Optimal Noise in Asset-Price Manipulation

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I study the optimal behavior of an insider who can manipulate asset prices by releasing private information to uninformed investors. The previous literature argues that without restrictive assumptions, information-based manipulation is not sustainable in the long run. I show that, by allowing the signal space to be continuous, long-run manipulation can easily exist under general assumptions. If the uninformed investors are boundedly rational, the insider has an even greater ability to manipulate prices.

Keywords: asset pricing, asset-price manipulation, asymmetric information

INTRODUCTION

In this paper, I use a heterogeneous-agent dynamic model to study the optimal behavior of an insider who can manipulate the price of a risky asset. The insider is assumed to have private information about a future dividend shock and releases a noisy signal to uninformed investors. Uninformed investors choose whether to use the signal or not. In the long run, it turns out that the insider optimally chooses the variance of the noise to distort the asset price and earn extra profit. In addition, I analyze the optimal behavior of the insider when uninformed investors are boundedly rational. With bounded rationality, beliefs of the uninformed investors can temporarily deviate from rational expectations. This strengthens the manipulation ability of the insider.

This paper contributes to the existing literature about information-based manipulation in several aspects. Firstly, it shows that information-based manipulation is sustainable in the long run even without any restrictive assumptions. This finding contradicts Benabou and Laroque (1992) and Bommel (2003) who suggest that manipulation is not sustainable in the long run, since uninformed investors gradually learn that the signal sender is a manipulator[1]. This is because the current paper relaxes the assumption of discrete manipulation strategies in Benabou and Laroque (1992) and Bommel (2003), e.g., sending a bad signal when the prospect is good. In the current paper, the signals are continuous. For example, if the dividend shock of the next period is 0.6, the insider can send a signal of 0.7 or -1.1 by adding a noise of 0.1 or -1.7, respectively. This noisy signal releases partial information which is beneficial to the uninformed investors, but also provides extra manipulation profit for the insider by distorting the asset price. This mutual benefit makes manipulation sustainable in the long run. Secondly, the current paper shows that even if the investment horizon is short, the insider can earn a large manipulation profit. This is because continuous signals allow the insider to distort the price heavily in one period. This conclusion contrasts with Schmidt (2020) who reports that the insider manipulates only if the investment horizon is long. Thirdly, the conclusion of this paper is also different from Eren and Ozsoylev (2006) who show that manipulation is possible only if there is at least one boundedly rational investor. The current paper suggests that manipulation is sustainable because of asymmetric information. Therefore, it is plausible even under a rational expectation equilibrium.

In the model of this paper, there is one insider who possesses private information about the dividend shock in the next period. Following Glosten and Milgram (1985) this private information includes insider information and the superior knowledge, skills, and technology to process publicly available information. Since this paper analyzes information-based manipulation, I assume that the market power of the insider is extremely small in the sense that his or her trading has no price impact on the asset. The insider sends a signal about the dividend shock in the next period to uninformed investors. Uninformed investors maximize a mean-variance utility function and choose whether to be Type 1, who use the signal, or Type 2, who do not use the signal.

I find that the insider has an incentive to add noise to the signal to earn extra profit. One example is that when the dividend shock in the next period is positive, the insider knows that the asset price in the next period will be \$10. In the current period, the insider may buy the asset at the current price, say \$9. Then he or she sends a signal with a positive noise, i.e., exaggerating the dividend shock, to mislead the uninformed investors to bid up the price to \$11. At \$11, the insider can sell the asset to realize a profit. He or she can then short sell the asset at this price. In the next period, the asset price will decrease to the fundamental value \$10 when the actual dividend shock is revealed. The insider can earn another profit by buying the asset to cover the short position. The profits from these two trading rounds are higher than the profit without manipulation. It is because if the insider does not manipulate, he can only buy the asset at \$9 in the current period and sell the asset at \$10 in the next period. Since the insider randomizes the noise, in some situations, there is no extra profit. However, in all situations, the manipulation profit will never be less than the profit without manipulation.

There are two factors restricting the variance of the noise. Firstly, under a rational expectation equilibrium, as the noise becomes larger, Type 1 investors will have weaker beliefs and become less responsive to the signal. Secondly, if the signal is too noisy, it will decrease the profit of Type 1 investors relative to Type 2 investors. Thus, investors will shift away from Type 1 to Type 2 if the expected utility of Type 1 is lower than the expected utility of Type 2. Both factors can decrease the price impact of the signal and the manipulation ability of the insider. However, this paper shows that under the assumed parameter values, the utility of Type 1 investors is always greater than the utility of Type 2 investors. Therefore, it is the first factor makes the insider choose an optimal variance of the noise.

I also study the optimal noise of the signal when uninformed investors are boundedly rational. Specifically, they need to gradually learn the correlation coefficient between the signal and the dividend. Under this assumption, beliefs of investors can deviate from rational expectations temporarily. Thus, the insider tends to add a large noise to the signal in one period. The first reason is that when the insider makes the signal very noisy in the current period, the estimated coefficient in the beliefs does not change immediately, though they will change substantially in the following periods. This increases the manipulation ability of the insider compared to the assumption of rational expectations. In a rational expectation equilibrium, the correlation coefficient in the beliefs of Type 1 investors will change immediately when the signal is noisier. The second reason is that small noises have lower probabilities to make the asset price undervalued or overvalued. For example, if the dividend shock is 1, the noise needs to be lower than -1 to change the sign of the signal and make the asset price undervalued. This finding corresponds to the literature of benchmark manipulation (Carhart et al, 2002; Ni et al., 2005; and Ben-David et al., 2013) which suggests that manipulations tend to cluster around some benchmark periods, such as settlement dates of contracts or reporting dates of hedge funds.

The model of this paper draws new conclusions by using a different assumption of the manipulation strategy. Benabou and Laroque (1992) document that an opportunistic insider can manipulate the asset price only if he or she may exogenously change the type to be an honest insider, otherwise his or her reputation will vanish in the long run. Benabou and Laroque assume that the insider can only send binary signals, good or bad prospect, to uninformed investors. In the model of the current paper, I allow the insider to send continuous signals. As a result, the insider can release useful information and misleading noise simultaneously. Even if uninformed investors know the signal is not precise, the manipulation ability of the insider does not vanish because the signal still provides useful information. Moreover, if the insider only sends binary signals, the price distortion is limited. With continuous signal, the insider can add a great noise to the signal in one single period to earn a large manipulation profit.

Bommel (2003) suggests that a manipulation equilibrium is not sustainable if the insider only uses a pure strategy by sending a false signal, i.e., sending a buy signal when the liquidation value is negative. A signal transmits information only in a honest strategy, i.e., sending a buy signal when the liquidation value is positive, and in a bluff strategy, sending a signal without knowing any private information. Bommel also considers imprecise information with a restrictive format such as “the liquidation value is higher than 0.8165 or within -0.8165 and 0”. By contrast, the insider in the current paper can persistently send noisy but not entirely misleading signals to uninformed investors. This indirectly relaxes the restrictive assumption of using pure strategies because the continuous signal space in the current paper includes all pure strategies in Bommel (2003). The format of signals in this paper is also more practical compared to a signal with two unconnected ranges.

Eren and Ozsoylev (2006) report that if there is at least one naive trader who takes the face value of the signal, while others are sophisticated in the sense that they can infer the true liquidation value from the signal, profitable information-based manipulation is possible. In the current paper, even investors have rational expectations about the correlation between the noisy signal and the actual dividend, manipulation exists. Therefore, it is asymmetric information but not boundedly rationality allows manipulation.

Schmidt (2020) shows that with a binary signal, a short investment horizon limits the incentive of manipulation. However, I show that if the signal is continuous, this result does not hold. The reason is that under binary signals, the insider can increase the probability of manipulation but not the magnitude of distortion. When manipulation is more often, the beliefs of investors become weaker. In the model of the current paper, when the insider increases the magnitude of the noise, beliefs of investors will shrink, but the information distortion is larger. Therefore, a shorter investment horizon does not drive out manipulation.

Empirical studies, such as Frieder and Zittrain (2007), Hanke and Hauser (2008), Chen et al. (2014) and Bartov et al. (2018), indirectly show that information-based manipulation is possible. Frieder and Zittrain (2007) find that a manipulator can send spam e-mails to mislead the market. If the manipulator buys shares before sending spam-emails and reverses the holding in the next day, he or she makes a return of 4.9%. Hanke and Hauser (2008) show that spam e-mails are followed by unusual returns, volatility, bid-ask spread, and trading volume. Chen et al. (2014) and Bartov et al. (2018) document that future earnings and stock returns are correlated with forecasts on social media websites.

This paper proceeds as follows. The next section introduces and develops the model. The optimal variance section analyzes the optimal variance of the noise chosen by the insider. The sensitivity analyses section presents numerical analyses with different parameter values. The bounded rationality section discusses implications under different assumptions. The last section concludes.

AN ASSET PRICING MODEL WITH MANIPULATION

I follow the mean-variance linear asset pricing model of DeLong et al. (1990) and Branch and Evans (2010). Two types of investors j = 1, 2 with population $0 \leq n_1 \leq 1$ and $n_2 = 1 - n_1$ respectively, maximize the following utility function,

$$\max_{z_{j,t}} E_t^j W_{j,t+1} - \frac{a}{2} E_t^j Var_{j,t} W_{j,t+1}, \tag{1}$$

subject to,

$$W_{j,t+1} = W_{j,t} + (p_{t+1} + y_{t+1} - R p_t) z_{j,t}.$$

$W_{j,t}$ denotes the wealth of a Type j investor at period t, $R > 1$ represents the rate of return of a risk-free asset, p_t and y_t are the price and dividend of a risky asset at period t, respectively, $z_{j,t}$ indicates the holding of the risky asset of a Type *j* investor at period *t*. It is negative if the investor takes a short position. $E_{j,t}$ is the conditional expectation operator based on the information of a Type *j* investor at period *t* which may not be rational. $a > 0$ is the risk averse coefficient. I assume that the dividend of the risky asset follows an AR(1) process,

$$y_t = \rho y_{t-1} + e_t \tag{2}$$

where $0 < \rho < 1$ is the auto-correlation coefficient and e_t is a normally distributed i.i.d. noise with a mean of zero and a variance of σ_e^2. Rearranging the maximization problem, it becomes,

$$\max_{z_{j,t}} RW_{j,t} + E_t^j(p_{t+1} + y_{t+1} - Rp_t)z_{j,t} - \frac{a}{2}\sigma_{j,t}^2 z_{j,t}^2, \tag{3}$$

where $\sigma_{j,t}^2 = Var\left(p_{t+1} + y_{t+1} - E_t^j p_{t+1} - E_t^j y_{t+1}\right)$ is the perceived variance of the excess returns of a Type *j* investor. The first-order condition gives the optimal holding of a Type *j* investor,

$$z_{j,t} = \frac{1}{a\sigma_{j,t}^2} E_t^j(p_{t+1} + y_{t+1} - Rp_t). \tag{4}$$

Let $z_{s,t}$ denote the aggregate holding of noisy traders who trade the risky asset for liquidity reasons. The aggregate holding of noisy traders $z_{s,t}$ is i.i.d. normally distributed with a mean of zero and a variance o fσ_z^2. When the demand and supply equal, the market equilibrium implies,

$$n_1 z_{1,t} + n_2 z_{2,t} = z_{s,t}. \tag{5}$$

Substituting equation (4) into (5), the equilibrium asset price at period *t* is,

$$p_t = \left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{1}{R}\left[\frac{n_1}{a\sigma_{1,t}^2}(E_t^1 p_{t+1} + E_t^1 y_{t+1}) + \frac{n_2}{a\sigma_{2,t}^2}(E_t^2 p_{t+1} + E_t^2 y_{t+1}) - z_{s,t}\right]. \tag{6}$$

At period *t*, an insider sends a signal $\tilde{e}_{t+1} = e_{t+1} + \xi_{t+1}$ about the dividend shock of next period, i.e., e_{t+1}, to the market. ξ_{t+1} is i.i.d. normally distributed with a mean of zero and a variance of σ_ξ^2. ξ_{t+1} is assumed to be independent of e_{t+1}. Investors choose whether to be Type 1, who use the signal, or Type 2, who do not use the signal. The dividend at period *t*, i.e., y_t, is public information. Therefore, Type 1 investors use both $\tilde{e}_{t+1}$ and y_t to forecast y_{t+1} and p_{t+1}. The beliefs of Type 1 investors about the dynamics of the asset price and the dividend are,

$$p_t = \theta_{11} y_t + \Theta_1 \tilde{e}_{t+1} + \eta_{1,t}^p, \tag{7}$$

$$y_t = \rho_1 y_{t-1} + \gamma_1 \tilde{e}_t + \eta_{1,t}^y, \tag{8}$$

where $\eta_{1,t}^p$ and $\eta_{1,t}^y$ are residuals of the price and the dividend equations, respectively, of Type 1 investors. Substituting equation (8) into (7) and shifting one period forward, the expectations of Type 1 investors are,

$$\begin{aligned} E_t^1 p_{t+1} &= \theta_{11} E_t^1 y_{t+1} + \Theta_1 E_t^1 \tilde{e}_{t+2} + E_t^1 \eta_{1,t+1}^p \\ &= \theta_{11}\rho_1 y_t + \theta_{11}\gamma_1 \tilde{e}_{t+1}, \end{aligned} \tag{9}$$

$$\begin{aligned} E_t^1 y_{t+1} &= \rho_1 y_t + \gamma_1 \tilde{e}_{t+1} + E_t^1 \eta_{1,t+1}^y \\ &= \rho_1 y_t + \gamma_1 \tilde{e}_{t+1}. \end{aligned} \tag{10}$$

The second lines of (9) and (10) are because $E_t^1 \tilde{e}_{t+2}$, $E_t^1 \eta_{1,t+1}^p$, and $E_t^1 \eta_{1,t+1}^y$ are zero. Type 2 investors do not use the signal. They forecast y_{t+1} and p_{t+1} by using only the public information y_t. Therefore, the beliefs of Type 2 investors about the dynamics of the asset price and the dividend are,

$$p_t = \theta_{12} y_t + \eta_{2,t}^p, \tag{11}$$

$$y_t = \rho_2 y_{t-1} + \eta_{2,t}^y, \tag{12}$$

where $\eta_{2,t}^p$ and $\eta_{2,t}^y$ are the residuals of the price and dividend equations, respectively, of Type 2 investors. Substituting equation (12) into (11) and shifting one period forward, the expectations of Type 2 investors are,

$$\begin{aligned} E_t^2 p_{t+1} &= \theta_{12} E_t^2 y_{t+1} + E_t^2 \eta_{2,t+1}^p \\ &= \theta_{12} \rho_2 y_t, \end{aligned} \tag{13}$$

$$\begin{aligned} E_t^2 y_{t+1} &= \rho_2 y_t + E_t^2 \eta_{2,t+1}^y \\ &= \rho_2 y_t. \end{aligned} \tag{14}$$

The second lines of (13) and (14) are because both $E_t^2 \eta_{2,t+1}^p$ and $E_t^2 \eta_{2,t+1}^y$ are zero. Substituting (9), (10), (13), and (14) into (6) gives the equilibrium dynamic of the asset price,

$$\begin{aligned} p_t = &\left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{1}{R}\left[\frac{n_1}{a\sigma_{1,t}^2}(\theta_{11}\rho_1 y_t + \theta_{11}\gamma_1 \tilde{e}_{t+1} + \rho_1 y_t + \gamma_1 \tilde{e}_{t+1}) \right. \\ &\left. + \frac{n_2}{a\sigma_{2,t}^2}(\theta_{12}\rho_2 y_t + \rho_2 y_t) - z_{s,t}\right] \\ = &\ \theta_1 y_t + \Theta \tilde{e}_{t+1} - \psi z_{s,t} \end{aligned} \tag{15}$$

where,

$$\Theta = \frac{1}{R}\left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{n_1}{a\sigma_{1,t}^2}\gamma_1(\theta_1 + 1)$$

and,

$$\psi = \frac{1}{R}\left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}$$

The exogenous dividend process guarantees that (10) and (14) are rational expectations with $\rho_1 = \rho_2 = \rho$. I use the conjecture of $\theta_{11} = \theta_{12}$ in the last line of (15). Since y_t is uncorrelated with $\tilde{e}_{t+1}$, the solutions of θ_{11} in (7) and θ_{12} in (11) under the rational expectation equilibrium can be solved by the method of undetermined coefficients. Let $\theta_1 = \theta_{11} = \theta_{12}$, it can be shown that,

$$\theta_1 = \frac{1}{R}(\theta_1 \rho + \rho)$$

$$\theta_1 = \frac{\rho}{R - \rho}$$

Under the rational expectation equilibrium Θ_1 in (7) equals Θ in (15). From (15), the equilibrium asset price of one period ahead is,

$$p_{t+1} = \theta_1 y_{t+1} + \Theta \tilde{e}_{t+2} - \psi z_{s,t+1} \tag{16}$$

Moreover, comparing (16) with (9) and (13), the expectations on the asset price of Type 1 and Type 2 investors are both rational according to their different information sets. Using (2) and (16), the actual excess returns of both types of investors for each share of the risky asset is,

$$\begin{aligned} p_{t+1} + y_{t+1} - Rp_t &= \left(\theta_1 y_{t+1} + \Theta \tilde{e}_{t+2} - \psi z_{s,t+1}\right) + y_{t+1} \\ &\quad -(\theta_1 \rho + \rho) y_t - R\Theta \tilde{e}_{t+1} + R\psi z_{s,t} \\ &= (\theta_1 \rho + \rho) y_t + (\theta_1 + 1) e_{t+1} + \Theta \tilde{e}_{t+2} - \psi z_{s,t+1} \\ &\quad -(\theta_1 \rho + \rho) y_t - R\Theta \tilde{e}_{t+1} + R\psi z_{s,t} \\ &= (\theta_1 + 1) e_{t+1} - R\Theta \tilde{e}_{t+1} + \Theta \tilde{e}_{t+2} - \psi z_{s,t+1} + R\psi z_{s,t} \\ &= [(\theta_1 + 1) - R\Theta] e_{t+1} - R\Theta \xi_{t+1} + \Theta \tilde{e}_{t+2} - \psi z_{s,t+1} + R\psi z_{s,t} \end{aligned} \tag{17}$$

The actual excess returns of each share depend on three components. The first component is e_{t+1}, which is the realized shock of the dividend at period *t + 1*. The second component is ξ_{t+1}, which is the noise of the signal added by the insider. The third component is $\Theta \tilde{e}_{t+2} - \psi z_{s,t+1} + R\psi z_{s,t}$, which is the combination of the information on the dividend shock at period *t + 2* and the supply shocks of period t and *t + 1*.

Since investors can take long or short positions, we need to know the actual holdings of different types of investors in order to calculate their actual profits. The holding of a Type 1 investor is,

$$\begin{aligned} z_{1,t} &= \frac{1}{a\sigma_{1,t}^2} E_t^1 (p_{t+1} + y_{t+1} - Rp_t) \\ &= \frac{1}{a\sigma_{1,t}^2} \left[(\theta_1 \rho + \rho) y_t + \gamma_1 (\theta_1 + 1) \tilde{e}_{t+1} - (\theta_1 \rho + \rho) y_t - R\Theta \tilde{e}_{t+1} + R\psi z_{s,t}\right] \\ &= \frac{1}{a\sigma_{1,t}^2} \left[(\gamma_1 (\theta_1 + 1) - R\Theta) \tilde{e}_{t+1} + R\psi z_{s,t}\right] \end{aligned} \tag{18}$$

The holding of a Type 2 investor is,

$$\begin{aligned} z_{2,t} &= \frac{1}{a\sigma_{2,t}^2} E_t^2 (p_{t+1} + y_{t+1} - Rp_t) \\ &= \frac{1}{a\sigma_{2,t}^2} \left[(\theta_1 \rho + \rho) y_t - (\theta_1 \rho + \rho) y_t - R\Theta \tilde{e}_{t+1} + R\psi z_{s,t}\right] \\ &= \frac{1}{a\sigma_{2,t}^2} \left[-R\Theta \tilde{e}_{t+1} + R\psi z_{s,t}\right] \end{aligned} \tag{19}$$

Note that the holding of a Type 1 investor is increasing in the signal $\tilde{e}_{t+1}$ due to the expectation of higher dividends in the following periods, but the opposite is true for a Type 2 investor since they are uninformed about $\tilde{e}_{t+1}$. Type 2 investors will lower their holdings when $\tilde{e}_{t+1}$ is positive because they believe that the current price is too high and the expected returns are too low.

Combining (17) and (18) the expected profit of a Type 1 investor is,

$$\begin{aligned} &E\left[(p_{t+1} + y_{t+1} - Rp_t) z_{1,t}\right] \\ &= \frac{1}{a\sigma_{1,t}^2} \left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{n_2}{a\sigma_{2,t}^2} \left[1 - \left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{n_1}{a\sigma_{1,t}^2} \gamma_1\right] \gamma_1 (\theta_1 + 1)^2 E(e_{t+1}^2) \\ &\quad - \frac{1}{a\sigma_{1,t}^2} \left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{n_2}{a\sigma_{2,t}^2} \left(\frac{n_1}{a\sigma_{1,t}^2} + \frac{n_2}{a\sigma_{2,t}^2}\right)^{-1} \frac{n_1}{a\sigma_{1,t}^2} \gamma_1^2 (\theta_1 + 1)^2 E(\xi_{t+1}^2) \end{aligned} \tag{20}$$

$$+\frac{1}{a\sigma_{1,t}^2}\left(\frac{n_1}{a\sigma_{1,t}^2}+\frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}E\left(z_{s,t}^2\right)$$

Holding other endogenous parameters constant, the expected profit of a Type 1 investor is positively related to $E(e_{t+1}^2)$, which is the variance of the realized dividend shock at period $t+1$, but negatively related to the variance of the noise $E(\xi_{t+1}^2)$. The intuition is that Type 1 investors are informed about e_{t+1} through the signal $\tilde{e}_{t+1}$ and utilize this information to make profit. However, at the same time, Type 1 investors suffer from the noise of the signal by establishing sub-optimal holdings.

Combining (17) and (19) the expected profit of a Type 2 investor is,

$$\begin{aligned} &E\left[(p_{t+1}+y_{t+1}-Rp_t)z_{1,t}\right] \\ &=-\frac{1}{a\sigma_{2,t}^2}\left(\frac{n_1}{a\sigma_{1,t}^2}+\frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}\frac{n_1}{a\sigma_{1,t}^2}\left[1-\left(\frac{n_1}{a\sigma_{1,t}^2}+\frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}\frac{n_1}{a\sigma_{1,t}^2}\gamma_1\right]\gamma_1(\theta_1+1)^2E(e_{t+1}^2) \\ &\quad+\frac{1}{a\sigma_{2,t}^2}\left(\frac{n_1}{a\sigma_{1,t}^2}+\frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}\left(\frac{n_1}{a\sigma_{1,t}^2}\right)^2\gamma_1^2(\theta_1+1)^2E(\xi_{t+1}^2) \\ &\quad+\frac{1}{a\sigma_{2,t}^2}\left(\frac{n_1}{a\sigma_{1,t}^2}+\frac{n_2}{a\sigma_{2,t}^2}\right)^{-1}E\left(z_{s,t}^2\right) \end{aligned} \tag{21}$$

Type 2 investors suffer from the inferior information relative to Type 1 investors but benefit from the noise used by Type 1 investors. Holding other endogenous parameters constant, the expected profit of Type 2 investors is decreasing in the variance of the realized dividend shock $E(e_{t+1}^2)$ and increasing in the variance of the noise $E(\xi_{t+1}^2)$. Actually, profits related to the signal and the noise are a zero-sum game between the two types of investors.

The perceived variance of the excess returns of a Type 1 investor for each share is,

$$\begin{aligned} \sigma_{1,t}^2 &= E(p_{t+1}+y_{t+1}-E_t^1p_{t+1}-E_t^1y_{t+1})^2 \\ &=\left[(\theta_1\rho+\rho)y_t+(\theta_1+1)e_{t+1}+\Theta\tilde{e}_{t+2}-\psi z_{s,t+1}-(\theta_1\rho+\rho)y_t-\gamma_1(\theta_1+1)\tilde{e}_{t+1}\right]^2 \\ &=(1-\gamma_1)^2(\theta_1+1)^2E(e_{t+1}^2)+\gamma_1{}^2(\theta_1+1)^2E(\xi_{t+1}^2)+\Theta^2E(\tilde{e}_{t+2}^2)+\psi^2E\left(z_{s,t+1}^2\right) \end{aligned} \tag{22}$$

The perceived variance of the excess returns of a Type 2 investor for each share is,

$$\begin{aligned} \sigma_{2,t}^2 &= E(p_{t+1}+y_{t+1}-E_t^2p_{t+1}-E_t^2y_{t+1})^2 \\ &=E\left[(\theta_1\rho+\rho)y_t+(\theta_1+1)e_{t+1}+\Theta\tilde{e}_{t+2}-\psi z_{s,t+1}-(\theta_1\rho+\rho)y_t\right]^2 \\ &=(\theta_1+1)^2E(e_{t+1}^2)+\Theta^2E(\tilde{e}_{t+2}^2)+\psi^2E\left(z_{s,t+1}^2\right) \end{aligned} \tag{23}$$

OPTIMAL VARIANCE OF NOISE WITH FIXED n_1 AND n_2

Optimal Variance of Noise Chosen by the Insider

The first factor restricting the noise variance of the signal sent by the insider is that when the signal is noisier, Type 1 investors will assign a smaller γ_1 in (8). This will lower the price impact of the signal because Θ in (15) becomes smaller. Although a larger noise can distort the price more, a lower price impact decreases the manipulation ability.

To analyze the profit of the insider from manipulation, I decompose the price of the risky asset from period t to $t+1$ into three stages. From (15), at the first stage of period t, before noisy traders trade and the insider releases the signal, only y_t is announced, the price of the risky asset is,

$$p_t^0=\theta_1y_t-\psi z_{s,t-1}$$

At the second stage of period t, the noisy traders trade and the equilibrium price becomes,

$$p_t^1 = \theta_1 y_t - \psi z_{s,t}$$

At the third stage of period t, the insider releases the signal to the market, the equilibrium price becomes,

$$p_t^2 = \theta_1 y_t + \Theta \tilde{e}_{t+1} - \psi z_{s,t}$$

Without loss of generality, this paper assumes that the trading capacity of the insider is one share, and the insider trades after the noisy traders. The profit of the insider from stage 2 to stage 3 of period *t* is,

$$\begin{aligned}\pi_t^1 &= (p_t^2 - p_t^1) I(p_t^2 - p_t^1 \geq 0) \\ &= |\Theta \tilde{e}_{t+1}|\end{aligned}$$

where $I(p_t^2 - p_t^1 \geq 0) = 1$ if $p_t^2 - p_t^1 \geq 0$ and $I(p_t^2 - p_t^1 \geq 0) = -1$ otherwise.

The expected profit of the insider from stage 3 of period t to stage 1 of period *t + 1* is,

$$\begin{aligned}\pi_{t+1}^0 &= E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) I(E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) \geq 0) \\ &= \left|(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1}) - (1 - R)\psi z_{s,t}\right|\end{aligned}$$

where E_{It} is the conditional expectation operator of the insider based on his or her information at period *t*. The indicator function $I(E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) \geq 0) = 1$ if $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) \geq 0$ and $I(E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) \geq 0) = -1$ otherwise. The trading position of noisy traders $z_{s,t}$ has impacts on the expected profit and the trading position of the insider. However, assuming $z_{s,t} = 0$ can focus on the impacts of the signal and the noise. With this assumption, the expected profit of the insider from stage 3 of period t to stage 1 of period *t + 1* is,

$$\pi_{t+1}^0 = |(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1})|$$

The total expected profit of the insider from the second stage of period *t* to the first stage of period *t + 1* is $\Pi_{t|t+1} = \pi_{t+1}^0 + R\pi_t^1$.

As a benchmark for comparison, suppose the insider does not add any noise to the signal, i.e., $\sigma_\xi^2 = 0$, it can be shown that the expected manipulation profit from period t to *t + 1* is,

$$\begin{aligned}\Pi_{t|t+1} &= \pi_{t+1}^0 + R\pi_t^1 \\ &= |(\theta_1 + 1)e_{t+1}|\end{aligned}$$

If the insider adds a noise to the signal, i.e., $\sigma_\xi^2 > 0$, several scenarios need to be considered. The first scenario is when e_{t+1} and $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2)$ have the same sign, i.e., the insider exaggerates the dividend shock but that is not enough to make the price overvalued. This may be because the proportion of Type 1 investors n_1 is small, or Type 1 investors believe that the signal is not reliable, i.e., γ_1 is small. In this situation, the insider will keep his or her holding position unchanged between the second stage of period *t* and the first stage of period *t + 1*. Without loss of generality, suppose $e_{t+1} > 0$, $\xi_{t+1} > 0$, and $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) > 0$. This implies that the expected manipulation profit from period *t* to *t + 1* is,

$$\begin{aligned}\Pi_{t|t+1}^1 &= [(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1})](1) + R\Theta(e_{t+1} + \xi_{t+1})(1) \\ &= (\theta_1 + 1)e_{t+1}\end{aligned}$$

which is the same as the profit when the insider does not manipulate.

The second scenario is when e_{t+1} and ξ_{t+1} have the same sign, but $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2)$ and e_{t+1} have opposite signs, i.e., the insider exaggerates the dividend shock and it is enough to make the asset overvalued or undervalued. Therefore, the insider will have reversed holding positions between the second stage of period *t* and the first stage of period *t + 1*. Without loss of generality, suppose $e_{t+1} > 0$, $\xi_{t+1} > 0$, and $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) < 0$, the expected profit of the insider from manipulation is,

$$\begin{aligned}\Pi_{t|t+1}^2 &= [(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1})](-1) + R\Theta(e_{t+1} + \xi_{t+1})(1) \\ &= R\Theta(e_{t+1} + \xi_{t+1}) - (\theta_1 + 1)e_{t+1} + R\Theta(e_{t+1} + \xi_{t+1}) \\ &> (\theta_1 + 1)e_{t+1}\end{aligned}$$

The last inequality is due to $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) < 0$, which implies $R\Theta(e_{t+1} + \xi_{t+1}) - (\theta_1 + 1)e_{t+1} > 0$.

The third scenario is when e_{t+1} and ξ_{t+1} have opposite signs, $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2)$ and e_{t+1} have the same sign, and $|e_{t+1}| > |\xi_{t+1}|$. That is the insider understates the dividend shock but the sign of the signal is still true. In this scenario, the insider will keep his or her holding position unchanged between the second stage of period *t* and the first stage of period *t + 1*. Without loss of generality, suppose $e_{t+1} > 0$, $\xi_{t+1} < 0$ and $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) > 0$, the expected profit of the insider from manipulation is,

$$\begin{aligned}\Pi_{t|t+1}^3 &= [(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1})](1) + R\Theta(e_{t+1} + \xi_{t+1})(1) \\ &= (\theta_1 + 1)e_{t+1}\end{aligned}$$

which is the same as the profit when the insider does not manipulate.

The fourth scenario is when e_{t+1} and ξ_{t+1} have opposite signs, $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2)$ and e_{t+1} have the same sign, and $|e_{t+1}| < |\xi_{t+1}|$, i.e., the sign of the signal is opposite to the direction of the price movement. Without loss of generality, suppose $e_{t+1} > 0$, $\xi_{t+1} < 0$ and $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2) > 0$, if the insider sells the asset at stage 2 of period *t* and purchases the asset at stage 3 of period *t*, and sell the asset again when the dividend shock is revealed at the first stage of period *t + 1*, the expected profit of the insider from manipulation is,

$$\begin{aligned}\Pi_{t|t+1}^4 &= [(\theta_1 + 1)e_{t+1} - R\Theta(e_{t+1} + \xi_{t+1})](1) + R\Theta(e_{t+1} + \xi_{t+1})(-1) \\ &= (\theta_1 + 1)e_{t+1} - 2R\Theta(e_{t+1} + \xi_{t+1}) \\ &> (\theta_1 + 1)e_{t+1}\end{aligned}$$

Since $e_{t+1} + \xi_{t+1} < 0$, the expected manipulation profit in the fourth scenario is greater than the expected profit without manipulation.

The expected profits in the first and third scenarios are the same as the expected profit without manipulation, but the expected profits in the second and fourth scenarios are larger than the expected profit without manipulation. Thus, the insider maximizes the extra expected profit in the second and fourth scenarios by choosing the variance of the noise σ_ξ^2. The expected extra profit of the second scenario compared to no manipulation is,

$$P^2 = 2\int_0^\infty \frac{1}{\sqrt{2\pi\sigma_e^2}} exp\left(-\frac{1}{2}\frac{e_{t+1}^2}{\sigma_e^2}\right) \int_{\frac{(\theta_1+1)-R\Theta}{R\Theta}e_{t+1}}^\infty \frac{1}{\sqrt{2\pi\sigma_\xi^2}} exp\left(-\frac{1}{2}\frac{\xi_{t+1}^2}{\sigma_\xi^2}\right) \left(\Pi_{t|t+1}^2 - (\theta_1 + 1)e_{t+1}\right) de_{t+1} d\xi_{t+1}$$

The expected extra profit of the fourth scenario compared to no manipulation is,

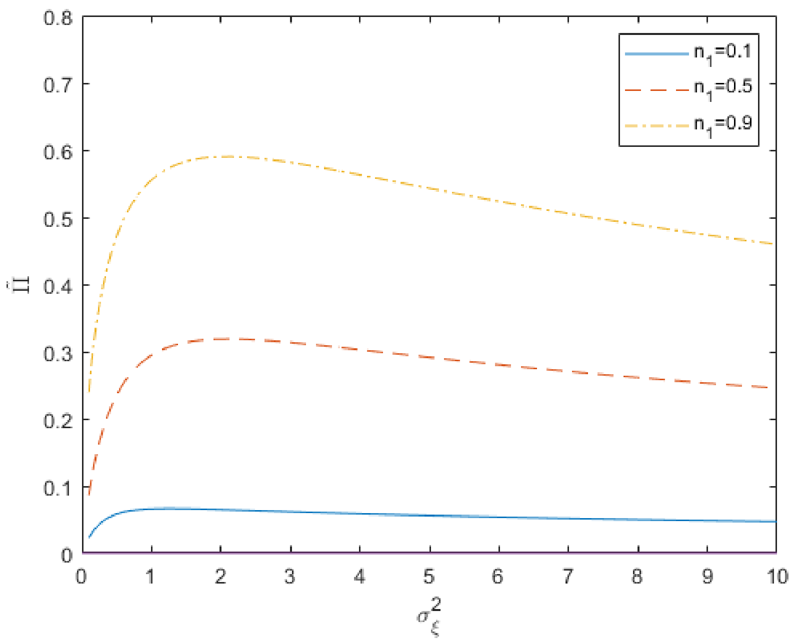
$$P^4 = 2\int_0^{\infty} \frac{1}{\sqrt{2\pi\sigma_e^2}} exp\left(-\frac{1}{2}\frac{e_{t+1}^2}{\sigma_e^2}\right) \int_{-\infty}^{e_{t+1}} \frac{1}{\sqrt{2\pi\sigma_\xi^2}} exp\left(-\frac{1}{2}\frac{\xi_{t+1}^2}{\sigma_\xi^2}\right) \left(\Pi_{t|t+1}^4 - (\theta_1 + 1)e_{t+1}\right) de_{t+1} d\xi_{t+1}$$

The objective function of the insider is,

$$\max_{\sigma_\xi^2} \tilde{\Pi} = P^2 + P^4$$

The term Θ in $\Pi_{t|t+1}^2$ and $\Pi_{t|t+1}^4$ includes the endogenous perceived variances $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$. The rational expectation solutions of $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ are roots of a fourth-degree polynomial. I use the numerical method to calculate the solutions of $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ with different variances of the signal noise, i.e., σ_ξ^2, and different values of other parameters. I follow Branch and Evans (2010) to select the smaller positive real roots of $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ because they are the equilibrium under real time learning. Then the extra profit from manipulation, i.e., $\tilde{\Pi}$, is computed by using the solutions of $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$. FIGURE 1 shows the relationship between $\tilde{\Pi}$ and σ_ξ^2 with three different proportions of Type 1 investors. The optimal σ_ξ^2 is larger when the proportion of Type 1 investors increases.

FIGURE 1
THE EXPECTED EXTRA PROFIT OF THE INSIDER FROM MANIPULATION WITH $R = 1.01$, $\rho = 0.3$, $\sigma_e^2 = 2$, $\sigma_z^2 = 1$, AND $a = 0.1$



Incentive Compatibility Constraint of Type 1 Investors

The second factor restricting the variance of the noise is the incentive compatibility constraint. To manipulate the asset price, the insider must ensure that $n_1 > 0$, i.e., there are uninformed investors using the signal. If the signal is not used, he or she cannot manipulate the price. Uninformed investors choose to use the signal if the utility of Type 1 (using the signal) is higher than the utility of Type 2 (not using the signal). Assuming the current wealth of the two types are the same, i.e., $W_{1,t} = W_{2,t}$, uninformed investors compare the expected risk-adjusted utility of being Type 1 or Type 2. Uninformed investors choose to be Type 1 if,

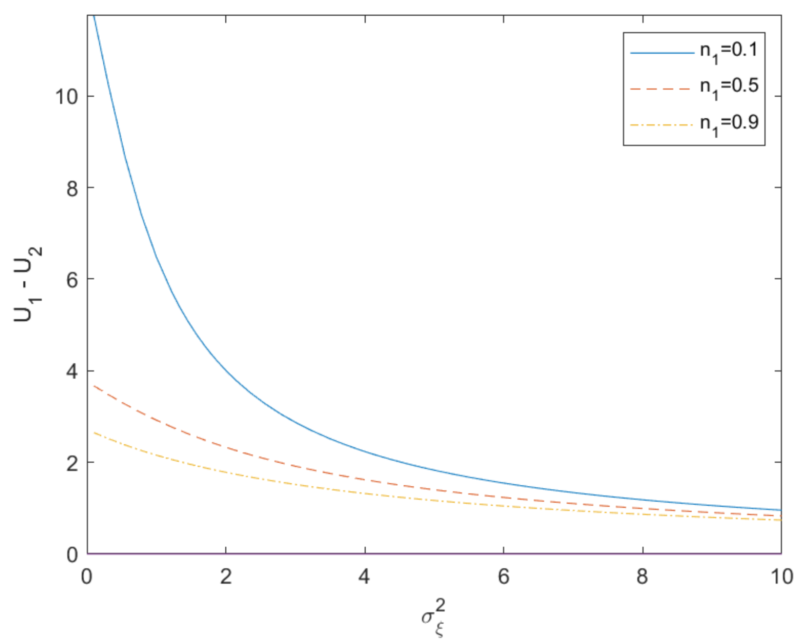
$$U_1 - U_2 > 0$$

where,

$$U_j = E_t\left[(p_{t+1} + y_{t+1} - Rp_t)z_{j,t}\right] - \frac{a}{2}E_t\left[\sigma_{j,t}^2 z_{j,t}^2\right]$$

$\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ are roots of a high-order polynomial which depends on the variance of the noise, σ_ξ^2. From equations (21), (22), (23), and (24), the direct effect of an increase in σ_ξ^2 is negative on $U_1 - U_2$. However, when σ_ξ^2 increases, the values of other endogenous parameters will change. Therefore, the indirect effect is not clear. If the direct effect is larger than the indirect effect, i.e., the total effect of an increase in σ_ξ^2 is negative on $U_1 - U_2$, uninformed investors will shift away from Type 1 to Type 2 when the noise is too large. FIGURE 2 shows the relationship between $U_1 - U_2$ and σ_ξ^2. For three different values of n_1, $U_1 - U_2$ is decreasing in σ_ξ^2 but greater than zero. The incentive compatibility constraint is always satisfied. Nevertheless, since profits from the insider signal are a zero-sum game between Type 1 and Type 2 investors. When n_1 gets larger, a smaller proportion of Type 2 investors implies that Type 1 investors earn a thinner profit.

FIGURE 2
THE DIFFERENCE OF THE EXPECTED UTILITIES BETWEEN TYPE 1 AND TYPE 2 UNINFORMED INVESTORS WITH $R = 1.01$, $\rho = 0.3$, $\sigma_e^2 = 2$, $\sigma_z^2 = 1$, AND $a = 0.1$



SENSITIVITY ANALYSES

I also perform sensitivity analyses by changing the values of parameters. One important parameter is the variance of the dividend shock σ_e^2. FIGURE 3 indicates that as σ_e^2 increases, utilities of Type 1 investors becomes larger relative to Type 2 investors. The intuition is that the insider signal contains information about the variation of the dividend shock. As the volatility of the dividend shock increases, the signal becomes more valuable. FIGURE 4 shows that as σ_e^2 increases, the extra expected profit of the insider from manipulation is also improved. This is because the manipulation profit depends heavily on the signal to noise ratio $\gamma_1 = \sigma_e^2/\sigma_e^2 + \sigma_\xi^2$. As γ_1 increases, the price impact of the noisy signal and the manipulation ability of the insider become stronger.

The impacts of the supply shock variance σ_z^2 on the relative utilities of the two types $U_1 - U_2$ and the insider's extra profit from manipulation $\widetilde{\Pi}$ are not clear. σ_z^2 does not directly enter $U_1 - U_2$ and $\widetilde{\Pi}$. It affects these two variables indirectly through $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$. FIGURE 5 shows that a larger σ_z^2 makes $U_1 - U_2$ slightly higher when σ_ξ^2 is small. However, as σ_ξ^2 increases, a larger σ_z^2 does not change $U_1 - U_2$ substantially. FIGURE 6 also suggests that σ_z^2 does not have a significant impact on $\widetilde{\Pi}$.

FIGURE 7 shows that a higher persistence of the dividend shock ρ increases $U_1 - U_2$ when σ_ξ^2 is small. When σ_ξ^2 is large, a higher persistence of dividend shock does not have a significant impact on $U_1 - U_2$. One possible reason is that when the noise of the signal is small, a higher persistence will make the signal more valuable because the effect of the dividend shock lasts longer. However, when the noise of the signal is large, this benefit decreases. FIGURE 8 shows that a higher persistence monotonically increases the expected extra profit of the insider from manipulation. The intuition is that when the dividend shock is more persistent, Type 1 investors take the signal more seriously. This increases the manipulation ability of the insider.

FIGURE 9 indicates that a higher risk aversion parameter a lowers the utilities of Type 1 investors relative to Type 2 investors. The intuition is that the noise makes the expected wealth of Type 1 investors more volatile. Therefore, the utility of Type 1 investors is lower when they are more risk averse. FIGURE 10 shows that when the noise is small, the profit of the insider from manipulation is lower when the risk aversion is high. The magnitude of this effect reduces when the noise is large. A possible reason is that a higher risk aversion parameter decreases the utility of Type 1 investors more than the utility of Type 2 investors. This will decrease the holding of Type 1 investors relative to Type 2 investors. Therefore, the price impact of the signal will be smaller when investors are more risk averse.

FIGURE 3
THE DIFFERENCE OF THE EXPECTED UTILITIES BETWEEN TYPE 1 AND TYPE 2 UNINFORMED INVESTORS WITH $\boldsymbol{R = 1.01, \rho = 0.3, n_1 = 0.5, \sigma_z^2 = 1}$, AND $\boldsymbol{a = 0.1}$

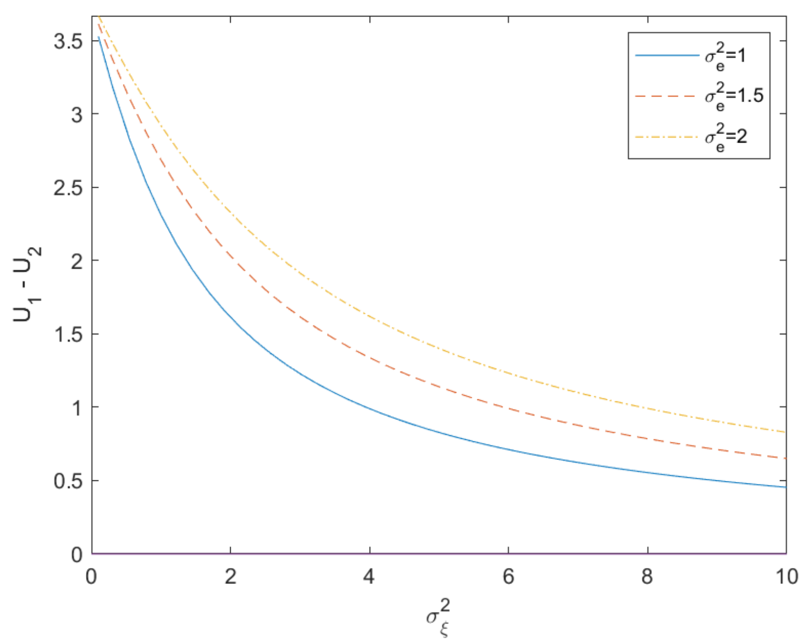


FIGURE 4
THE EXPECTED EXTRA PROFIT OF THE INSIDER FROM MANIPULATION WITH $R = 1.01, \rho = 0.3, n_1 = 0.5, \sigma_z^2 = 1$, AND $a = 0.1$

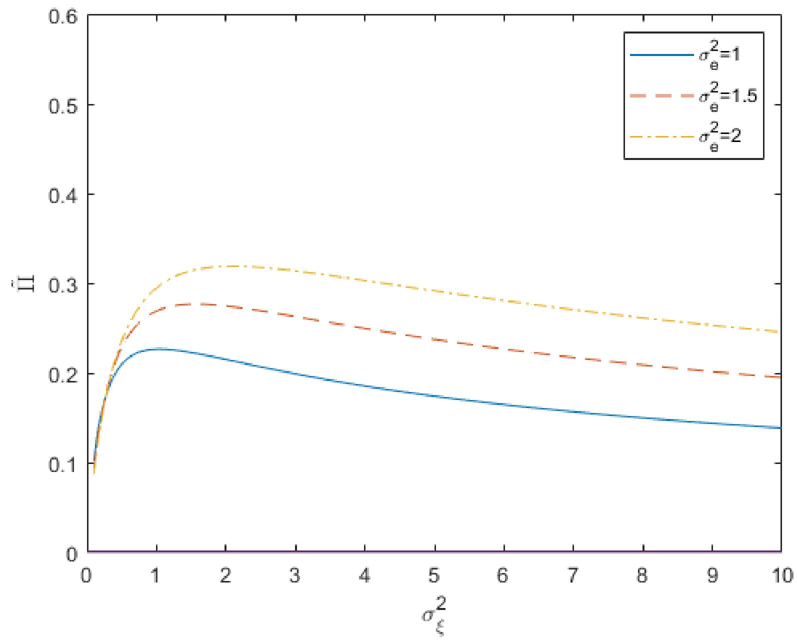


FIGURE 5
THE DIFFERENCE OF THE EXPECTED UTILITIES BETWEEN TYPE 1 AND TYPE 2 UNINFORMED INVESTORS WITH $R = 1.01, \rho = 0.3, \sigma_e^2 = 2, n_1 = 0.5$, AND $a = 0.1$

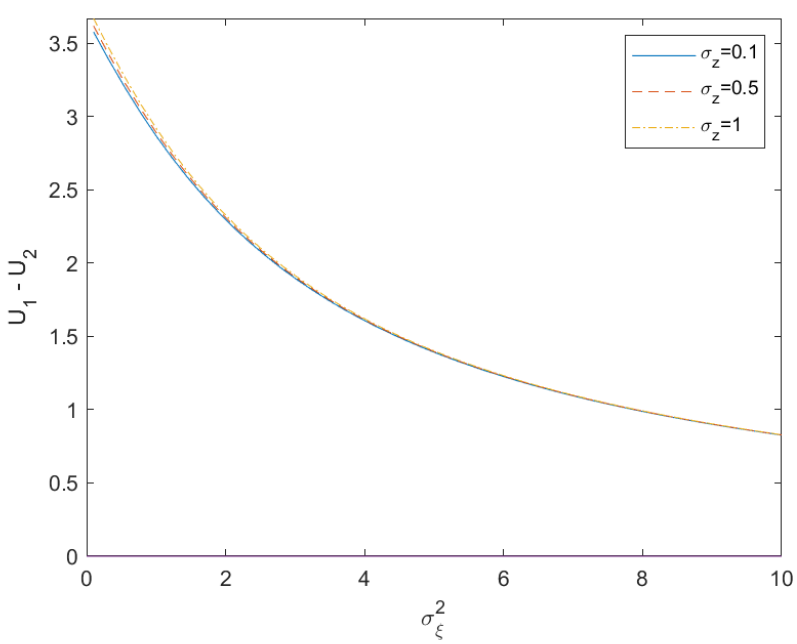


FIGURE 6
THE EXPECTED EXTRA PROFIT OF THE INSIDER FROM MANIPULATION WITH $R = 1.01$, $\rho = 0.3$, $\sigma_e^2 = 2$, $n_1 = 0.5$, AND $a = 0.1$

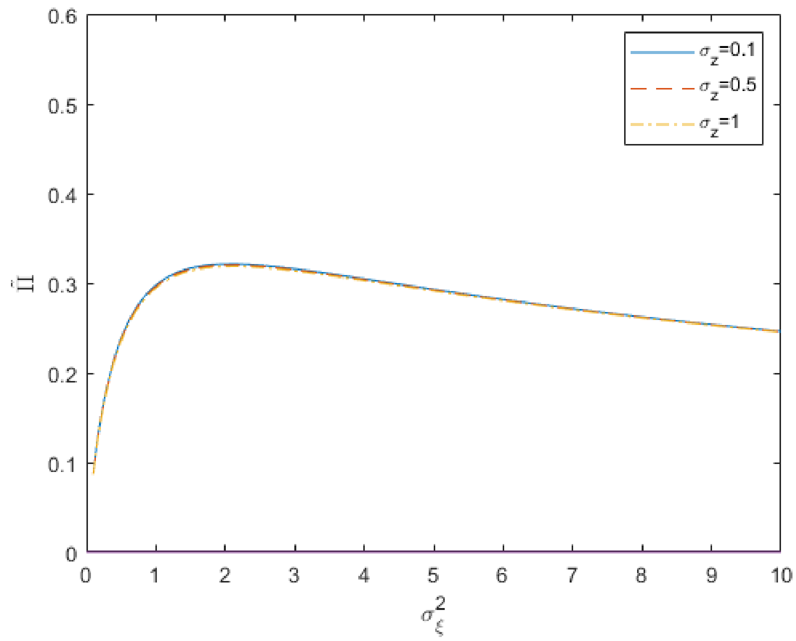


FIGURE 7
THE DIFFERENCE OF THE EXPECTED UTILITIES BETWEEN TYPE 1 AND TYPE 2 UNINFORMED INVESTORS WITH $R = 1.01$, $n_1 = 0.5$, $\sigma_e^2 = 2$, $\sigma_z^2 = 1$, AND $a = 0.1$

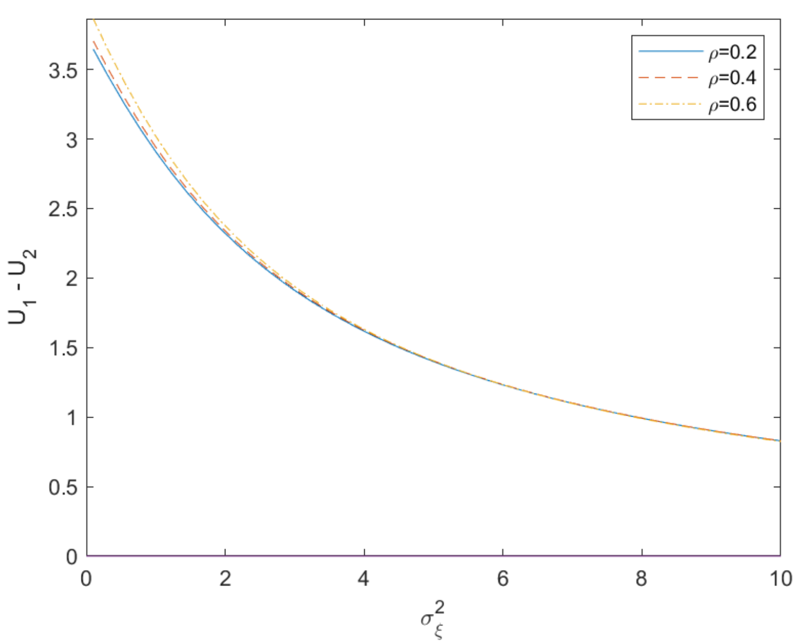


FIGURE 8
THE EXPECTED EXTRA PROFIT OF THE INSIDER FROM MANIPULATION WITH $R = 1.01, n_1 = 0.5, \sigma_e^2 = 2, \sigma_z^2 = 1$, AND $a = 0.1$

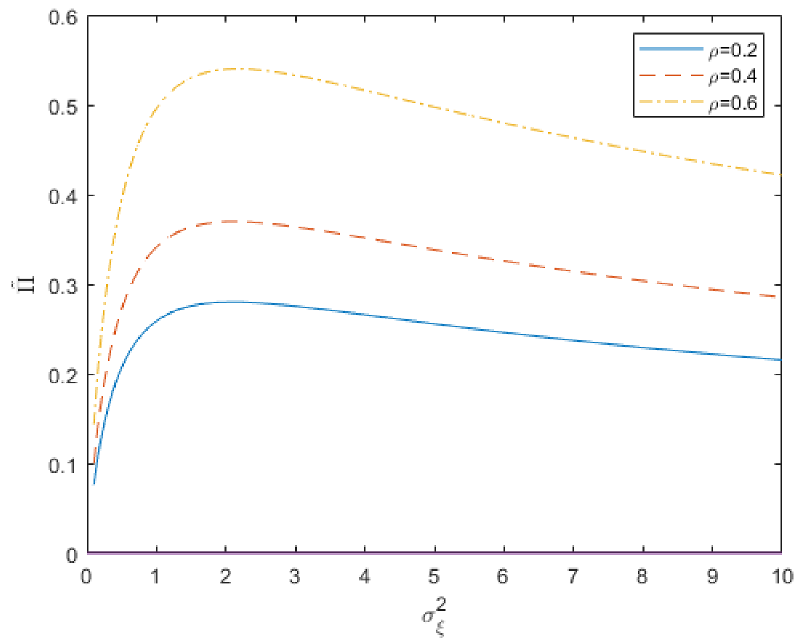


FIGURE 9
THE DIFFERENCE OF THE EXPECTED UTILITIES BETWEEN TYPE 1 AND TYPE 2 UNINFORMED INVESTORS WITH $R = 1.01, \rho = 0.3, \sigma_e^2 = 2, \sigma_z^2 = 1$, AND $n_1 = 0.5$

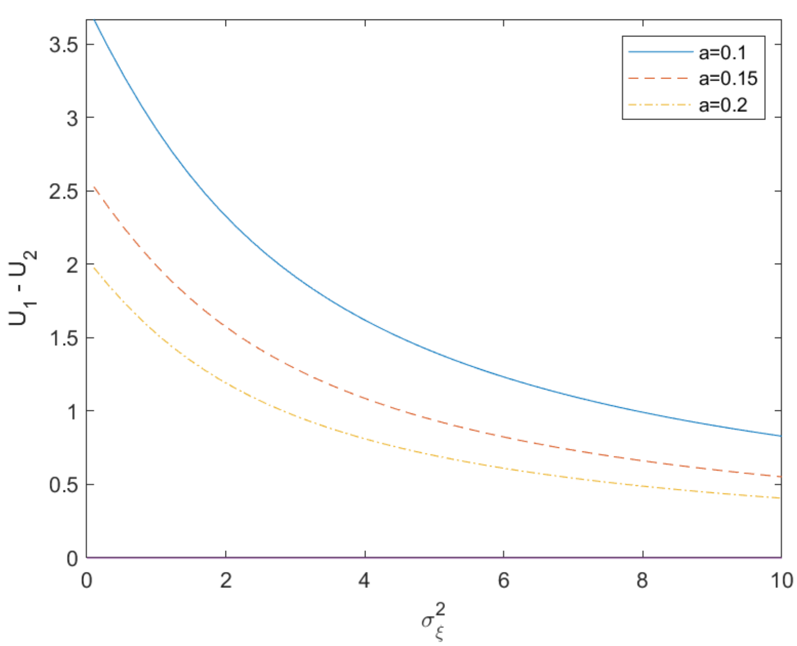
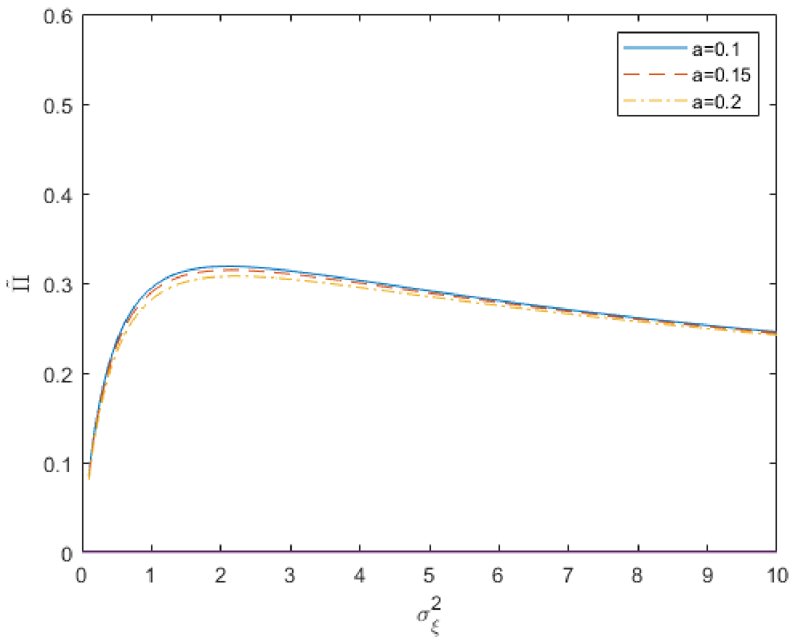


FIGURE 10
THE EXPECTED EXTRA PROFIT OF THE INSIDER FROM MANIPULATION WITH $R = 1.01, \rho = 0.3, \sigma_e^2 = 2, \sigma_z^2 = 1$, AND $n_1 = 0.5$



IMPLICATION OF BOUNDED RATIONALITY

In the previous section, I only analyze the rational expectation equilibria. Under this setting, when the insider changes the variance of the noise, other parameters change immediately to the values of rational expectations. This will lower the manipulation ability of the insider. However, under boundedly rationality, Type 1 investors learn the coefficient γ_1 in (8) gradually. Therefore, when the insider makes the signal noisier, the beliefs of Type 1 investors remain unchanged temporarily. Thus, the insider can add a large noise to the signal in one period to manipulate the price without lowering the price impact.

Another reason for the insider to add a large noise in one period is that it makes the second scenario, i.e., the price is overshot when e_{t+1} and ξ_{t+1} have the same sign, but $E_{It}(p_{t+1}^0 + y_{t+1} - Rp_t^2)$ has an opposite sign with the signal e_{t+1}, and the fourth scenario, i.e., the sign of the signal is opposite to the dividend shock when e_{t+1} and ξ_{t+1} have opposite signs but $|e_{t+1}| < |\xi_{t+1}|$, more likely. These are the two scenarios under which the insider earns extra manipulation profit.

CONCLUSION

This paper shows that information-based manipulation exists even without any restrictive assumption. The insider only needs to add a random noise to the signal that he or she sends to the market. Since the noisy signal contains valuable information, uninformed investors are attracted to use the signal. The price impact of these uninformed investors thus provides manipulation ability to the insider. This outcome is beneficial to both the insider and the uninformed investors. Compared to the previous studies which assume that the insider sends discrete signals under some restrictive assumptions, the manipulation strategy in the current paper is more flexible and practical. Moreover, the uninformed investors in the current paper use simple linear beliefs to make forecasts. To evaluate the reliability of the signal, they only need to know the correlation coefficient between the signal and the dividend. When the signal is noisier, the coefficient will be lower, reflecting a weaker reliability, vice versa. This is less cognitive consuming than Bayesian learning assumed in the previous literature, which requires uninformed investors to understand the complex probability calculation and know the distributions of the related random variables.

Another important finding is that the insider can make huge profit in a short investment horizon if uninformed investors are boundedly rational. This is because boundedly rational investors learn the reliability of the signal gradually. Their beliefs are constant in the short run. Therefore, even if the insider distorts the asset price by sending a very noisy signal in one period, the beliefs do not become weaker. However, several questions remain unanswered. Though the insider tends to add a large noise in one period, it is not known when this period is. Suppose the beliefs of the uninformed investors are weak in the current period, is that more profitable to add a large noise in the current period or keep sending precise signals to strengthen the beliefs and add a large noise in a later period? Another question is, how fast the beliefs can recover after noises are added to signals? Under different learning algorithms, the beliefs can recover faster or slower when a large noise is added in one period than when several noises are spread into multiple periods. Lastly, this paper assumes that the proportion of Type 1 and Type 2 uninformed investors are fixed. If uninformed investors move from type to type gradually by using a multinomial logit approach (Brock and Hommes, 1997, 1998), the speed of belief recovery may be different. This may change the manipulation strategy of the insider.

ACKNOWLEDGEMENTS

I am very grateful to Florian Kuhn, David Slichter, and Wei Xiao for providing useful comments.

ENDNOTE

1. In Benabou and Laroque (1992), manipulation is only sustainable in the long run if the nature can change the insider between being an opportunistic manipulator or an honest signal sender. The insider in Bommel (2003) can only send an honest signal or pretends to be an insider even he or she does not have any private information. A false signal manipulation strategy is not sustainable. In the current paper, a continuous signal space allows the insider to engage in any kind of manipulation even without other restrictive assumptions.

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