $8,000,000, will grow 5% annually					
Depreciation:	5% of fixed assets					
Stock:	$3,000,000, 20% firm managers, 80% private equity fund					
Goodwill:	Initially $456,944 and is amortized annually for 5 years ($91,389 = $456,944 / 5)					
Income Statement:	**Year 0:**	**Year 1:**	**Year 2:**	**Year 3:**	**Year 4:**	**Year 5:**
Revenue:	2,000,000	2,400,000	3,120,000	4,368,000	5,678,400	6,814,080
Cost of goods sold (COGS):	900,000	1,080,000	1,404,000	1,965,600	2,555,280	3,066,336
SGA:	200,000	206,000	212,180	218,545	225,102	231,855
EBITDA[a]:		1,114,000	1,503,820	2,183,855	2,898,018	3,515,889
Depreciation:		420,000	441,000	463,050	486,203	510,513
Amortization:		91,389	91,389	91,389	91,389	91,389
EBIT[b]:		602,611	971,431	1,629,416	2,230,427	2,913,988
Senior debt interest:		400,000	432,000	466,560	503,885	544,196
Mezzanine debt interest:		105,000	116,025	128,208	141,669	156,545
EBT[c]:		97,611	423,406	1,034,648	1,674,873	2,213,247
Tax:		29,283	127,022	310,394	502,462	663,974
EAT[d]:		68,328	296,384	724,254	1,172,411	1,549,273
Balance Sheet (Assets):						
Cash:	200,000	816,106	1,649,404	2,867,797	4,575,534	6,746,445
Accounts receivable[e]:	305,556	366,667	476,667	667,333	867,533	1,041,040
Inventory[f]:	100,000	120,000	156,000	218,400	283,920	340,704
Total current assets:	605,556	1,302,772	2,282,070	3,753,531	5,726,987	8,128,189
Fixed assets:	8,000,000	8,400,000	8,820,000	9,261,000	9,724,050	10,210,253
Accumulated depreciation:	0	420,000	861,000	1,324,050	1,810,253	2,320,765
Goodwill:	456,944	365,556	274,167	182,778	91,389	0
Total fixed assets:	8,456,944	8,345,556	8,233,167	8,119,728	8,005,186	7,889,487
Total assets:	9,062,500	9,648,328	10,515,237	11,873,258	13,732,173	16,017,677
Balance Sheet (Liabilities):						
Accounts payable[g]:	62,500	75,000	97,500	136,500	177,450	212,940
Senior debt:	5,000,000	5,400,000	5,832,000	6,298,560	6,802,4445	7,346,640
Mezzanine debt:	1,000,000	1,105,000	1,221,025	1,349,233	1,490,902	1,647,447
Total liabilities:	6,062,500	6,580,000	7,150,525	7,784,293	8,470,797	9,207,027
Balance Sheet (Equity):						
Stock:	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000
Retained earnings:	0	68,328	364,712	1,088,966	2,261,377	3,810,650
Total equity:	3,000,000	3,068,328	3,364,712	4,088,966	5,261,377	6,810,650
Total liabilities and equity:	9,062,500	9,648,328	10,515,237	11,873,258	13,732,173	16,017,677

Statement of Cash Flow:					
EAT:	68,328	296,384	724,254	1,172,411	1,549,273
plus depreciation:	420,000	441,000	463,050	486,203	510,513
plus amortization:	91,389	91,389	91,389	91,389	91,389
less change in accounts receivable:	61,111	110,000	190,667	200,200	173,507
less change in inventory:	20,000	36,000	62,400	65,520	56,784
plus change in accounts payable:	12,500	22,500	39,000	40,950	35,490
less change in fixed assets:	400,000	420,000	441,000	463,050	486,203
plus change in senior debt:	400,000	432,000	466,560	503,885	544,196
plus change in mezzanine debt:	105,000	116,025	128,208	141,669	156,545
less dividends:	0	0	0	0	0
plus change in stock:	0	0	0	0	0
equals change in cash:	616,106	833,298	1,218,394	1,707,736	2,170,912

[a] EBITDA (Earnings before interest, taxes, depreciation, and amortization) = Revenue – COGS – SGA
[b] EBIT (Earnings before interest and taxes) = EBITDA – Depreciation – Amortization
[c] EBT (Earnings before interest and taxes) = EBIT – Senior debt interest – Mezzanine debt interest
[d] EAT (Earnings after taxes) = EBT – Taxes
[e] Accounts receivable = Revenue × Days in receivable / 360
[f] Inventory = COGS × Days in inventory / 360
[g] Accounts payable = COGS × Days in payables / 360

Assuming a sale of the firm in the fifth year based on an EBITDA multiple (EBITDAX) of 10.00, the firm will be sold for $35,158,891.85. Assuming the seller keeps 80% of the cash (i.e. the “cash sweep”), an additional $5,397,156.39 also becomes available to the seller. The proceeds of $40,556,048.24 (= $35,158,891.85 + $5,397,156.39) are distributed in the manner displayed is FIGURE 3 (i.e. the “waterfall”). The “waterfall” works in this manner:

- Determine the proceeds from the sale (selling price + cash sweep)
  Note: the “cash sweep” is the amount of cash the seller keeps upon selling the firm. It is a negotiated value that leaves enough cash for continuing operation of the firm after the sale of the firm.
- Retire/pay the senior and mezzanine debt
- Determine warrants as a percentage of the proceeds net of debt
- After warrants are paid, split the remaining amount proportionately to equity holders
- Preferred return calculations are based on the amount paid to the Fund based on its proportion of equity

Preferred return works in this manner:

- The Fund investors are entitled to receive a preferred return on their investment
  = investment × (1 + preferred return)^N, where “N” is the length of the investment in years
- If performance exceeds the preferred return, the Fund managers are entitled to “carried interest”, which is a certain percentage of the proceeds above the preferred return (C%).
  - First, a “step-up” provision calculation is performed. View the preferred return as an investment of “Z” which becomes Z × (1 + preferred return)^N. Let “Y” be the amount in excess of the investment “Z” [Y = Z × (1 + preferred return)^N – Z].The “step-up” calculation views Y as equivalent (1 – C%) of the proceeds associated with the preferred return requirement making 100% of the proceed associated with the preferred return equal to Y ÷ (1 – C%).The step-up calculation is (C%) × [Y ÷ (1 – C%)].For example, say Y = $80 and the %C is 20%.The step-up becomes $20 = 20% × [$80 ÷ (1 – 20%)].Essentially, the first $100 of proceeds beyond the initial investment is split between the Fund investors ($80) and the Fund managers ($20) in an 80% to 20%

manner. The $80 fulfills the preferred return requirement and the Fund managers are assessed 20% carried interest relative to the fulfillment of the preferred return.

- Second, proceeds less the initial investment with accrued preferred return less the step-up provision are split based on the percentage dictated by the carried interest (C%). Fund investors receive (1 – C%) and Fund managers receive C%.

FIGURE 3
PROCEEDS FROM THE SALE OF THE FIRM IN YEAR 5 (i.e. THE "WATERFALL")

EBITDA in Year 5:	$ 3,515,889.19
EBITDA multiplier (EBITDAX):	10.00
Selling price (EBITDA × EBITDAX):	$ 35,158,891.85
Proceeds from cash (Cash sweep, 80% of Year 5 Cash):	$ 5,397,156.39
Total proceeds from sale:	**$ 40,556,048.24**
Senior debt owed:	$ 7,346,640.38
Mezzanine debt owed:	$ 1,647,446.77
Proceeds net of debt:	**$ 31,561,961.09**
Mezzanine warrants (3% of proceeds net of debt):	$946,858.83
Remaining equity (proceeds net of debt less warrants):	**$ 30,615,102.26**
Firm managers portion of equity (20% of remaining equity):	$ 6,123,020.45
PE Fund's portion of equity (80% of remaining equity):	$ 24,492,081.81
Preferred return of 15% [= 80% × $3,000,000 × $(1 + 15\%)^5$]:	$ 4,827,257.25
Carried interest (Step-up provision)*:	$606,814.31
Carried interest (after Step-up provision)**:	$ 3,811,602.05
PE Fund's equity net carried interest:	$ 20,073,665.45
Mezzanine debt IRR:	21.01%
Firm manager's IRR:	59.13%
PE Fund IRR (net carried interest):	52.93%

IRR = [proceeds received from sale ÷ initial investment]$^{1/5}$ – 1

* = 80% × $3,000,000 × {$(1 + 15\%)^5$ – 1} × 20% ÷ (1 – 20%), see Zeisberger, Prahl, and White (2017) for a description of the step-up provision

** = 20% × {$24,492,081.81 – $4,827,257.25 – $606,814.31}

NOTE: Because the Fund equity is well above the preferred return calculation, one could simply split the Fund equity less the Fund investment 80%-20% to find the total value of the carried interest instead of working through the "step-up" calculation.
$22,092,081.81 = $24,492,081.81 – (80% × $3,000,000.00)
Carried interest = 20% × $22,092,081.81 = $4,418,416.36
Carried interest (Step-up provision) + Carried interest (after Step-up provision)
$4,298,416.36 = $606,814.31 + $3,811,602.05 = $4,418,416.36

However, the "step-up" process matters when the Fund equity is less than the preferred return + Step-up provision ($5,434,071.56 in this case). Say, the Fund equity is $5,000,000.00, the Fund investors receive the preferred return of $4,827,257.25 and the carried interest is $172,742.75. If the Fund equity is below the preferred return calculation, there will be no carried interest.

The algorithm derived in the next section, replicates the final values from Exhibit 2 that are applied in FIGURE 3. In other words, the algorithm generates the EBITDA in Year 5, the cash available in Year 5, the senior debt due in Year 5, and the mezzanine debt due in Year 5. This information in addition to the initial investments made by different entities and the incentives given to different entities (warrants and carried interest), then produces the "waterfall" calculations within the algorithm displayed in FIGURE 3.

DERIVATION OF THE ALGORITHM

As stated in the previous section, there are four components determined by the algorithm:

1. EBITDA in Year 5
2. Senior debt due in Year 5
3. Mezzanine debt due in Year 5
4. Cash available in Year 5

The first component is based on revenue, COGS and SGA. First, generate compounded revenue growth for five years using information from Exhibit 1 for the five annual growth rates applied to revenue:

$$3.40704 = (1 + 20\%) \times (1 + 30\%) \times (1 + 40\%) \times (1 + 30\%) \times (1 + 20\%) \tag{1}$$

The Year 5 revenue is the Year 0 revenue multiplied by the result of equation (1):

$$\$6{,}814{,}080 = \$2{,}000{,}000 \times 3.40704 \tag{2}$$

To find the revenue less the COGS in Year 5, multiply the result from equation (2) by the gross profit margin (set at 55% in FIGURE 2):

$$\$3{,}747{,}744 = \$6{,}814{,}080 \times 55\% \tag{3}$$

SGA grows 3% annually from its initial value of $200,000. Its Year 5 value is the initial value compounded at 3% for five years:

$$\$231{,}854.81 = \$200{,}000 \times (1 + 3\%)^5 \tag{4}$$

The Year 5 value for EBITDA becomes equation (3) less equation (4):

$$\$3{,}515{,}889.19 = \$3{,}747{,}744 - \$231{,}854.81 \tag{5}$$

The senior debt component and the mezzanine debt component are similar to the calculation for SGA, the initial value is compounded for five years using the respective interest rates.

$$\text{Senior debt in Year 5:} \$7{,}346{,}640.38 = \$5{,}000{,}000 \times (1 + 8\%)^5 \tag{6}$$

$$\text{Mezzanine debt in Year 5:} \$1{,}647{,}446.77 = \$1{,}000{,}000 \times (1 + 10.50\%)^5 \tag{7}$$

The cash component within the algorithm involves multiple accounts in the income statement and the balance sheet that need to be aggregated or summed for five years. In FIGURE 4, an "aggregated" growth rate is generated for revenue based on the annual revenue growth rates in FIGURE 2.

FIGURE 4
AGGREGATED GROWTH RATE FOR REVENUE

Year 1 growth rate:	1.20000 = (1 + 20%)
Year 2 growth rate:	1.56000 = (1 + 20%) × (1 + 30%)
Year 3 growth rate:	2.18400 = (1 + 20%) × (1 + 30%) × (1 + 40%)
Year 4 growth rate:	2.83920 = (1 + 20%) × (1 + 30%) × (1 + 40%) × (1 + 30%)
Year 5 growth rate:	3.40704 = (1 + 20%) × (1 + 30%) × (1 + 40%) × (1 + 30%) × (1 + 20%)
Aggregated growth rate:	11.19024 = 1.20000 + 1.56000 + 2.18400 + 2.83920 + 3.40704

The aggregated growth rate is the sum of the cumulative growth rates for each year.

If growth is a constant value "g," the aggregated growth rate becomes an annuity due calculation:

$= (1 / g) \times [(1 + g)^N - 1] \times (1 + g)$

The aggregated revenue value is the initial revenue multiplied by the aggregated growth rate for revenue:

$$\$22{,}380{,}480 = \$2{,}000{,}000 \times 11.19024 \tag{8}$$

To find the aggregated revenue less aggregated COGS, multiply equation (8) by the gross profit margin (55% from FIGURE 2):

$$\$12{,}309{,}264 = \$22{,}380{,}480 \times 55\% \tag{9}$$

As noted previously, SGA grows 3% annually. To aggregate SGA over five years requires the following calculation:

$$\begin{aligned}\$1{,}093{,}681.98 = {} & \$200{,}000 \times (1+3\%) + \$200{,}000 \times (1+3\%)^2 + \$200{,}000 \times (1+3\%)^3 + \\ & \$200{,}000 \times (1+3\%)^4 + \$200{,}000 \times (1+3\%)^5\end{aligned} \tag{10}$$

By viewing the equation in reverse, equation (10) can be seen as a future value annuity due.

$$\begin{aligned}\$1{,}093{,}681.98 = {} & \$200{,}000 \times (1+3\%)^5 + \$200{,}000 \times (1+3\%)^4 + \$200{,}000 \times (1+3\%)^3 + \\ & \$200{,}000 \times (1+3\%)^2 + \$200{,}000 \times (1+3\%) = (\$200{,}000/3\%) \\ & \times [(1+3\%)^5 - 1] \times (1+3\%)\end{aligned} \tag{11}$$

Because the future value annuity due will be used extensively, it will be symbolized as: FVA-DUE (rate (k), number of periods (n), and cash flow (CF)) or FVA-DUE (k, n, CF), for example, equation (11) can be symbolized as:

$$\$1{,}093{,}681.98 = FVA - DUE(3\%, 5, \$200{,}000) \tag{12}$$

Incorporating what has already been developed for the algorithm, FIGURE 5 displays the entire algorithm.

FIGURE 5
THE PRIVATE EQUITY (PE) ALGORITHM

Aggregated Income Statement Information		
Aggregated Revenue:	see equation (8)	\$22,380,480.00
Aggregated COGS:	[equation (8)] × (1 – 55%)[a]	\$10,071,216.00
Aggregated SGA:	see equation (10)	\$1,093,681.98
Aggregated Depreciation:	FVA-DUE (5%, 5, \$8,000,000) × 5%[b]	\$2,320,765.13
Aggregated Amortization:	5 × (\$456,944.44 ÷ 5)[c]	\$ 456,944.44
Accumulated Senior debt interest:	[equation (6)] – \$5,000,000	\$2,346,640.38
Accumulated Mezzanine debt interest:	[equation (7)] – \$1,000,000	\$ 647,446.77
Aggregated EAT: Note: with no dividend being paid, EAT is the amount by which Retained earnings increases	[Aggregated Revenue – Aggregated COGS – Aggregated SGA – Aggregated Depreciation – Aggregated Amortization – Accumulated Senior debt interest – Accumulated Mezzanine debt interest] × (1 – Tax rate)	\$3,810,649.77
Balance Sheet Information for Year 5		
Accounts receivable:	[equation (2)] × (55 / 360)[d]	\$1,041,040.00
Inventory:	[equation (2)] × (1 – 55%) × (40 / 360)[e]	\$ 340,704.00
Fixed assets[f]:	\$8,000,000 × $(1 + 5\%)^5$	\$10,210,252.50
Accumulated depreciation:	equals Aggregated depreciation	\$2,320,765.13
Goodwill:	Initial Goodwill – Aggregated Amortization	\$ 0
Year 5 Assets without cash:	Accounts receivable + Inventory + Fixed assets – Accumulated depreciation + Goodwill	\$9,271,231.37
Accounts payable:	[equation (2)] × (1 – 55%) × (25 / 360)[g]	\$ 212,940.00
Senior debt:	see equation (6)	**\$7,346,640.38**
Mezzanine debt:	see equation (7)	**\$1,647,466.77**
Common Stock:	set at initial value	\$3,000,000.00
Retained earnings:	Equals Aggregated EAT	\$3,810,649.77
Year 5 Liabilities and Equity:	Accounts payable + Senior debt + Mezzanine debt + Common stock + Retained earnings	\$16,017,676.86
Cash:	Year 5 Liabilities and Equity – Year 5 Assets without cash	**\$6,746,445.49**
EBITDA in Year 5:	see equation (5)	**\$3,515,889.19**
Values in bold are incorporated into the "waterfall" in Exhibit 3		

All of the following information is provided in Exhibit 1

[a] Gross profit margin is 55%
[b] Depreciation is 5% of fixed assets, fixed assets is initially $8,000,000 and grows 5% annually
[c] Goodwill is initially set at $456,944.44
[d] Days in receivables is 55 per 360-day year
[e] Days in inventory is 40 per 360-day year
[f] Fixed assets is initially $8,000,000 and grows 5% annually
[g] Days in payables is 25 per 360-day year

If necessary, the algorithm can be computed using a handheld calculator, but it can be readily developed in Excel or Google Sheets (see FIGURE 6).

FIGURE 6
PE ALGORITHM AND WATERFALL IN EXCEL
(EXCEL FORMULAS START WITH "=" AND ARE BENEATH ASSOCIATED VALUES)

	A	B	C
1	Initial Revenue:	**$2,000,00.00**	
2	Revenue growth Year 1:	**20%**	
3	Revenue growth Year 2:	**30%**	
4	Revenue growth Year 3:	**40%**	
5	Revenue growth Year 4:	**30%**	
6	Revenue growth Year 5:	**20%**	
7	Gross profit margin:	**55%**	
8	Initial SGA:	**$200,000.00**	
9	SGA annual growth:	**3%**	
10	Initial fixed assets (F/A):	**$8,000,000.00**	
11	Fixed assets annual growth:	**5%**	
12	Depreciation as percentage of F/A:	**5%**	
13	Initial goodwill:	**$456,944.44**	
14	Goodwill amortization periods:	**5**	
15	Senior debt:	**$5,000,000.00**	
16	Senior debt annual interest:	**8.00%**	
17	Mezzanine debt:	**$1,000,000.00**	
18	Mezzanine debt annual interest:	**10.50%**	
19	Mezzanine warrants:	**3.00%**	
20	Annual tax rate:	**30.00%**	
21	Days per year:	**360**	
22	Days in receivables:	**55.00**	
23	Days in inventory:	**40.00**	
24	Days in payables:	**25.00**	
25	Initial common stock:	**$3,000,000.00**	
26			
27	Years since initial sale:	**5**	
28			
29	EXIT Revenue:	$6,814,080.00 = B1*VLOOKUP(B27,A85:C89,2,false)	
30	EXIT COGS:	$3,066,336.00 = B29*(1 – B7)	

31	EXIT SGA:	$231,854.81 = FV(B9,B27,,-B8)	
32	EBITDA (EXIT):	$3,515,889.19 = B29 – B30 – B31	
33	Senior debt with interest:	$7,346,640.38 = FV(B16,B27,,-B15)	
34	Mezzanine debt with interest:	$1,647,446.77 = FV(B18,B27,,-B17)	
35			
36	Aggregated Revenue:	$22,380,480.00 = B1*SUMPRODUCT(B84:B88,C84:C88)	
37	Aggregated COGS:	$10,071,216.00 = B36*(1 – B7)	
38	Aggregated SGA:	$1,093,681.98 = FV(B9,B27,-B8,,1)	
39	Aggregated Depreciation:	$2,320,765.13 = FV(B11,B27,-B10,,1)*B12	
40	Aggregated Amortization:	$456,944.44 = B27*(B13/B14)	
41	Senior debt interest:	$2,346,640,38 = B33 – B15	
42	Mezzanine debt interest:	$647,446.77 = B34 – B17	
43	Aggregated EAT:	$3,810,649.72 = (B36-SUM(B37:B42))*(1 – B20)	
44	Accounts receivable:	$1,041,040.00 = B29*B22/B21	
45	Inventory:	$340,704.00 = B30*B23/B21	
46	Fixed assets:	$10,210,252.50 = FV(B11,B27,,-B10)	
47	Accumulated depreciation	$2,320,765.13 = B39	
48	Goodwill:	$0.00 = B13 – B27*(B13/B14)	
49	Accounts payable:	$212,940.00 = B30*B24/B21	
50	Senior debt:	$7,346,640.38 = B33	
51	Mezzanine debt:	$1,647,466.77 = B34	
52	Common Stock:	$3,000,000.00 = B25	
53	Retained earnings:	$3,810,649.71 = B43	
54	Assets without cash:	$9,271,231.37 = SUM(B44:B46) – B47 + B48	
55	Liabilities and equity:	$16,017,676.86 = SUM(B49:B53)	
56	Cash:	$6,746,445.49	

		= B55 – B54	
57			
58	Exit Proceeds:		
59	EDITDAX:	**10.00**	
60	Percentage of cash received:	**80.00%**	
61	Preferred return:	**15.00%**	
62	Carried Interest:	**20.00%**	
63	Percentage of firm manager equity:	**20.00%**	
64			
65	Selling price of firm:	$35,158,891.85 = B59*B32	
66	Proceeds from cash:	$5,397,156.39 = B60*B56	
67	Total proceeds from sale:	$40,556,048.24 = B65 + B66	
68	Repayment of senior debt:	$7,346,640.38 = B50	
69	Repayment of mezzanine debt:	$1,647,446.77 = B51	
70	Proceeds net of debt:	$31,561,961.09 = B67 – B68 – B69	
71	Warrants:	$946,858.83 = B70*B19	
72	Remaining equity:	$30,615,102.26 = B70 – B71	
73	Firm manager's equity:	$6,123,020.45 = B72*B63	
74	PE Fund equity:	$24,492,081.81 = B72 – B73	
75			
76	Preferred return:	$4,827,257.25 = FV(B61,B27,,-B25*(1 – B63))	
77	Carried interest (Step-up):	$606,814.31 =(B76 – B25*(1 – B63))*B62/(1 – B62)	
78	Carried interest:	$3,811,602.05 = (B74 – B76 – B77)*B62	
79	PE Fund equity net carried interest:	$20,073,665.45 = B74 – B77 – B78	
80			
81	Mezzanine IRR:	21.01% = ((B69 + B71)/B17)^(1/B27) - 1	
82	Firm manager IRR:	59.13% = (B73/(B63*B25))^(1/B27) - 1	
83	PE Fund IRR net carried interest:	52.93% = (B79/(B25*(1 – B63)))^(1/B27) - 1	
84			
85	**1**	1.20000 = 1 + B2	1 = IF(A84<=B27,1,0)

86	**2**	1.56000 = B84*(1 + B3)	1 = IF(A85<=B27,1,0)
87	**3**	2.18400 = B85*(1 + B4)	1 = IF(A86<=B27,1,0)
88	**4**	2.83920 = B86*(1 + B5)	1 = IF(A87<=B27,1,0)
89	**5**	3.40704 = B87*(1 + B6)	1 = IF(A88<=B27,1,0)

Note: Values in bold are user inputs

By changing the “Years since initial sale” value (cell B27 within the spreadsheet) to a value less than “5,” the programmed algorithm will still produce the appropriate analysis. This “flexibility” is programmed at the bottom of the spreadsheet (cells A85 through cells C89) and is referenced using the =VLOOKUP function and the =SUMPRODUCT function in cells B29 and C36 respectively. NOTE: The choice of 5 years between the initial and exit transaction is an arbitrary feature of this model treatment. The programming could easily adapt to allow for a longer planning horizon by incorporating longer-term expected revenue growth rates.

One does not necessarily need to understand how a given Excel function works, but will need to be very careful with the syntax of a given function. For example, duplicate commas, (, ,) within the FV function are part of the function syntax and not a typographical error. For Excel function coding, the interested reader is referred to McFedries (2019) and Alexander, Kusleika, and Walkenbach (2019).

ALGORITHM SCENARIO ANALYSIS AND IMPLIED INPUTS

A set of scenarios that may seem equivalent, but in reality, are not, is changing the sequence of the revenue growth rates. Changing the sequence of the revenue growth rates will not affect the final EBITDA calculation. Still, it will affect the aggregated revenue growth rate (see the calculation in FIGURE 4) and, consequently, cash accumulation and the IRR calculations (see FIGURE 7).

FIGURE 7
REVENUE GROWTH SEQUENCE ANALYSIS

Growth Sequence:	Mezzanine IRR:	Firm Manager IRR:	PE Fund net carried interest IRR:	Carried Interest:	Aggregated Revenue Growth Rate:	Proceeds from Cash:
Year 1: 40% Year 2: 20% Year 3: 30% Year 4: 30% Year 5: 20%	21.06%	59.33%	53.11%	$4,449,009.39	11.51024	$5,594,276.39
Year 1: 20% Year 2: 40% Year 3: 30% Year 4: 30% Year 5: 20%	21.03%	59.21%	53.00%	$4,429,888.75	11.31024	$5,471,076.39
Year 1: 20% **Year 2: 30%** **Year 3: 40%** **Year 4: 30%** **Year 5: 20%**	**21.01%**	**59.13%**	**52.93%**	**$4,418,416.36**	**11.19024**	**$5,397,156.39**
Year 1: 20% Year 2: 30% Year 3: 30% Year 4: 40% Year 5: 20%	20.98%	59.04%	52.84%	$4,403,502.26	11.03424	$5,301,060.39
Year 1: 20% Year 2: 30% Year 3: 30% Year 4: 20% Year 5: 40%	20.91%	58.78%	52.60%	$4,364,725.60	10.62684	$5,051,210.79

Bold values are the “base case” performed in the previous section

FIGURE 7 demonstrates that higher growth earlier in the firm is preferred because it allows cash to accumulate to greater levels, even when the growth over the five year period is unchanged. This is demonstrated within the algorithm by observing the aggregated revenue growth rate calculation (see FIGURE 4), which calculates cash at a specific point in time. A pro forma analysis cannot provide this direct of a connection between the revenue growth rate and cash.

Like a pro forma analysis, scenarios, where the EBITDAX and the timing of the future sale of the firm (i.e. “exit”) vary, can be produced using the algorithm (see FIGURE 8)

FIGURE 8
EBITDAX AND TIME OF FUTURE SALE (EXIT)

EBITDAX:	Exit Year:	Mezzanine IRR:	Firm Manager IRR:	PE Fund net carried interest IRR:
9.0	3	21.15%	66.60%	57.40%
	4	20.87%	62.28%	54.84%
	5	20.00%	55.42%	49.45%
9.5	3	21.89%	70.74%	61.11%
	4	21.48%	64.95%	57.30%
	5	20.51%	57.32%	51.23%
10.0	3	22.62%	74.68%	64.66%
	4	22.08%	67.50%	59.66%
	5	**21.01%**	**59.13%**	**52.93%**
10.5	3	23.34%	78.46%	68.06%
	4	22.68%	69.94%	61.91%
	5	21.49%	60.87%	54.55%
11.0	3	24.06%	82.08%	71.33%
	4	23.26%	72.28%	64.08%
	5	21.97%	62.53%	56.12%

Bold values are the "base case" performed in the previous section

This is a very common scenario analysis in which there is a trade-off between selling the firm earlier than expected or later at a higher EBITDAX. Clearly, if the EBITDAX is set at 10.0, exiting earlier will produce the best IRRs.

However, what is a more likely scenario is selling earlier, say in Year 4, at a lower EBITDAX than in the base case scenario of EBITDAX = 10.0 in Year 5. If the EBITDAX at Year 4 is 9.0 or 9.5, the firm managers and PE Fund investors benefit with higher IRRs than when compared to the base case. However, the mezzanine investors only benefit when the EBITDAX in Year 4 is 9.5.

Within a spreadsheet environment, the scenario analysis in Exhibit 8 can be performed equivalently using either the algorithm or a pro forma analysis. However, a pro forma analysis cannot transfer as easily as the algorithm outside of a spreadsheet environment, such as using a programming language. Although the algorithm is programmed in Excel in this presentation, it is not limited to only being programmed in Excel.

A single input that produces a certain outcome can be determined algebraically by working backward through the algorithm. Suppose the PE Fund has a target IRR, say 32.50% APR and the goal is to determine an initial annual revenue growth rate assuming the annual revenue growth rate for Years 2 through 5 is 15%.

Working backward through the Year 5 Waterfall calculations in Exhibit 3, sets a target "Total proceeds from the sale" of $24,007,323.54. This value combines the Year 5 EBITDA multiplied by 10.0 and the "Proceeds from cash." The Year 5 EBITDA solution comes directly from equations (1) through (4) with "g" denoting the initial annual revenue growth rate:

$$
\begin{aligned}
&= \$2{,}000{,}000 \times (1+g) \times (1+15\%)^4 \times 55\% - \$200{,}000 \times (1+\ 3\%)^5 \\
&= \ \$1{,}923{,}906.875 \times (1+g) - \$231{,}854.815 \qquad (13)
\end{aligned}
$$

Following FIGURE 4, the aggregated revenue growth rate becomes:

$$(1+g) \times \{1 + (1/15\%) \times [(1+15\%)4 - 1] \times (1+15\%)\} = (1+g) \times 6.742381 \qquad (14)$$

The algebra is messy, but certainly possible to find the “Proceeds from cash” as equation (14) multiplied by 473,903.84154:

$$\text{Proceeds from cash} = (1+g) \times 6.742381 \times 473{,}903.84154 \tag{15}$$

Set the target “Total proceeds from sale” to the sum of equations (13) multiplied by 10.0 and (15).

$$\$24{,}007{,}323.54 = [\$1{,}923{,}906.875 \times (1+g) - \$231{,}854.815] \times 10.0 + (1+g) \times 6.742381 \times 473{,}903.84154 \tag{16}$$

Simplifying to find “g”:

$$\frac{\$26{,}325{,}871.69}{(\$19{,}239{,}068.75 + 6.742381 \times 473{,}903.84154)} - 1 = 17.346\% = g \tag{17}$$

The benefit of an algebraic solution is that first and second derivatives can be solved or approximated relative to other inputs if desired. For example, if EBITDAX is inserted as a variable instead of 10.0, equation (17) becomes:

$$\frac{\$24{,}007{,}323.54 + \$231{,}854.815 \times \text{EBITDAX}}{(\$1{,}923{,}906.875 \times \text{EBITDAX} + 6.742381 \times 473{,}903.84154)} - 1 = g \tag{18}$$

The first derivative of g relative to EBITDAX can be calculated given the PE Fund target IRR of 32.50%. Computing these sensitivities to particular inputs can be used to focus performance efforts or to understand the potential consequences of future circumstances that may occur a private equity investment. Better understanding the latter situation will help to mitigate risk.

CONCLUSION

The private equity algorithm is demonstrated to be readily programmable in Excel; however, it is also readily programmable using other formats. As demonstrated in the previous section, the algorithm can illustrate issues that cannot be easily detected using a spreadsheet pro forma analysis and can provide a solution for a single input that creates a model that allows for calculating derivatives to determine sensitivities between inputs. Further, the algorithm can provide a validity check for a spreadsheet pro forma analysis.

The practitioner can use the algorithm to set performance goals and determine risk sources. Implementing Monte Carlo analysis to select variables is easy to implement within the algorithm in a spreadsheet or non-spreadsheet setting. However, one should be careful not to “over-randomize” the inputs. Some inputs are determined contractually (senior debt, mezzanine debt, and common stock). Some inputs are determined directly or indirectly by revenue (COGS, accounts receivable, inventory, and accounts payable). The remaining inputs grow at associated rates (SGA and fixed assets), but should be consistent with the level of revenue.

Consequently, “revenue” (or revenue growth) is a good choice to have as a random variable within a Monte Carlo simulation. COGS and fixed asset growth can also be randomized, but should be positively correlated with the revenue random variable. There is little benefit to having other inputs randomized.

From an empirical standpoint, the ability of the algorithm to provide a model for a particular input may be beneficial for the analysis of multiple private equity transactions. For a large empirical study, it may be best to simplify the algorithm by using an industry level EBITDA margin and other industry-level measures for IRRs. The transactions can then be analyzed for what level of growth was necessary to realize these industry standards and if there was actually capacity within the market to achieve such growth. Other possibilities exist to investigate other parameters within the algorithm, such as, EBITDAX. Consequently, the algorithm benefits the practitioner examining a single transaction and a researcher examining the results of many transactions.

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