Forecasting Volatility With Spot Index and Index Futures: Evidence From Taiwan

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In this article, we use the volatility of the Taiwan Stock Index (TAIEX) and its futures in the encompassing regression model to respectively make asynchronous forecasts of realized volatility (RV) and implied volatility (IV). Initially, we discover that, to obtain a stationary RV with a stable, long memory parameter, the optimal sampling intervals for the intraday return were 9 and 30 minutes. We uncover that the spot volatility is more predictive of RV than the futures volatility. To forecast IV, the volatility of futures has more information content, helping to improve overall forecast performance. The result implies that the underlying asset of the TAIEX options (TXO) is approximately the index futures rather than the spot index, owing mainly to the demands for hedging and arbitrage from the TXO holders. Finally, we confirm the forecasting capability of our various models via extensive forecasting error and simulation exercises.

Keywords: Bayesian ARFIMA, encompassing regression, forecasting, implied volatility, realized volatility, Taiwan

INTRODUCTION

The literature on volatility forecasting can be roughly grouped into two branches, one about realized volatility (RV) and another about implied volatility (IV). Realized volatility forecasting is conducted through historical volatility (HV) and implied volatility (IV), such as those studies carried out by Canina and Figlewski (1993), Christensen and Prabhala (1998), and Jiang and Tian (2005). These studies maintain that IV or perhaps HV could be an unbiased estimator of RV under the efficient-market hypothesis and can be used as the predictor variable for RV. Canina and Figlewski (1993) and Christensen and Prabhala (1998) find that implied volatility has virtually no correlation with future return volatility and does not appear to incorporate information contained in historical return volatility. However, Jiang and Tian (2005) provide support for the informational efficiency of the option markets.

Implied volatility forecasting is accomplished through the RV, HV or IV for at-the-money options[1], as done by Chan, Jha, and Kalimipalli (2009). The main contribution of above studies is that those models can forecast IV in real investment simulations. They compare real investment performance of different forecasting models. However, they do not compare the information content of volatility in the spot market with that in the futures market in the same time period. The current study intends to fill this void.

According to the efficient-market hypothesis, futures price should lead spot price when new information arrives, because futures market provides a function of price discovery. Empirically, many studies have documented that futures market incorporates new information more efficiently than spot market and, thus, futures returns overall tend to lead more often than lag spot index returns. For example, Kawaller, Koch, and Koch (1987) find that the S&P 500 index futures lead spot index by about 45 minutes. Stoll and Whaley (1990) also discover that return on S&P 500 and MMI index futures lead spot by about 5 minutes. Chan (1992) and Fleming, Ostdiek and Whaley (1996) report similar findings.

In Taiwan the first TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index Taiwan stock index) futures contract was launched by Taiwan Futures Exchange (TAIFEX) on July 21, 1998. The empirical studies about price relationship between spot and futures prices on TAIEX are mixed. Some of the evidence indicates that price discovery is still dominated by cash market while other evidence shows that the returns of TAIEX futures lead those of cash market. For example, Lin, Chen, Hwang and Lin, (2002) investigate the interaction of return and volatility between the TAIEX futures and the TAIEX spot markets. They found that the price discovery process is dominated by the TAIEX spot market in terms of return and volatility. Hsieh (2002) investigates information transmission between the TAIEX futures and the underlying spot index in terms of return. He finds that futures market dominants the spot market in price discovery. Jang and He (2004), employing a regression model, investigate the intraday price relationships between the spot, futures and options markets for the TAIEX. They find that the TAIEX futures returns lead the TAIEX returns by about 20 minutes. Overall, index futures tend to lead more often than lag the cash index in price discovery.

It seems that much of the literature does not discuss the leading effect of volatility with information content in futures market. If futures volatility with superior information leads spot volatility, futures volatility should possess superior forecasting ability than spot volatility, due mainly to the price discovery function in the futures market. In this paper, the TAIEX, the TAIEX futures (TX), and the TAIEX options (TXO) are used to calculate different volatilities to be incorporated into the encompassing regression model to forecast the RV and IV. Such an approach could contribute to the literature by comparing the information content of cash-based volatility with that of futures-based volatility.

Such an effort is interesting for a number of reasons. First, futures volatility is added as a predictor variable with incremental information to forecast IV. Second, forecast values are used for all predictor variables in the encompassing model to generate an asynchronous regression model so that the model would be realistic enough to be close to the real world. Finally, a simulation of the multiple of the smallest matching time for transaction is conducted to obtain stable intra-day data for RV calculation by using long-memory parameter and coefficient values synchronously estimated by Bayesian ARFIMA approach., which could reduce the bias created by the two-step maximum likelihood estimate (MLE) method.

We decide to adopt the encompassing regression model in light of the evolving literature. Many studies that examine volatility models have cited the methods and results of Lamoureux and Lastrapes (1993), who tracked 10 individual stock options to test several volatility models, as a benchmark for comparison in order to verify the accuracy of their empirical results. They criticize that their option data is outdated, their sample size is insufficient, and their methodology is incomplete. These deficiencies are responsible for the bias and inefficiency of the empirical results of Lamoureux and Lastrapes (1993). Canina and Figlewski (1993) find that the IV from the S&P100 index options is a poor forecast for the subsequent RV of the underlying index. They apply an encompassing regression analysis and find that IV has virtually no correlation with future RV and thus does not incorporate information contained in historical volatility.

Likewise, our use of options data follows the literature. However, according to Rubinstein (1994), the US option market has undergone a structural change since 1987. He contends that only high liquidity options possess valuable information. Several related studies consistently discover that the implied volatility of high liquidity options has better information content. Mayhew (1995) suggests using nearly at-the-money option to estimate implied volatility for pricing option with identical maturity. The empirical method shows that the at-the-money implied volatility is an important variable. After reviewing 93 papers on volatility, Poon and Granger (2003) conclude that implied volatility with more relevant information has the best forecasting ability and high-liquidity at-the-money option has the least error. Accordingly, our research sample comprises of only nearly at-the-money option contracts.

Our empirical results and findings are as follows. Trading frequency and transaction matching time are conducive to a stable RV. The multiple of 45 seconds (6, 9, 15, and 30 minutes) transaction matching time helps calculating Taiwan stock index RV with stability. We observe a stable long memory parameter in 9 and 30-minutes RV values. In forecasting ability, we find that RV with 9 minutes has the least forecasting error. Spot HV has higher RV forecasting ability than futures HV. Furthermore, we find that futures volatility has higher forecasting ability for IV. Therefore, the futures market leads the spot market in Taiwan. Our empirical results imply that the underlying asset for implied TXO is approximately TX, not TAIEX. It is due to the hedging and arbitraging needs from option holders. Finally, through trading simulations of a delta-neutral straddle portfolio using various IV forecasts, we observe not only long memory but also jump tendency in the return of TAIEX. However, combinations of jump and leverage in ARFIMA model for forecasting IV have the lowest forecasting error and best profitability within stock crash period.

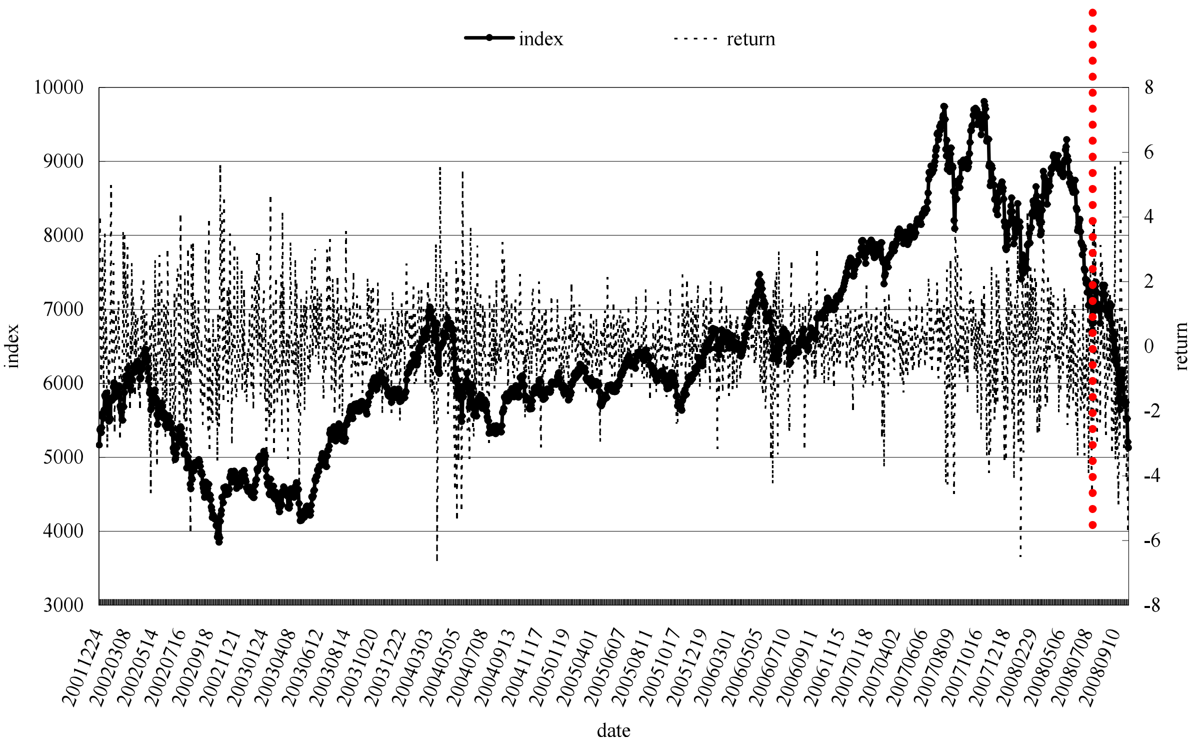
The remainder of the paper is organized as follows. Section 2 describes data and methodology. Section 3 introduces the estimation methods of volatility forecasts. Section 4 discusses the main results and checks their robustness. Section 5 concludes.

DATA AND METHODOLOGY

Data

Our samples include Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), TAIEX futures (TX) and TAIEX options (TXO) traded on the Taiwan security market over the period December 24, 2001 to May 20, 2008. The daily and intraday data are provided by Taiwan Economic Journal (TEJ) database, TAIFEX and TWSE (Taiwan Stock Exchange). We restrict our data to pre-May 20, 2008 as TAIEX plunged significantly after the Presidential election. During the months after the Presidential election, the stock market collapsed on May 21, 2008. To test the effectiveness of the forecasting models, we use an out-of-sample period from December 19, 2007 to May 20, 2008. Figure 1 presents TAIEX and its return during the sample period. The patterns of TAIEX in Fig. 1 (period before the vertical dashed line) shows that our sample period contains both bull and bear markets. Furthermore, the pattern (period after the vertical dashed line) of TAIEX shows a sharp decrease following the Presidential election. As such, we use another post-election out-of-sample period, May 21, 2008 to October 9, 2008, for robustness test.

FIGURE 1
INDEX LEVEL AND RETURN OF TAIEX FROM 12/24/2001 TO 10/09/2008



Note: The post-Presidential election period is after the vertical dashed line.

We use nearby contracts owing to the low turnover of the distant contracts. Additionally, many prior studies suggest that the last few days before the expiration of the futures contracts reveal unusual trading activities and higher volatility. Contracts are thus rolled over to the next nearby contract on the 7th day before expiration to mitigate the expiration effects. Thus, our samples of futures and options contracts will be nearby contracts with a maturity period of 8 to 30 days. In addition, if the implied volatility is negative or greater than 100%, the contract is eliminated. Finally, we proxy for the risk-free interest rate using 1-month time deposit rate from the First Bank of Taiwan.

The stocks are traded on the Taiwan Stock Exchange during trading hours from 9:00 am to 1:30 pm, Monday to Friday. However, the trading hours for the TAIEX futures and options extend from 8:45 am to 1:45 pm. We thus truncate the first and last 15 minutes of the data on futures trading to match the trading hours for the futures and stocks.

We compute moneyness following Bakshi, Cao, and Chen (1997). Owing to the non-tradability of the TAIEX and the non-synchronous problems, we employ the TAIEX futures index in computing moneyness (M):

$$Moneyness = \frac{TX}{K}, \tag{1}$$

where TX is daily closing price with a nearby contract (if no closing price on the day, we replace the closing price with settlement price), K is the strike price. Contracts with an absolute value of M from 0.97 to 1.03 are the placed into the at-the-money (ATM) categories. Options with absolute moneyness below 0.02 or above 0.98 are excluded due to the distortion caused by price discreteness.

Models

Studies on volatility relationships have generally emerged from the perception that estimated volatility is an informationally efficient predictor. A particularly simple and intuitive approach for testing this conjecture is to run the Mincer and Zarnowitz (1969) predictive regressions. This methodology is still by far the dominant approach in the literature addressing the efficiency and bias issue of volatility forecasts. The setting for Mincer and Zarnowitz (1969)'s model accommodates traditional univariate predictive regressions as well as extended encompassing regressions. The generic notation for the regressor and regressands emphasizes the fact that the exposition applies across alternative transformations of volatility measures and forecasts, such as basic variances, standard deviations or volatilities and log-volatilities, which each may have desirable empirical or theoretical properties.

The encompassing regression analysis can well explain dependent variables from the information content contained in independent variables. In a study of the forecasting ability of volatility models, it is quite obvious that selecting independent variables with strong forecasting ability is an important task. Clemen (1989) points out that a mixture of at least two types of volatility forecasts can have better forecasting performance for RV than a single type of volatility. Following prior research (e.g., Canina and Figlewski, 1993, and Christensen and Prabhala, 1998), we employ encompassing regressions to examine the ability to forecast RV and IV. In encompassing regression model, realized and implied volatilities are regressed against three volatility forecasts respectively in order to distinguish which one has the highest explanatory power.[2] In addition, the information contents of spot volatility and futures volatility are compared.

Dumas, Fleming, and Whaley (1998) utilizes deterministic volatility function (DVF) to estimate the relationship of implied volatility and five endogenous variables. The model has a very high explanatory power implying that the volatility formed by DVF has indeed incremental information. In view of the criticism raised by Lamoureux and Lastrapes (1993) and Canina and Figlewski (1993) with regards to the informational superiority of IV, we extend the model to include the DVF implied volatility (DVFIV).

First, we combine HV, IV, and DVFIV as three proxy volatility variables to forecast RV, (σ_t^{RV}), the realized volatility. The model (RV model) is expressed as follows:

$$Ln(\sigma_t^{RV}) = \beta_0 + \beta_1 Ln(\sigma_{t\,|\,\Omega\ t-1}^{HV^e}) + \beta_2 Ln(\sigma_{t\,|\,\Omega\ t-1}^{IV.ATM^e}) + \beta_3 Ln(\sigma_{t\,|\,\Omega\ t-1}^{DVFIV^e}) + \varepsilon_t, \quad (2)$$

where β's and ε are respectively the regression coefficients and error term, $\sigma_{t\,|\,\Omega_{t-1}}^{HV^e}$, $\sigma_{t\,|\,\Omega_{t-1}}^{IV\,ATM^e}$ and $\sigma_{t\,|\,\Omega_{t-1}}^{DVFIV^e}$ are respectively the day-t predictors of historical volatility (HV), ATM implied volatility (IV) and implied volatility computed by the deterministic volatility models (DVFIV), conditional on information at day t-1. The superscript e stands for expected value. Equation (2) contains multiple forecasts and is, thus, often called an "encompassing regression"[3]. We use both spot index and index futures separately to estimate historical volatility and to ascertain that both HV estimates could be unbiased estimators of RV. Furthermore, we forecast IV using HV and RV obtained from spot index and index futures. Thus, we set up the following encompassing regression (IV-RV) model:

$$Ln(\sigma_t^{IV}) = \beta_0 + \beta_1 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX-HV^e}) + \beta_2 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX-RV^e}) + \beta_3 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TXF-HV^e}) + \beta_4 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TXF-RV^e}) + \varepsilon_t, \quad (3)$$

where σ_t^{IV} is implied volatility. σ_t^{TX-HV} and σ_t^{TX-RV} are respectively historical volatility and realized volatility of TAIEX. σ_t^{TX-HV} and σ_t^{TX-RV} are respectively historical volatility and realized volatility of TX. Following Christensen and Prabhala (1998)'s hypotheses test in a univariate model, there are some hypotheses to be tested in our encompassing regression models. The first is about the efficiency of the volatility forecast. We test whether the volatility forecast subsumes all the information contained in realized or implied volatilities. In an affirmative case the slope coefficient of volatility forecast should be equal to zero. Moreover, as a joint test of information content and efficiency we test in equation (2) and (3) if the slope coefficients of all volatility are equal to zero and one respectively. Following Jiang and Tian (2005), we ignore the intercept in the latter null hypothesis, and if our null hypothesis is verified, we interpret the volatility forecast as unbiased after a constant adjustment. Differing from other papers that use spot and options to forecast volatility, we apply not only spot and option variables but also futures variable.

In our encompassing regression model, we expect to use spot volatility or futures volatility as an informational proxy variable to forecast IV. In the meantime, we could identify options traders' information source. The novelty vis-á-vis the previous literature is that forecasted values are used for explanatory variables in the encompassing regression model. Therefore, we have to forecast explanatory proxy variables before executing encompassing regressions.

The concept of forecasting realized, and implied volatilities can be expressed as follows:

$$\sigma_t \Leftarrow \hat{\sigma}_{t\,|\,\Omega\ t-1}, \qquad (4)$$

where σ_t is the forecasted volatility at time t, and $\hat{\sigma}_{t\,|\,\Omega\ t-1}$ is the volatility forecasted at time t using all information (Ω) collected at time t-1. Our volatility forecasts will include information content coming from one-day-ahead (t-1), five-day-ahead (t-5) and 20-day-ahead (t-20). We then average across these estimated volatilities to obtain the volatility forecasts at time t.

Encompassing regression tests need to address several methodological issues such as possible measurement errors, telescoping errors, orthogonal errors, and biases (Fleming, 1998; Christensen and Prabhala, 1998 and Neely, 2009). Because our samples come from higher liquidity nearby futures and options contracts, the measurement errors should be low. Moreover, we estimate the coefficient of the encompassing regressions using the generalized method of moments (GMM) approach, which can minimize potential telescoping overlapping data problems. At the same time, any remaining measurement errors could lead to a descending bias in the encompassing regression.

Following many prior studies on volatility forecasts, we employ the following three metrics to compare the pricing performance of alternative volatility models: MSE (mean squared error), MAE (mean absolute error) and MAPE (mean absolute percentage error).

Trading Strategies and Simulation

Since the TAIEX is a non-traded asset, we use out-of-sample data to simulate pricing error for the value of forecasted IV in order to compare the forecasting ability of IV forecast model. Therefore, we use Equation (3) as the forecasting model. Furthermore, a trading strategy using options in portfolio could be formulated. Then we could observe whether the volatility forecast model could profit from abnormal returns. We use competing out-of-sample volatility forecasts to trade in nearly at-the-money delta-neutral straddles. In a trading rule, the theoretical strike price in a delta-neutral straddle can be solved as $N(d_1)+[N(d_1)-1]$. Furthermore, we determine that traded straddle combination whose strike price is closest to the theoretical strike price. Therefore, we form delta neutral ATM straddles each day in out-of-sample period, price them based on alternate volatility forecasts and then buy (sell) them on each day depending on whether they are underpriced (overpriced). When the straddle is a long (short) position, we assume that we can borrow (lend) funds at a risk-free rate. We apply the bid-ask spread as a filter for all the trades. Then we trade only when the absolute-value difference between the model and market price exceeds the average bid-ask spread on day t-1. Moreover, we also apply NT$80/contract trading cost[4]. In measuring the risk-return tradeoff for option trading, we only consider robust Sharpe ratios, considering the underlying non-normal distributions.

ESTIMATION METHODS OF VOLATILITY FORECASTS

Historical Volatility (σ^{HV})

To estimate historical volatility, we apply the well-known formula of Parkinson (1980) to daily high and low stock prices as follows:

$$\sigma_t^{HV} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\frac{1}{4Ln(2)}\left[Ln\left(\frac{H_t}{L_t}\right)\right]^2} \times \sqrt{day}, \tag{5}$$

where H_t and L_t are the highest and lowest index level for day t, respectively. The *n* and *day* are the number of observed days and business days, respectively. Following Gemmill (1986) and Chiras and Manaster (1978), we take the historical 20-trading-day average as the historical volatility to mitigate the estimation noise.

According to the prior literature on volatility forecast, HV has rather poor forecasting ability. To enhance the ability of HV forecasts, we compute the predictor of historical volatility using the ARMA (p, q) model, as defined below:

$$\varphi_p(B)(Z_t - \mu) = \theta_q(B)a_t, \tag{6}$$

where $\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$, and $\theta_q(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$, B is backward shift operator. The p and q order of ARMA model can be determined by sample autocorrelation function (ACF), sample partial autocorrelation function (PACF) and sample extended autocorrelation function (EACF). Furthermore, we can obtain the predictor of HV forecasts by the property ARMA model.

Implied Volatility (σ^{IV})

We estimate the implied volatility through the Newton-Raphson method on the Black-Scholes (BS, 1973) model. The Newton-Raphson method makes a guess for volatility, and then re-adjusts value by Vega. It uses the tangent to approximate the value, making use of the comparative bi-secant method, with faster convergence speed. Furthermore, depending on moneyness, we average one-day-ahead (t-1), five-day-ahead (t-5) and 20-day-ahead (t-20) ATM implied volatility as the ATM implied volatility forecasts for day t.

Instead of using the BS model to estimate IV, Dumas, Fleming, and Whaley (1998) develop a deterministic volatility function (DVF) option valuation model that has the potential of fitting the observed cross section of option price exactly. Following Dumas, Fleming, and Whaley (1998), we use the equation below:

$$Ln(\sigma_t^{IV}) = \alpha_0 + \alpha_1(\frac{TXF}{K})_t + \alpha_2(\frac{TXF}{K})_t^2 + \alpha_3 T_t + \alpha_4 T_t^2 + \alpha_5 T_t \cdot (\frac{TXF}{K})_t + \varepsilon_t, \tag{7}$$

where α and ε are the regression coefficients and error term, respectively. Following Whaley (1982) and Lamoureux and Lastrapes (1993), the daily IV for each option class is obtained by minimizing the mean squared error between the market ($Price^{Market}$) and theoretical price ($Price^{BSmodel}$) of the BS model for all N number of observed option i on day t:

$$\underset{\alpha}{Min}\left\{\frac{1}{N}\sum_{i=1}^{N}\left[Price_{t,i}^{Market} - Price_{t,i}^{BSmodel}\left(F_t, K_{t,i}, r_t, T_{t,i}, \sigma_{t,i}^{IV}\right)\right]^2\right\}. \tag{8}$$

By minimizing the mean squared error, we can obtain a vector of parameter estimates, $\alpha = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$. Furthermore, suppose that we have an option trading on day t with a given moneyness M and maturity of T-t days, we can condition day t-k parameter vector α, and forecast IV on day t for T-k day horizon. As done usually in IV forecasts, we average across the one-day-ahead (t-1), five-day-ahead (t-5) and 20-day-ahead (t-20) volatility estimates as day-t DVF volatility forecasts.

Realized Volatility (σ^{RV})

Thanks to Canina and Figlewski (1993) discussion on unrelatedness between IV and future RV, many scholars use high frequency intraday data to calculate realized volatility in order to prove the forecasting ability of estimated volatility from historical data. Furthermore, comparative study of various calculated volatilities will illuminate more information content from future volatility. Andersen and Bollerslev (1997) discovers that high frequency data can construct more precise ex-post volatility estimates. Andersen, Bollerslev, Diebold, and Labys (2001, ABDL hereafter) argue that intraday returns provide better estimates of RV and 5-minute sampling is optimal, considering the impact of market microstructure factors on measures based on high-frequency data. Similarly, Blair, Poon, and Taylor (2001) and Aït-Sahalia, Mykland and Zhang (2005) argue that if the microstructure noise is unaccounted for, the optimal sampling frequency is finite.

In ABDL (2001), daily realized volatility of high-frequency data is defined by:

$$\sigma_t^{2,RV} = \sum_j^n r_j^2, \quad (9)$$

where r_j is the rate of return in interval j on day t and n is the number of intervals in a day. For 9-minute sampling frequency, n equals 30 in the Taiwan stock market. ABDL (2001) shows that five-minute interval can obtain much better realized volatility. However, their underlying asset is foreign exchange which is different from ours. According to the matching stipulate of the Taiwan Stock Exchange, their buy and sell matching time takes place at least every 45 seconds. Therefore, we think the best decision interval for RV is 45 seconds. We will use multiples of 45 seconds to simulate stable realize volatility estimates.

The volatility especially realized volatility, of many financial data series exhibits characteristics consistent with long memory behavior as discovered in previous studies. Although many stochastic processes could potentially exhibit the long memory property, the most widely used such process is the ARFIMA model [Granger and Joyeux (1980), Granger (1980), and Hosking (1981)]. Andersen, Bollerslev, Diebold, and Labys (2001, 2003) and Andersen, Bollerslev, Diebold, and Ebens (2001) suggest that one can obtain better realized volatility through long-memory ARFIMA (autoregressive fractionally integrated moving average) time series model. Koopman, Jungbacker, and Hol (2005) also discovers that an ARFIMA model can predict RV better than other models. Therefore, we use the ARFIMA to describe the path of RVs to avoid estimation errors and low forecasting ability.

In the process of estimating an ARFIMA(p,d,q) model, the values of p and q must be determined before the value of d. Prior studies indicate that higher order of p and q can lead to high standard deviation of coefficients in the model. Some research uses ARMA(1,1) or ARMA(1,0) to describe RV with good fit. To avoid higher order p and q lowering the test power, we use an ARFIMA(1,d,0)[5] as the forecasting model for RV, where a stable value of d is obtained through Bayesian estimation. Therefore, the ARFIMA (1,d,0) model can be specified as:

$$(1-B)^d Ln(\sigma_t^{RV}) = \omega_0 + \omega_1 Ln(\sigma_{t-1}^{RV}) + e_t, \quad (10)$$

where B is backward shift operator, σ_t^{RV} is the realized volatility, ω and e are the coefficient and error term, respectively. The fractional parameter d (between zero and one) represents a long memory structure, implying slow hyperbolic decay in autocorrelations.

Black (1976), Christie (1982), Schwert and Seguin (1990) and Cheung and Ng (1992) point out that there is a negative contemporary relationship between volatility change and rate of return, which can be explained by the leverage effect. Therefore, for the leverage effect, we set up a dummy variable I, which is equal to 1 if the rate of return (r_{t-1}) in the previous period is less than zero, and zero otherwise (see Equation 12). Moreover, many studies also find that information with jump phenomenon will lead to unstable parameter estimation. Anderson, Bollerslev, Diebold, and Labys (2003) uses non-parametric method of the bi-power variation (BV) measure to isolate the jump element, which is non-negative, from the intraday data. The BV measure can be calculated as follows:

$$BV_t(\Delta) \equiv (\sqrt{2/\pi})^{-2} \sum_{j=2}^{1/\Delta} \left| r_{t+j\Delta,\Delta}^2 \right| \left| r_{t+(j-1)\Delta,\Delta}^2 \right|, \tag{11}$$

where Δ[6] is the sampling frequency. Following the concept of $\sigma_{t+1}^{RV}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{t<s<t+1} \kappa^2(s)$, we can estimate Jump by $\kappa(t)$. Therefore, the Jump component J is defined according to Andersen, Bollerslev and Diebold (2007)[7] as:

$$J_t = \text{Max}\,[\,\sigma_t^{RV} - BV_t\,,0\,]. \tag{12}$$

Thus, we specify an ARFIMA model that respectively incorporates leverage (ARFIMA+L), jump (ARFIMA+J), and both leverage and jump (ARFIMA+L/J) as follows:

$$\text{ARFIMA+L: } (1-B)^d Ln(\sigma_t^{RV}) = \omega_0 + \omega_1 Ln(\sigma_{t-1}^{RV}) + \omega_2 Ln(\sigma_{t-1}^{RV}) \times I_{r_{t-1}<0} + e_t, \tag{13}$$

$$\text{ARFIMA+J: } (1-B)^d Ln(\sigma_t^{RV}) = \omega_0 + \omega_1 Ln(\sigma_{t-1}^{RV}) + \beta^{Jump} J_{t-1} + e_t, \tag{14}$$

$$\text{ARFIMA+LJ: } (1-B)^d Ln(\sigma_t^{RV}) = \omega_0 + \omega_1 Ln(\sigma_{t-1}^{RV}) + \omega_2 Ln(\sigma_{t-1}^{RV}) \times I_{r_{t-1}<0} + \beta^{Jump} J_{t-1} + e_t, \tag{15}$$

where ω, β^{Jump}, e, B, I and r are the coefficient, jump coefficient, error term, backward shift operator, dummy variable and index return, respectively. Hosking (1981) investigates the likely value of long-memory parameter for ARFIMA model and discovers that for $-0.5 < d < 0.5$ the process is stationary and invertible. If $-0.5 < d < 0$, the process is said to exhibit anti-persistence because the autocorrelations are negative. If $0<d \leqq 0.5$, the process exhibits long-memory. The most prevalent method[8] for estimating the fractional differencing parameter is the two-step procedure proposed by Geweke and Porter-Hudak (GPH, 1983). The value of d should be estimated first and then used as an input variable to estimate all parameter values in the model, because it is wrong to treat endogenous variables as input variables in the model. Koop, Ley, Osiewalski and Steel (1997) suggest that using Bayesian method to estimate all parameter values simultaneously provides more reliable estimates. Therefore, we use Bayesian method to estimate the value of d in the ARFIMA model.

EMPIRICAL RESULTS

Volatility and Its Forecasts

While some prior studies [e.g., ABDL (2003), Pong et al. (2004)] use a 30-minute interval to estimate realized volatility, we use the interval of one minute in the very last year of the sample period. The RV value is close to 12%. The RV for the non-overlapping 45- second multiples of 6 (RV6), 9 (RV9), 12 (RV12), and 30 (RV30) minutes interval appears to be quite consistent and stable, as in Figure 2. Table 1 lists all summary statistics for all realized volatilities. We find that the estimate of RV3 (3-minute interval) differs the most between index futures and spot index, while the estimate of RV30 differs the least and is highly stable.

FIGURE 2
THE TAIEX REALIZED VOLATILITY OVER DIFFERENT SAMPLING FREQUENCIES

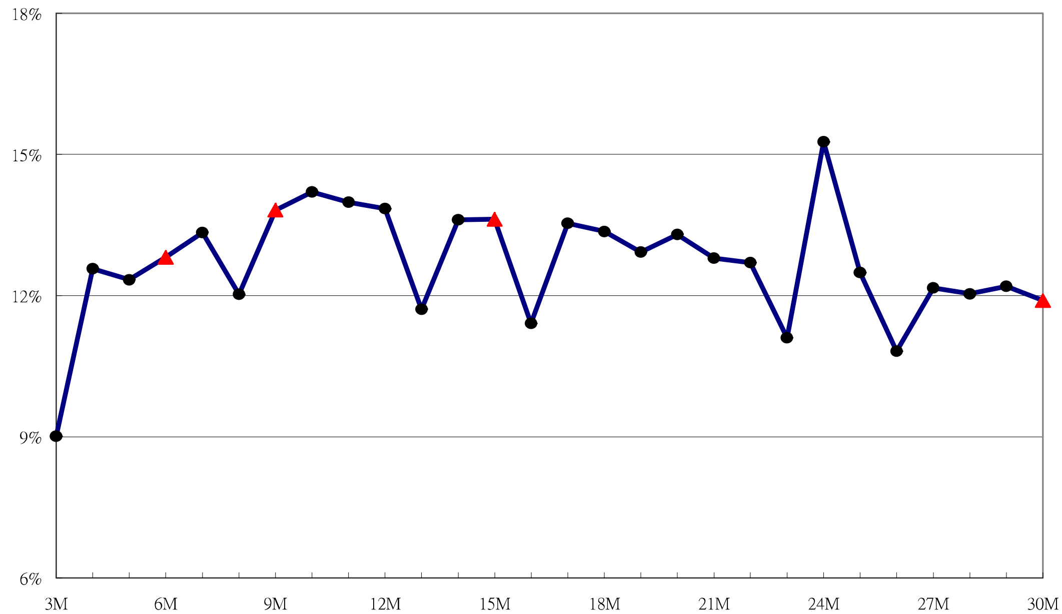


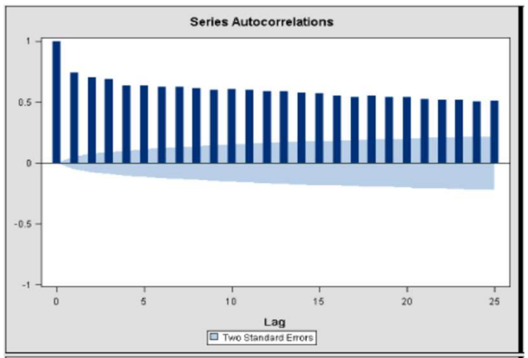
TABLE 1
SUMMARY STATISTICS OF REALIZED VOLATILITY AND HISTORICAL VOLATILITY FOR TAIEX AND TX

This table provides summary statistics for realized volatility of TAIEX and TX across five sample interval period (RV3, RV6, RV9, RV15 and RV30) and historical volatility (HV) of TAIEX and TX.

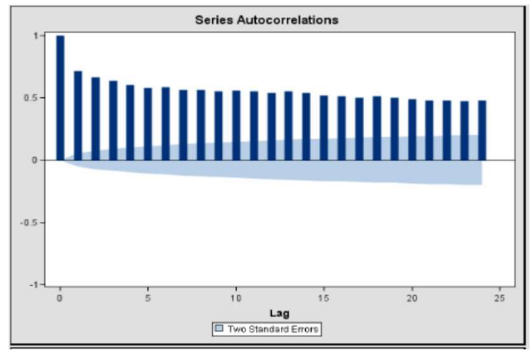
Index	Variable	RV3	RV6	RV9	RV15	RV30	HV
TAIEX	Means(1)	9.37%	12.36%	13.27%	13.10%	11.91%	20.84%
	Volatility	3.82%	5.63%	6.11%	6.15%	5.66%	2.65%
	Maximum	26.06%	40.10%	43.57%	45.77%	50.75%	26.15%
	Minimum	2.25%	2.19%	1.85%	2.22%	1.88%	15.10%
	Median	9.06%	11.05%	11.85%	11.61%	9.60%	20.65%
	Skew	0.5971	1.0539	1.1512	1.2593	1.7316	-0.0325
	Kurt	0.1147	1.2631	1.6276	2.0129	5.2991	-0.6868
TX	Means(2)	22.97%	19.62%	18.17%	15.82%	12.50%	24.18%
	Volatility	17.14%	14.55%	13.37%	11.65%	9.26%	4.23%
	MAX	98.95%	99.53%	96.30%	89.54%	75.51%	33.94%
	MIN	4.40%	4.20%	3.86%	0.00%	0.00%	14.63%
	Median	17.06%	14.72%	13.76%	12.31%	10.00%	24.93%
	Skew	1.8382	2.2753	2.2636	2.4699	2.4934	-0.2176
	Kurt	3.4545	6.3858	6.5181	8.1449	9.1175	0.1643
DIFF(2)-(1)		13.59%	7.26%	4.90%	2.72%	1.59%	3.34%

We find from Figure 3 that the ACF (autocorrelation function) for four realized volatilities diminish to zero much more slowly than the ARMA process. This is consistent with Hosking's (1981) finding about the long memory property. To obtain a stable value of d (the fractional difference), we use an ARFIMA(1,d,0) model with in-sample data every half year to calculate the moving average of the d value. As shown in Figure 4, 12 moving averages of the d values are obtained. The moving average of the d values appear to be quite stable from number 6 to number 10. The average d value is 0.406 for 6 minutes; 0.4018 for 9 minutes; 0.345 for 15 minutes; and 0.2133 for 30 minutes. They are all in the long memory region of 0 < d < 0.5. We can thus confirm the long memory property for the TAIEX volatility through Bayesian estimation of the d values. Comparing d values of the four realized volatilities, we also find that RV9 and RV30 volatilities are relatively lower. As such, we will continue to use RV9 and RV30 in further tests.

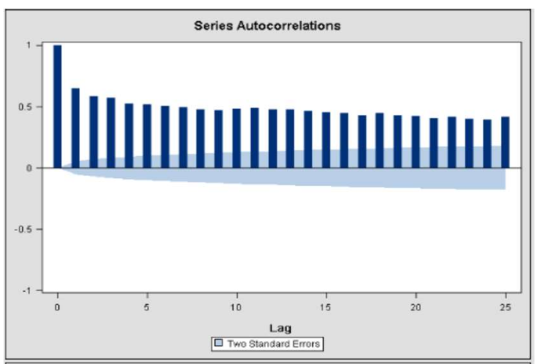
FIGURE 3
THE AUTOCORRELATION FUNCTION OF REALIZED VOLATILITY



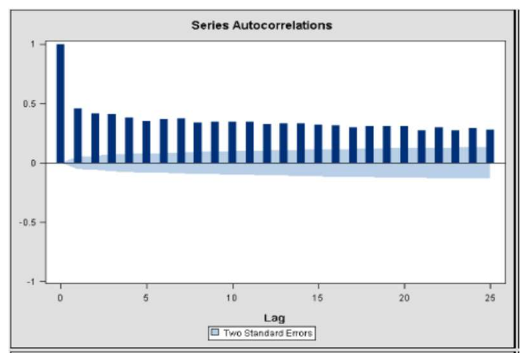
RV6



RV9



RV15



RV30

FIGURE 4
THE ESTIMATED VALUES OF PARAMETER D IN THE ARFIMA (1, D, 0) MODEL FOR DIFFERENT REALIZED VOLATILITIES IN VARIOUS MOVING PERIODS

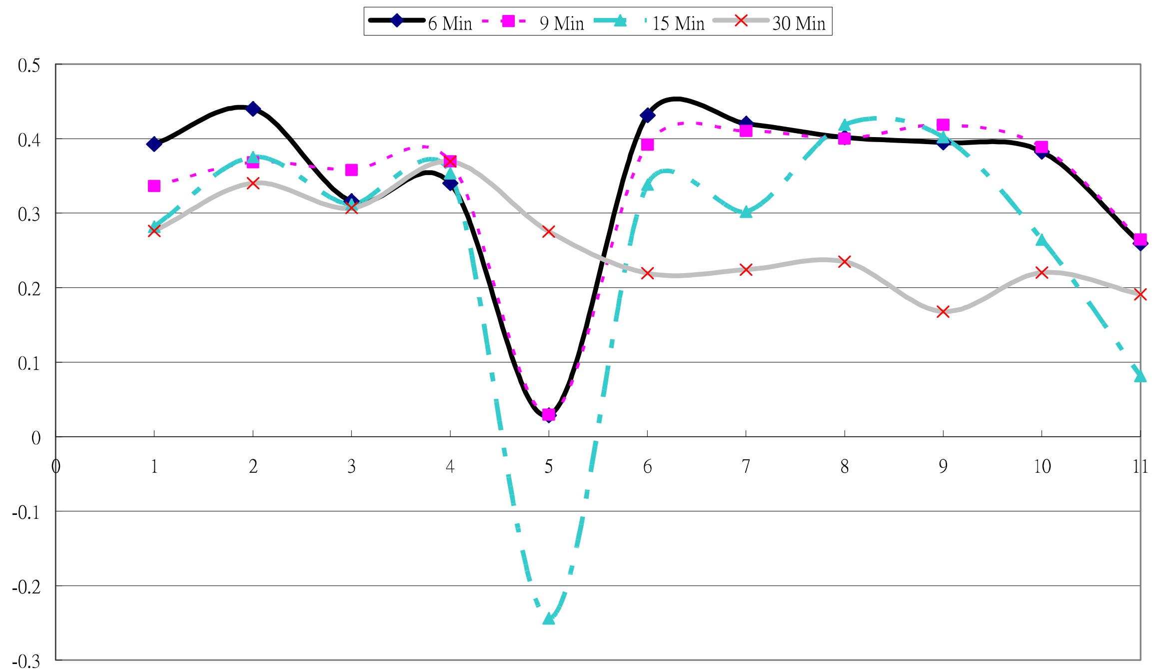


Table 2 gives all statistics for the IV average values. It shows higher IV estimates and standard deviations for the nearby contracts. These volatilities exhibit much more information content with the skewness and kurtosis approximating normal distribution, as typically assumed in the options literature.

TABLE 2
SUMMARY STATISTICS OF IMPLIED VOLATILITY FROM CALL AND PUT OPTIONS WITH DIFFERENT MONEYNESS

This table provides summary statistics for implied volatility across two moneyness and two maturities in TXO. ATM are options where 0.97 < M < 1.03. Nearby are options contract with a maturity period of 8 to 30 days. Options with maturity longer than 2 months are excluded because of light trading activities.

Maturity	Moneyness	All			ATM		
	Variables	σ^{IV}	σ^{IV}_{Call}	σ^{IV}_{Put}	σ^{IV}	σ^{IV}_{Call}	σ^{IV}_{Put}
All	Means	29.47%	29.78%	29.14%	28.91%	28.67%	29.15%
	Volatility	4.12%	4.20%	4.23%	4.28%	4.58%	4.29%
	Maximum	43.34%	43.69%	43.05%	44.61%	44.61%	44.61%
	Minimum	21.84%	21.70%	20.73%	21.48%	19.67%	20.22%
	Median	29.03%	29.90%	28.35%	28.67%	28.92%	28.24%
	Skew	0.9391	0.6571	1.0735	0.7922	0.4393	0.9614
	Kurtosis	1.5318	0.9318	1.8735	1.3347	0.8764	1.6162
Nearby	Means	31.01%	30.44%	31.38%	29.10%	28.79%	29.42%
	Volatility	4.75%	5.09%	5.08%	4.35%	4.68%	4.38%
	Maximum	45.73%	45.23%	46.38%	44.61%	44.61%	44.61%
	Minimum	22.14%	20.16%	21.30%	21.48%	20.29%	20.22%

Median	30.26%	30.37%	30.51%	28.72%	28.92%	28.45%
Skew	0.8876	0.5492	0.9728	0.7167	0.3629	0.8884
Kurtosis	1.0046	0.6112	1.0261	0.9934	0.5954	1.1592

Table 3 presents estimated values for various volatilities. These average estimated values are observed on t-1, t-5, and t-20 days. They are the proxy predictor variables for all information known up to day t. We find that the standard deviation of forecasted values is less than that of estimated value. It is because of previous information on average values contained in forecasted values. From the kurtosis and skewness coefficients of the forecasted values, we find that most time series exhibit normal distribution except RV9's forecasted values for the spot index and index futures, which shows right skewness and high kurtosis.

TABLE 3
SUMMARY STATISTIC OF FORECASTING ESTIMATORS ON VARIOUS VOLATILITIES

This table provides summary statistics for estimated values of volatility based on volatility forecasts from different forecasting methods. These estimated values are observed on t-1, t-5, and t-20 days from different forecasting method. ATM are options where 0.97 < M < 1.03. All options are with a maturity period of 8 to 30 days. Options with maturity longer than 2 months are excluded because of light trading activities. Estimated RV employs the 9-minute interval (RV9) and 30-minute interval (RV30).

Statistics	$\sigma^{IV}_{ATM-All}$	$\sigma^{IV}_{ATM-Call}$	$\sigma^{IV}_{ATM-Put}$	σ^{DVF}_{All}	σ^{DVF}_{Call}	σ^{DVF}_{Put}
Means	29.04%	28.66%	30.81%	29.99%	29.69%	30.55%
Volatility	2.39%	3.06%	2.52%	3.16%	4.07%	3.04%
Maximum	35.88%	37.84%	39.05%	39.75%	42.36%	41.38%
Minimum	22.74%	21.54%	25.54%	23.49%	15.98%	24.55%
Median	29.02%	28.86%	30.53%	29.79%	29.86%	30.43%
Skew	0.0753	0.1183	0.7275	0.3058	-0.2783	0.6093
Kurtosis	-0.0499	0.4737	0.6232	0.1189	1.0188	0.6675
Statistics	σ^{RV9}_{TAIEX}	σ^{RV30}_{TAIEX}	σ^{HV}_{TAIEX}	σ^{RV9}_{TX}	σ^{RV30}_{TX}	σ^{HV}_{TX}
Means	19.97%	16.24%	20.25%	22.18%	17.36%	23.26%
Volatility	3.32%	3.57%	1.50%	5.28%	4.48%	2.94%
Maximum	32.71%	28.74%	23.22%	40.65%	30.39%	29.94%
Minimum	11.96%	9.15%	16.96%	12.04%	8.05%	16.94%
Median	19.46%	15.65%	20.02%	21.51%	17.10%	24.31%
Skew	1.0069	0.6443	-0.1299	1.1041	0.6008	-0.4640
Kurtosis	2.4335	0.4126	-0.5497	2.1102	0.3746	-0.6084

Table 4 shows forecasted values of realized volatility using a fitted ARFIMA model. We find higher RV values, especially higher RV9. The RV series using the fitted ARFIMA follows the normal distribution.

TABLE 4
SUMMARY STATISTIC OF FORECASTING ESTIMATOR FOR RV USING ARFIMA (1, D, 0) RELATED MODELS

This table provides summary statistics for estimated RV from TAIEX and TX based on various ARFIMA related models.

Samples		Estimator	ARFIMA	ARFIMA +Leverage	ARFIMA +Jump	ARFIMA + Leverage + Jump
TAIEX	9 Min	Means	0.2817	0.2825	0.2818	0.2815
		Volatility	2.50%	3.23%	2.53%	1.33%
		Maximum	0.5503	0.6152	0.6906	0.6835
		Minimum	0.0184	0.0085	0.0155	0.0125
		Skew	-0.0028	0.3946	-0.0188	0.0946
		Kurtosis	-0.8007	-0.0611	-0.7702	-0.0726
	30 Min	Means	0.2471	0.2478	0.2472	0.2477
		Volatility	3.65%	3.89%	3.94%	2.98%
		Maximum	0.6104	0.6454	0.6108	0.6191
		Minimum	0.5650	0.5536	0.5367	0.5159
		Skew	-0.0232	0.3914	-0.7311	-0.4508
		Kurtosis	-0.5990	-0.1393	-0.9425	-0.0964
TX	9 Min	Means	0.2956	0.3057	0.3008	0.3025
		Volatility	5.65%	5.98%	5.48%	5.01%
		Maximum	0.6508	0.6954	0.7011	0.7152
		Minimum	0.0298	0.0350	0.0300	0.0201
		Skew	-0.0018	0.3598	-0.0145	0.1146
		Kurtosis	-0.9116	-0.0784	-0.8510	-0.0756
	30 Min	Means	0.2800	0.3000	0.2900	0.2988
		Volatility	5.87%	6.21%	6.45%	6.15%
		Maximum	0.6956	0.7011	0.7135	0.7013
		Minimum	0.0199	0.0100	0.0018	0.0101
		Skew	-0.0119	0.4544	-0.1568	0.1987
		Kurtosis	-0.9975	-0.0985	-0.9570	-0.0987

Encompassing Regression Results

In an encompassing regression, if the information contained in independent variables can completely explain the dependent variable, the market is deemed efficient. So, all regression coefficients are significant, positive, and less than one, and should sum up to one. It means the dependent variable is completely explained by the independent variables. It also implies that the encompassing regression models are good quality volatility forecasting models. Moreover, we can observe the information contents in different volatility via regression coefficients.

The majority of the prior studies on volatility forecasting concludes that the forecasting ability of historical volatility (HV) is inferior to that of implied volatility (IV). Surprisingly, Table 5 shows that HV comes first in power of explanation, followed by IV, in forecasting RV. It seems that DVFIV does not have superior forecasting power for RV. This agrees with Dumas, Fleming, and Whaley (1998) with regards to DVFIV as described above.

TABLE 5
ENCOMPASSING REGRESSION TESTS FOR REALIZED VOLATILITY REGRESSED ON VARIOUS FORECASTING ESTIMATORS

The encompassing regression model is specified as follows (RV model): $Ln(\sigma_t^{RV}) = \beta_0 + \beta_1 Ln(\sigma_{t|\Omega_{t-1}}^{HV^e}) + \beta_2 Ln(\sigma_{t|\Omega_{t-1}}^{IV,ATM^e}) + \beta_3 Ln(\sigma_{t|\Omega_{t-1}}^{DVFIV^e}) + \varepsilon_t$, where β, ε and Ω_{t-1} are the regression coefficients, error term, and information collected at time t-1, respectively. $\sigma_{t|\Omega_{t-1}}^{HV^e}$, $\sigma_{t|\Omega_{t-1}}^{IV\,ATM^e}$ and $\sigma_{t|\Omega_{t-1}}^{DVFIV^e}$ respectively are the day-t predictors of historical volatility (HV), ATM implied volatility (IV) and implied volatility computed by the deterministic volatility models (DVFIV), conditional on information at day t-1. σ_t^{RV} is realized volatility (RV). The superscript e stands for forecast value. Dependent variables (RV) are estimated using the 9-minute interval and 30-minute interval.

Dependent	β_0	$\beta_{1(TAIEX)}$	$\beta_{1(TX)}$	β_2	β_3	F	R^2	σ^{option}
RV9	-0.168 ***	**0.498** ***		0.332 ***	0.116 ***	410.38 ***	0.47	All
	(-2.73)	(11.57)		(4.53)	(2.86)			
	-0.171 ***		0.412 ***	0.339 ***	0.110 ***	409.11 ***	0.47	All
	(-3.51)		(12.04)	(4.12)	(2.62)			
RV9	-0.113 **	**0.503** ***		0.318 ***	0.260 ***	436.35 ***	0.49	Call
	(-1.98)	(12.96)		(6.31)	(3.53)			
	-0.106 **		0.495 ***	0.326 ***	0.198 ***	395.07 ***	0.46	Call
	(-1.88)		(12.71)	(4.55)	(2.69)			
RV9	-0.197 ***	**0.680** ***		0.205 ***	0.071	389.15 ***	0.46	Put
	(-3.14)	(18.77)		(3.46)	(1.09)			
	-0.189 ***		0.601 ***	**0.197** ***	0.090	451.87 ***	0.51	Put
	(-2.94)		(19.53)	(4.69)	(1.17)			
RV30	-0.723 ***	0.358 ***		**0.477** ***	0.020	194.71 ***	0.30	All
	(-9.40)	(7.09)		(5.19)	(0.21)			
	-0.657 ***		0.312 ***	0.485 ***	0.091	201.55 ***	0.29	All
	(-9.06)		(6.67)	(4.57)	(0.10)			
RV30	-0.622 ***	**0.426** ***		0.338 ***	0.141	200.87 ***	0.30	Call
	(-8.62)	(8.95)		(5.31)	(1.51)			
	-0.604 ***		0.405 ***	0.342 ***	0.101	171.61 ***	0.24	Call
	(-8.71)		(6.54)	(4.28)	(1.03)			
RV30	-0.741 ***	**0.544** ***		0.307 ***	-0.075	185.78 ***	0.29	Put
	(-9.51)	(12.45)		(4.18)	(-0.93)			
	-0.669 ***		0.517 ***	0.301 ***	0.021	222.34 ***	0.32	Put
	(-9.17)		(13.38)	(4.59)	(0.16)			

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

We believe the high explanatory power of HV derives from better HV estimates obtained from the ARMA model. The prior literature suggests that futures market leads spot market. Our empirical results show that the spot market HV can explain RV much better than the futures market HV. This is anticipated owing to the fact that we use the same sample to estimate RV and spot market HV. IV can explain RV as well as HV when call options are lumped together with put options. However, when a call option is separated from a put option, IV has lower ability to explain RV, while HV possesses higher ability in explaining RV. Table 5 implies that HV of spot and futures market is an unbiased estimator of RV. It is very important for option traders whether HV and RV are unbiased estimators of IV. Table 6 compares spot index volatility and index futures volatility in explaining IV. However, spot volatility becomes less predictive of IV once futures market HV and RV are included. Of all the explanatory variables, futures market HV is the best in explaining IV, while the coefficient for spot market HV becomes negative. Neither spot market RV nor futures market RV can forecast IV better than HV, even though the spot market RV performs less satisfactorily than futures market RV as an independent variable.

Since some estimated RV explains IV poorly, we use the ARFIMA model to have better RV estimates (labeled as ARV) and use those RV estimates in running the encompassing regression as specified in Equation (3) (labeled as IV-ARV model). As Table 7 shows, ARV can explain IV much better. It is very noticeable that the spot ARV explains the call option IV quite well, as is true for futures ARV in forecasting the put option IV. Of the four volatility variables, futures HV is the best in explaining and forecasting power. The spot HV cannot forecast IV because it lacks the information contents provided by futures volatility. Overall, the RV estimated in the 9-minute interval via encompassing regression is superior to that estimated in the 30-minute interval. Results in Table 6 lead to similar conclusions. Furthermore, comparing the performance of different ARFIMA models in the 9-minute interval via encompassing regression analysis, we find that RV9 fitted with ARFIMA+J model (ARV9+J) has high power of explanation for IV. This result implies that Taiwan stock market return does follow a jump path.

TABLE 6
ENCOMPASSING REGRESSION TESTS FOR IMPLIED VOLATILITY REGRESSED ON VARIOUS FORECASTING ESTIMATORS

The encompassing regression model is specified as follows (IV-RV model): $Ln(\sigma_t^{IV}) = \beta_0 + \beta_1 Ln(\sigma_{t|\Omega_{t-1}}^{TAIEX,HV^e}) + \beta_2 Ln(\sigma_{t|\Omega_{t-1}}^{TAIEX,RV^e}) + \beta_3 Ln(\sigma_{t|\Omega_{t-1}}^{TX,HV^e}) + \beta_4 Ln(\sigma_{t|\Omega_{t-1}}^{TX,RV^e}) + \varepsilon_t$, where β, ε and Ω_{t-1} are the regression coefficients, error term, and information collected at time t-1, respectively. σ_t^{IV} is implied volatility. σ_t^{TX-HV} and σ_t^{TX-RV} are respectively historical volatility and realized volatility of TAIEX. σ_t^{TX-HV} and σ_t^{TX-RV} are respectively historical volatility and realized volatility of TX. The superscript e stands for forecast value. RVs are estimated using the 9-minute interval and 30-minute interval. All options are with a maturity of 8 to 30 days.

IV	RV	β_0	β_1	β_2	β_3	β_4	F	R^2
All	RV9	-0.259***	**0.161*****	0.155***			461.96***	0.39
		(-7.31)	**(10.72)**	(5.42)				
		-0.27***			**0.375*****	0.164***	583.47***	0.45
		(-8.63)			**(14.22)**	(7.01)		
		-0.393***		0.151***		**0.285*****	432.81***	0.37
		(-12.35)		(4.91)		**(9.33)**		
		-0.370***	-0.407***	0.149***	**0.690*****	0.081***	304.00***	0.46
		(-9.99)	(-5.25)	(3.79)	**(10.28)**	(2.41)		
	RV30	-0.192***	**0.178*****	0.156***			475.92***	0.40
		(-5.20)	**(14.17)**	(6.83)				
		-0.264***			**0.421*****	0.114***	577.96***	0.45
		(-8.29)			**(18.84)**	(6.54)		
		-0.411***		**0.219*****		0.170***	351.04***	0.33
		(-11.63)		**(6.95)**		(6.21)		
		-0.314***	-0.312***	0.127***	**0.672*****	0.039	299.62***	0.46
		(-8.48)	(-4.59)	(3.73)	**(10.79)**	(1.44)		
Call	RV9	-0.213***	**0.103*****	0.263***			527.55***	0.42
		(-5.72)	**(8.56)**	(8.76)				
		-0.305***			**0.397*****	0.159***	496.42***	0.41
		(-8.72)			**(13.46)**	(6.09)		
		-0.345***		0.124***		**0.209*****	384.17***	0.37
		(-10.07)		(3.71)		**(5.46)**		
		-0.309***	-0.196**	0.244***	**0.465*****	-0.062*	280.58***	0.44
		(-7.58)	(-2.29)	(8.05)	**(6.30)**	(-1.70)		
	RV30	-0.131***	**0.188*****	0.202***			521.73***	0.42
		(-3.34)	**(13.73)**	(8.35)				
		-0.304***			**0.447*****	0.105***	489.63***	0.41
		(-8.52)			**(17.88)**	(5.37)		
		-0.411***		**0.207*****		0.167***	305.98***	0.34
		(-11.63)		**(5.81)**		(5.54)		
		-0.198***	0.022	0.260***	**0.363*****	-0.057*	270.80***	0.43
		(-4.84)	(0.29)	(6.91)	**(5.27)**	(-1.90)		
Put	RV9	-0.258***	**0.123*****	0.082**			323.92***	0.31
		(-6.30)	**(10.88)**	(2.49)				
		-0.213***			**0.378*****	0.175***	478.37***	0.40
		(-6.00)			**(12.64)**	(6.60)		
		-0.385***		-0.016		**0.443*****	348.22***	0.32
		(-10.72)		(-0.47)		**(12.84)**		
		-0.381***	-0.554***	0.006	**0.865*****	0.191***	265.88***	0.43
		(-9.17)	(-6.37)	(0.15)	**(11.51)**	(5.09)		
	RV30	-0.197***	**0.187*****	0.133***			337.87***	0.32
		(-4.63)	**(12.57)**	(5.05)				
		-0.205***			**0.424*****	0.125***	475.39***	0.40
		(-5.68)			**(16.76)**	(6.31)		
		-0.419***		0.098***		**0.273*****	278.88***	0.28
		(-10.56)		(2.78)		**(8.85)**		
		-0.362***	-0.559***	0.047	**0.918*****	0.102***	262.13***	0.42
		(-8.74)	(-7.35)	(1.24)	**(13.18)**	(3.36)		

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

TABLE 7
ENCOMPASSING REGRESSION TESTS FOR IMPLIED VOLATILITY REGRESSED ON VARIOUS FORECASTING ESTIMATORS USING THE IV-ARV MODEL

The encompassing regression model is specified as follows (IV-ARV model): $Ln(\sigma_t^{IV}) = \beta_0 + \beta_1 Ln(\sigma_{t|\Omega_{t-1}}^{TAIEX.HV^e}) + \beta_2 Ln(\sigma_{t|\Omega_{t-1}}^{TAIEX.ARV^e}) + \beta_3 Ln(\sigma_{t|\Omega_{t-1}}^{TX.HV^e}) + \beta_4 Ln(\sigma_{t|\Omega_{t-1}}^{TX.ARV^e}) + \varepsilon_t$, where β, ε and Ω_{t-1} are the regression coefficients, error term, and information collected at time t-1, respectively. σ_t^{IV} is implied volatility. σ_t^{TX-HV} and σ_t^{TX-ARV} are respectively historical volatility and realized volatility estimated by the ARFIMA model of TAIEX. σ_t^{TX-HV} and σ_t^{TX-ARV} are respectively historical volatility and realized volatility estimated by the ARFIMA model of TX. The superscript e stands for forecast value. RVs are estimated using the 9-minute interval and 30-minute interval. All options are with a maturity of 8 to 30 days.

IV	ARV variables	β_0	β_1	β_2	β_3	β_4	F	R^2
All	ARV9	-0.359*** (-5.15)	-0.012 (-1.27)	0.188** (2.45)	0.417*** (9.58)	0.199* (2.00)	411.25***	0.35
	ARV9+J	-0.339*** (-5.54)	-0.019 (-1.12)	0.201*** (3.99)	0.356*** (9.12)	0.371** (2.54)	498.19***	0.40
	ARV9+L	-0.398*** (-5.89)	-0.011 (-1.50)	0.149* (1.90)	0.519*** (9.26)	0.175 (1.10)	300.35***	0.31
	ARV9+J/L	-0.342*** (-5.10)	-0.017 (-1.17)	0.104* (1.81)	0.490*** (9.10)	0.093* (1.89)	351.01***	0.35
	ARRV30	-0.304*** (-4.41)	-0.201* (-1.79)	0.180* (1.93)	**0.451*** (10.19)**	0.101* (2.10)	256.75***	0.21
	ARV30+J	-0.249*** (-3.94)	-0.100 (-0.59)	0.198** (2.54)	**0.380*** (9.51)**	0.305** (3.58)	278.02***	0.25
	ARV30+L	-0.298*** (-3.82)	-0.215* (-2.01)	0.101* (1.89)	**0.592*** (10.16)**	0.109 (0.92)	200.54***	0.19
	ARV30+J/L	-0.208*** (-3.95)	-0.156 (-1.06)	0.128* (1.90)	**0.490*** (10.01)**	0.093* (1.87)	208.46***	0.20
Call	ARV9	-0.310*** (-5.10)	0.026 (0.01)	0.350*** (8.05)	**0.450*** (5.31)**	0.089 (1.00)	251.51***	0.30
	ARV9+J	-0.311*** (-5.72)	0.027 (0.09)	0.398*** (8.05)	**0.455*** (5.56)**	0.081 (1.11)	256.57***	0.35
	ARV9+L	-0.326*** (-5.00)	-0.030 (-0.02)	0.344*** (8.05)	**0.450*** (5.01)**	0.029 (0.20)	249.01***	0.28
	ARV9+J/L	-0.327***	-0.031	0.351***	**0.452*****	0.072	250.27***	0.29

		(-5.01)	(-0.15)	(8.05)	**(5.09)**	(0.95)		
	ARRV30	-0.195*** (-3.00)	0.001 (0.18)	0.311*** (7.05)	**0.370*** (5.25)**	-0.055 (1.00)	201.12***	0.25
	ARV30+J	-0.187*** (-3.18)	0.015 (0.12)	0.351*** (7.11)	**0.364*** (5.10)**	-0.060* (1.94)	216.77***	0.28
	ARV30+L	-0.192*** (-3.08)	-0.010 (-0.21)	0.336*** (7.12)	**0.350*** (4.98)**	-0.050* (2.03)	200.14***	0.16
	ARV30+J/L	-0.191*** (-3.84)	-0.022 (-0.29)	0.349*** (7.81)	0.310*** (4.55)	-0.057* (-1.76)	200.46***	0.20
Put	ARV9	-0.391*** (-8.01)	-0.021* (-3.10)	0.010 (1.15)	0.611*** (10.15)	0.290*** (6.18)	391.10***	0.41
	ARV9+J	-0.314*** (-9.57)	-0.023* (-3.56)	0.015* (1.79)	0.602*** (10.69)	0.290*** (6.91)	401.28***	0.45
	ARV9+L	-0.364*** (-9.91)	-0.022* (-3.08)	0.016 (1.01)	0.695*** (10.34)	0.201* (1.87)	305.73***	0.40
	ARV9+J/L	-0.335*** (-9.00)	-0.029* (-3.81)	0.011 (1.50)	0.656*** (10.10)	0.274*** (5.89)	361.61***	0.42
	ARRV30	-0.385*** (-8.55)	-0.218* (-2.05)	0.011 (0.01)	0.754*** (13.18)	0.152** (2.10)	280.13***	0.30
	ARV30+J	-0.302*** (-8.09)	-0.201 (-1.04)	0.016 (0.29)	0.700*** (13.18)	0.113** (2.26)	291.07***	0.33
	ARV30+L	-0.345*** (-8.48)	-0.219 (-1.49)	0.011 (0.78)	0.810*** (13.18)	0.105 (1.49)	201.22***	0.28
	ARV30+J/L	-0.328*** (-8.66)	-0.203 (-1.37)	0.012 (0.54)	0.766*** (13.18)	0.110* (2.06)	205.87***	0.29

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Forecasting Error

Through in-sample and out-of-sample forecast error, we should be able to delineate the forecasting ability of different volatility forecasting models. Tables 8, 9, and 10 contain all in-sample and out-of-sample forecast errors from the encompassing regression analysis. There are three measures of forecast errors, i.e., MSE, MAE and MAPE. In general, the forecast error from RV9 is smaller than those from RV30, indicating a high correlation between matching time interval and the forecasting ability of volatility models. Table 8 contains the error in forecasting RV via the encompassing regression, as in Table 5. Regardless of if it's in-sample or out-of-sample, the RV9 forecast error is the smallest when information on futures HV and put option IV are provided. The second best in-sample forecast encompasses information from spot HV and call option IV. The second best out-of-sample forecast comes from index futures HV and all option IV. Panel C of Table 8 reports the robustness test results and confirms that the RV9 forecast error remains the smallest when we use futures HV and call option IV embedded in information proxy variable in the forecasting model. But even in this case, the forecast error from the pre-Presidential election sample is smaller than that from the post-Presidential election sample. The forecast error with call option IV as a forecasting variable is smaller than that with put option IV in the post-Presidential election period, due possibly to the post-election stock market collapse in Taiwan.

TABLE 8
FORECASTING ERRORS OF THE RV MODEL

$$Ln(\sigma_t^{RV}) = \beta_0 + \beta_1 Ln(\sigma_{t\,|\,\Omega\ t-1}^{HV^e}) + \beta_2 Ln(\sigma_{t\,|\,\Omega\ t-1}^{IV,ATM^e}) + \beta_3 Ln(\sigma_{t\,|\,\Omega\ t-1}^{DVFIV^e}) + \varepsilon_t$$

In Panel A, B and C respectively are the in-sample, pre-Presidential election out-of-sample and post-Presidential election out-of-sample forecast errors from the encompassing regression analysis. MSE is the mean square error. MAE is the mean absolute error. MAPE is the mean absolute percentage error. RVs are estimated using the 9-minute interval and 30-minute interval. All options are with a maturity of 8 to 30 days.

HV variables		σ_{TAIEX}^{HV}			σ_{TX}^{HV}		
RV	σ_{ATM}^{IV}	MSE	MAE	MAPE	MSE	MAE	MAPE
A. In-sample							
RV9	All	0.0974	0.2785	0.7183	0.0978	0.2790	0.7185
	Call	0.0946	0.2780	0.7179	0.0951	0.2782	0.7180
	Put	0.0991	0.2831	0.7200	0.0910	0.2771	0.7278
RV30	All	0.1159	0.3087	0.7319	0.1155	0.3055	0.7316
	Call	0.1118	0.3028	0.7314	0.1198	0.3112	0.7328
	Put	0.1167	0.3090	0.7321	0.1087	0.2991	0.7305
B. Pre-Presidential election out-of-sample							
RV9	All	0.1256	0.3194	0.7339	0.1211	0.3129	0.7331
	Call	0.1296	0.3247	0.7351	0.1216	0.3132	0.7336
	Put	0.1294	0.3242	0.7349	0.1201	0.3128	0.7329
RV30	All	0.1299	0.3249	0.7352	0.1281	0.3237	0.7345
	Call	0.1309	0.3261	0.7362	0.1305	0.3258	0.7356
	Put	0.1319	0.3273	0.7387	0.1294	0.3243	0.7349
C. Post-Presidential election out-of-sample							
RV9	All	0.1275	0.3229	0.7397	0.1250	0.3172	0.7395
	Call	0.1298	0.3275	0.7392	0.1242	0.3171	0.7382
	Put	0.1299	0.3280	0.7405	0.1251	0.3175	0.7399
RV30	All	0.1299	0.3231	0.7405	0.1261	0.3192	0.7399
	Call	0.1301	0.3284	0.7403	0.1251	0.3190	0.7398
	Put	0.1305	0.3298	0.7411	0.1265	0.3194	0.7405

Table 9 shows the in-sample and out-of-sample forecast error from encompassing regression model (3). The smallest forecast error for IV occurs when there are four predictor variables containing volatility information in the regression model. In Panel A and B, the smallest in-sample MSE is 0.0384 and the smallest out-of-sample MSE is 0.0675. The forecast error for IV appears inconsistent with regards to call option and put option. Panel C of Table 9 reports similar empirical results from the robustness tests.

TABLE 9
FORECASTING ERRORS OF THE IV-RV MODEL

$$Ln(\sigma_t^{IV}) = \beta_0 + \beta_1 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TAIEX.HV^e}) + \beta_2 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TAIEX.RV^e}) + \beta_3 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX.HV^e}) + \beta_4 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX.RV^e}) + \varepsilon_t$$

In Panel A, B and C respectively are the in-sample, pre-Presidential election out-of-sample and post-Presidential election out-of-sample forecast errors from the encompassing regression analysis. MSE is the mean square error. MAE is the mean absolute error. MAPE is the mean absolute percentage error. RVs are estimated using the 9-minute interval and 30-minute interval. All options are with a maturity of 8 to 30 days.

Independent variable		All			Call			Put		
		MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
A. In-sample										
9 min	TX-HV, TX-RV	0.0432	0.1649	0.5350	0.0461	0.1690	0.5363	0.0577	0.1915	0.5986
	TXF-HV, TXF-RV	0.0391	0.1555	0.5264	0.0489	0.1751	0.5478	0.0503	0.1781	0.5614
	TX-RV, TXF-RV	0.0411	0.1605	0.5326	0.0488	0.1746	0.5415	0.0550	0.1881	0.5725
	Full	0.0384	0.1540	0.5251	0.0465	0.1693	0.5399	0.0482	0.1715	0.5401
30 min	TX-HV, TX-RV	0.0498	0.1766	0.5550	0.0511	0.1815	0.5664	0.0598	0.1950	0.6181
	TXF-HV, TXF-RV	0.0412	0.1612	0.5347	0.0556	0.1899	0.5784	0.0578	0.1924	0.6079
	TX-RV, TXF-RV	0.0455	0.1685	0.5351	0.0512	0.1827	0.5701	0.0511	0.1825	0.5699
	Full	0.0399	0.1598	0.5318	0.0510	0.1801	0.5651	0.0555	0.1885	0.5771
B. Pre-Presidential election out-of-sample										
9 min	TX-HV, TX-RV	0.0735	0.2460	0.6359	0.0766	0.2503	0.6399	0.0879	0.2728	0.6997
	TXF-HV, TXF-RV	0.0690	0.2384	0.6274	0.0795	0.2563	0.6481	0.0805	0.2593	0.6624
	TX-RV, TXF-RV	0.0710	0.2417	0.6341	0.0790	0.2554	0.6466	0.0852	0.2693	0.6763
	Full	0.0675	0.2355	0.6269	0.0771	0.2505	0.6409	0.0788	0.2527	0.6411
30 min	TX-HV, TX-RV	0.0800	0.2581	0.6560	0.0813	0.2627	0.6675	0.0900	0.2762	0.7177
	TXF-HV, TXF-RV	0.0714	0.2424	0.6357	0.0861	0.2711	0.6794	0.0880	0.2744	0.7092
	TX-RV, TXF-RV	0.0757	0.2497	0.6382	0.0815	0.2639	0.6711	0.0813	0.2637	0.6709
	Full	0.0707	0.2410	0.6328	0.0812	0.2621	0.6662	0.0857	0.2699	0.6781
C. Post-Presidential election out-of-sample										
9 min	TX-HV, TX-RV	0.0739	0.2468	0.6366	0.0799	0.2561	0.6415	0.0896	0.2731	0.7001
	TXF-HV, TXF-RV	0.0694	0.2389	0.6299	0.0799	0.2570	0.6488	0.0811	0.2599	0.6631
	TX-RV, TXF-RV	0.0712	0.2422	0.6348	0.0798	0.2560	0.6471	0.0857	0.2700	0.6771
	Full	0.0685	0.2359	0.6281	0.0789	0.2514	0.6411	0.0791	0.2531	0.6420
30 min	TX-HV, TX-RV	0.0805	0.2590	0.6566	0.0851	0.2658	0.6796	0.0905	0.2769	0.7183
	TXF-HV, TXF-RV	0.0719	0.2428	0.6361	0.0866	0.2712	0.6797	0.0893	0.2750	0.7098
	TX-RV, TXF-RV	0.0761	0.2500	0.6388	0.0822	0.2641	0.6720	0.0819	0.2641	0.6713
	Full	0.0710	0.2418	0.6332	0.0820	0.2629	0.6669	0.0861	0.2702	0.6786

Compared with the results in Table 9, those in Table 10 are rather consistent. All results but that for call option, as in Panel B of Table 10, point to lower forecast errors. It suggests that the ARV model can have more information content in forecasting RV, especially in forecasting put option IV before the

Presidential election. Whether it is in-sample or out-of-sample, the IV-ARV+J always renders smallest forecast error for forecasting IV. The forecast error for the put option IV is smaller than that for the call option IV. In the robustness checks, we find that the smallest forecast error in forecasting the IV comes from the IV-ARV+J/L model instead of the IV-ARV+J model. This result implies that it is necessary to incorporate the leverage effect into the ARFIMA model during a stock market crash.

TABLE 10
FORECASTING ERRORS FOR IV-ARV MODEL

$$Ln(\sigma_t^{IV}) = \beta_0 + \beta_1 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TAIEX.HV^e}) + \beta_2 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TAIEX.ARV^e}) + \beta_3 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX.HV^e}) + \beta_4 Ln(\sigma_{t\,|\,\Omega\ t-1}^{TX.ARV^e}) + \varepsilon_t$$

In Panel A, B and C respectively are the in-sample, pre-Presidential election out-of-sample and post-Presidential election out-of-sample forecast errors from the encompassing regression analysis. MSE is the mean square error. MAE is the mean absolute error. MAPE is the mean absolute percentage error. RVs are estimated using the 9-minute interval and 30-minute interval. All options are with a maturity of 8 to 30 days.

ARV variable		ALL			Call			Put		
		MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
A. In-sample										
9 min	ARV	0.0301	0.1375	0.5006	0.0498	0.1769	0.5579	0.0371	0.1451	0.5195
	ARV+ J	0.0289	0.1321	0.4982	0.0490	0.1754	0.5502	0.0350	0.1425	0.5187
	ARV+L	0.0578	0.1925	0.6099	0.0674	0.2301	0.6251	0.0597	0.1942	0.6162
	ARV+J/L	0.0328	0.1402	0.5152	0.0498	0.1770	0.5594	0.0384	0.1541	0.5257
30 min	ARV	0.0390	0.1553	0.5260	0.0589	0.1937	0.6127	0.0425	0.1643	0.5350
	ARV+ J	0.0381	0.1500	0.5222	0.0521	0.1849	0.5719	0.0401	0.1602	0.5325
	ARV+L	0.0599	0.2011	0.6190	0.0710	0.2422	0.6344	0.0611	0.2078	0.6205
	ARV+J/L	0.0399	0.1599	0.5320	0.0577	0.1916	0.5991	0.0435	0.1666	0.5351
B. Pre-Presidential election out-of-sample										
9 min	ARV	0.0640	0.2168	0.6211	0.0783	0.2521	0.6410	0.0656	0.2197	0.6224
	ARV+ J	0.0574	0.1910	0.5785	0.0775	0.2510	0.6410	0.0597	0.1944	0.6170
	ARV+L	0.0951	0.2782	0.7179	0.0997	0.2840	0.7224	0.0989	0.2799	0.7186
	ARV+J/L	0.0625	0.2121	0.6209	0.0795	0.2564	0.6501	0.0669	0.2236	0.6233
30 min	ARV	0.0675	0.2357	0.6270	0.0874	0.2722	0.6908	0.0710	0.2423	0.6347
	ARV+J	0.0671	0.2279	0.6248	0.0806	0.2599	0.6643	0.0686	0.2371	0.6274
	ARV+L	0.0990	0.2827	0.7199	0.1061	0.2949	0.7291	0.0995	0.2835	0.7201
	ARV+J/L	0.0684	0.2369	0.6273	0.0862	0.2715	0.6805	0.0720	0.2438	0.6358
C. Post-Presidential election out-of-sample										
9 min	ARV	0.0798	0.2566	0.6416	0.0812	0.2620	0.6651	0.0807	0.2611	0.6601
	ARV+ J	0.0688	0.2359	0.6295	0.0791	0.2560	0.6411	0.0710	0.2415	0.6371
	ARV+L	0.0715	0.2498	0.6395	0.0800	0.2589	0.6561	0.0754	0.2501	0.6393
	ARV+J/L	0.0678	0.2187	0.6270	0.0780	0.2500	0.6402	0.0699	0.2387	0.6298
30 min	ARV	0.0991	0.2830	0.7210	0.1098	0.3005	0.7302	0.1116	0.3157	0.7391
	ARV+J	0.0710	0.2490	0.6391	0.0821	0.2657	0.6691	0.0860	0.2701	0.6781
	ARV+L	0.0798	0.2560	0.6471	0.0894	0.2729	0.7000	0.0901	0.2763	0.7196
	ARV+J/L	0.0701	0.2400	0.6311	0.0817	0.2621	0.6660	0.0816	0.2620	0.6659

Performance of Portfolio

To demonstrate that our volatility forecasting model is able to forecast accurately for actual trading, we conduct trading simulations of various forecasting models to compare their ability to forecast and select price.

The investment strategy involves a straddle portfolio with nearby at-the-money option contracts. Volatility is estimated by a volatility forecasting model. Out-of-sample investment portfolio is formed by observing pricing error. Our trading rule requires the bid-ask spread to be higher than the transaction cost of NT$80. There will be no trading if bid-ask spread is less than the transaction cost. To test whether the investment performance is significantly different from zero, following Bollen and Whaley (2004), we perform the modified Johnson (1978) t-test statistics specified below to accommodate the asymmetrical distribution of profit from short positions in options:

$$t_{\text{Johnson}} = \left[(\tilde{x} - u) + \frac{u_3}{6\sigma^2 N} + \frac{u_3}{3\sigma^4 N}(\tilde{x} - u)^2\right][s_2/N]^{-\frac{1}{2}}, \tag{16}$$

where $\tilde{x}$ is sample mean, μ, σ^2 and u_3 are the first, second and third central moments, respectively, of the population, s_2 is the sample variance, and N is the sample size. In the mean tests, μ is set to zero, σ^2 and u_3 are estimated by the sample variance and skewness, respectively. Finally, we also calculate the Sharpe ratio more precisely. For robustness, we use two simulation periods: one trading day and five trading days (a week).

Table 11 presents the results from trading simulation using delta-neutral straddle portfolios based on various volatility forecast models. The highest abnormal returns in both directions occur in the IV-RV model, implying that the profitability from the IR-RV model is highly uncertain. Overall, the average rate of return is highest from the IV-ARV+J model. These empirical results demonstrate that the profitability from the IV-ARV+J model is stable. Ignoring transaction costs, as done in the Sharpe ratio, the RV-full model performs better than the ARFIMA+J model. With transaction cost, as reported in Panel B of Table 11, the ARFIMA+J model does the best with 13.1% rate of return for each 1% risk undertaken in the one-week holding period. The rate of return in the rest of the models is seriously eroded by transaction costs, especially for short terms.

TABLE 11
THE PERFORMANCE OF TRADING SIMULATIONS BEFORE THE PRESIDENTIAL ELECTION

This table presents the results from trading simulations using delta-neutral straddle portfolios based on various volatility forecast models. Daily straddle trading is based on one day-ahead volatility forecasts from competing models. Daily returns have been adjusted for bid-ask spread. "+" and "−" Return are respectively positive returns and negative returns during the period.

Model		IV-RV	IV-ARV	IV-ARV + J	IV-ARV +L	IV-ARV +J/L
A. Without trading cost						
1-day	+ Return	20.82	18.73	19.43	16.44	17.51
	− Return	-16.95	-14.03	-13.97	-15.20	-14.27
	Return	**2.79*****	**3.25*****	**5.81*****	**1.08** ***	**2.91*****
	Johnson T	11.15	11.88	10.67	9.59	10.15
	Sharpe Ratio	**0.2204**	**0.1987**	**0.2107**	**0.0898**	**0.1055**
1-week	+ Return	60.80	61.23	67.12	61.37	68.04
	− Return	-28.43	-24.49	-24.05	-26.86	-26.77

	Return	**8.21** *	**9.93** ***	**13.08** ***	**5.57** **	**9.91** **
	Johnson T	1.71	2.81	2.74	-2.21	2.84
	Sharpe Ratio	**0.2855**	**0.2574**	**0.2801**	**0.0914**	**0.0912**
B. With trading cost						
1-day	+ Return	18.13	16.05	17.12	15.12	15.12
	− Return	-18.54	-15.59	-15.01	-16.91	-15.27
	Return	**-0.72** ***	**-0.83** ***	**1.18** ***	**-1.11** ***	**-0.04** ***
	Johnson T	2.59	2.82	2.47	2.71	2.57
	Sharpe Ratio	**-0.0502**	**-0.0418**	**0.1154**	**-0.0013**	**-0.0028**
1-week	+ Return	54.01	52.91	53.58	50.75	50.04
	− Return	-59.43	-52.11	-50.34	-54.28	-50.00
	Return	**-1.21** *	**0.07** *	**1.81** *	**-1.38**	**0.17**
	Johnson T	0.71	1.80	1.72	-0.21	0.49
	Sharpe Ratio	**-0.1237**	**0.0067**	**0.1310**	**-0.0981**	**0.0899**

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Finally, to gauge the impact of the stock market crash in our volatility forecasting model, we repeat the simulations using out-of-sample data from the post-Presidential election period as a robustness test. In Table 12, we can find that the rates of return during the stock market collapse are lower than in Table 11. These returns are significantly positive only in the one-day holding period. Furthermore, while the IV-ARV+J/L model has a negative Sharpe ratio in the one-week holding period with transaction cost, it has a higher risk adjusted return than all other models. That is, the ARFIMA model with leverage and jump effects outperforms all others in the stock market crash. Consistent with the findings in Table 11, the rate of return is seriously eroded by transaction costs. Although the rate of return is negative from the IV-ARV+J/L model, it is not statistically significant.

TABLE 12
THE PERFORMANCE OF TRADING SIMULATIONS AFTER THE PRESIDENTIAL ELECTION

This table presents the results from trading simulations using delta-neutral straddle portfolios based on various volatility forecast models. Daily straddle trading is based on one day-ahead volatility forecasts from competing models. Daily returns have been adjusted for bid-ask spread. "+" and "-" Return are respectively positive returns and negative returns during the period.

Model		IV-RV	IV-ARV	IV-ARV + J	IV-ARV +L	IV-ARV +J/L
A. Without trading cost						
1-day	+ Return	20.91	20.87	20.43	20.51	18.09
	− Return	-20.74	-20.68	-19.01	-19.20	-15.43
	Return	**0.01** ***	**0.02** ***	**1.82** ***	**1.09** ***	**1.99** ***
	Johnson T	8.01	8.55	9.01	9.50	9.55
	Sharpe Ratio	**0.1845**	**0.1800**	**0.1897**	**0.1884**	**0.1988**
1-week	+ Return	24.87	24.91	31.34	30.57	32.60
	− Return	-27.16	-27.38	-32.21	-31.96	-31.99
	Return	**-1.08** *	**-1.51** *	**-0.58**	**-0.62**	**0.11** ***
	Johnson T	-1.61	-1.79	-1.28	-1.01	2.84
	Sharpe Ratio	**0.1998**	**0.1915**	**0.1993**	**0.1990**	**0.2014**

B. With trading cost						
1-day	+ Return	18.09	18.29	18.50	18.57	17.51
	− Return	-21.01	-21.22	-20.06	-20.17	-17.40
	Return	**-1.19** *	**-1.06** *	**-0.21** *	**-0.38** *	**0.05**
	Johnson T	-1.66	-1.79	-1.70	-1.61	1.25
	Sharpe Ratio	**-0.054**	**-0.068**	**-0.015**	**-0.020**	**0.0951**
1-week	+ Return	44.09	44.98	46.21	46.00	48.51
	− Return	-49.53	-48.54	-47.56	-48.15	-48.87
	Return	**-2.01**	**-1.90**	**-1.05**	**-1.28**	**-0.94**
	Johnson T	-0.98	-0.91	-1.15	-1.19	-1.22
	Sharpe Ratio	**-0.2013**	**-0.1901**	**-0.1601**	**-0.1654**	**-0.0975**

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

SUMMARY

The paper employs the volatility in the spot and futures markets in the encompassing regression model to forecast RV and IV respectively. Our study differs most from the literature in that we use asynchronous variables in the forecasting model. Moreover, these asynchronous predictors are all their forecasted values, rendering our forecasts much closer to the real world.

While RV can be non-stationary with high sample frequency (ABDL, 2001), a stationary RV also depends on trade-matching time. Our empirical evidence suggests that a stationary RV of TAIEX index is calculated from 6, 9, 15 and 30 minutes, which are all multiples of 45 seconds. Furthermore, the Bayesian estimates of the long-memory parameter confirm that the RVs of the TAIEX index possess long memory. Among all estimates of the long-memory parameter, those from - RV9 and RV30 are more stationary.

While the prior literature contends that HV is less predictive of RV than IV, our results show that HV explains RV the best, owing mainly to the fact that we use HV forecasts from an ARMA model. Compared with the HV of futures, the HV of spot has more information content, which helps to explain RV. A relatively surprising finding is that DVF seems to have no information advantage over RV. Not surprisingly, the RV forecast error decreases with the number of predictor variables used, no matter if it is in sample or out, suggesting that multi-information content can reduce forecasting errors. While RV9 attains the smallest forecasting errors, implied RV30 shows more variation.

By using the volatility of spot and futures simultaneously in the encompassing regression model, the volatility of futures is significantly positively correlated with the forecast IV, suggesting that futures leads spot and that the underlying asset of the TXO is approximately TX instead of TAIEX. Given the non-tradability of TAIEX, option holders in Taiwan can only trade TX for hedging and arbitraging purposes, leading to better information content of the futures volatility. Nonetheless, the explanatory power of RV variable in the model is inferior to HV variable. But if we predict IV with ARV, the forecasting ability would enhance significantly through ARV of spot (for IV of calls) and ARV of futures (for IV of puts). The forecasting performance of the IV-ARV+J model in group RV9 is better, indicating the jump tendency in Taiwan stock returns.

Despite the fact that RV is less predictive of IV than ARV in the encompassing regression model, in simulated investments, the performances of the IV-RV model are superior to the IV-ARV+J model in the pre-Presidential election period. When we consider transaction costs, rate of return in most models becomes negative. Yet, the returns from the IV-ARV+J model remain positive and the highest, showing that the IV-ARV+J model approaches the real world more closely. In the post-Presidential election period, we find that combinations of jump and leverage in the ARFIMA model for forecasting IV have the lowest forecasting error and best profitability when the stock market crashed.

ENDNOTES

1. Some prior studies also adopt the single-variable time series models to forecast IV.
2. This is the main advantage of the model while a potential shortcoming is that when the number of independent variable increases, the model's testing power may decline.
3. This model is discussed in Fair and Shiller (1990), and used by Lamoureux and Lastrapes (1993), Jorion (1995), Christensen and Prabhala (1998), Campa and Chang (1998), Jiang and Tian (2005), etc.
4. Refer to FCM's (Futures Commission Merchant) Income Statement of Taiwan, - 2008.
5. Testing the ACF and PACF of the realized volatility shows that there is a good fit at p=1/q=0.
6. For example, for realized volatility constructed in the 9-minute intervals, eliminating effects from the first and last time period, there are 28 regions, then $\Delta = 1/28 = 0.0357$.
7. "J" is a jump measure estimated using high frequency data according to the theory of quadratic variations (see Andersen, Bollerslev and Diebold, 2007).
8. See Geweke and Porter-Hudak (1983) and Janacek (1982) for details.

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