Introducing Learning by Doing into the Break-Even Analysis Model

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Break-even analysis is widely used in helping managers anticipate how output changes affect profitability. Warnings abound, however, in relying too heavily upon implications drawn because strong assumptions in the model may limit applicability. This paper focuses on relaxing the assumption that variable cost per unit is constant, inconsistent with ergonomic emphasis on learning.

The analysis below indicates the effect of learning on output needed to: (1) break-even, (2) achieve a target profit, and (3) choose the make rather than buy decision. Including learning does complicate the analysis, but as shown, it need not be much more difficult to use.

Keywords: breakeven analysis , cost-volume -profit , target profit , outsourcing , learning by doing , experiential learning

THE TRADITIONAL BREAK-EVEN, TARGET PROFIT, OUTSOURCING MODEL

Early works by Hess (1903) and Mann (1903, 1907) developed the analysis traditionally known as the Cost-Volume-Profit (CVP) model from a managerial accounting perspective. First, they recognized operating profit as the difference between revenue from sales of goods produced and production cost. The latter was then separated into two categories, variable cost for inputs whose usage depends on volume (notably production labor and materials), and fixed cost independent of volume, such as rent, mortgage, interest expense, and payments to non-production employees. Total variable cost, in turn, was viewed as consisting of two components, quantity of output produced, and variable cost per unit of output. The last term could be measured using accounting data as the ratio of the firm's total variable cost to quantity. Cost-volume projections could then be made under the condition that variable cost per unit remains unchanged over the range of production considered.

Total revenue was also decomposed into components of quantity produced, and the ratio of total revenue to quantity, that is, average revenue. A simplifying assumption often made is average revenue is unaffected by volume, which implies incremental revenue per unit is constant. When so, average revenue can be regarded as constant unit price. Ability to actually sell all output at that price was subsequently noted as a source of demand uncertainty, not directly considered, but introduced as a matter requiring expert judgment.

The model, today is often referred to as "break-even" analysis, i.e. the absence of loss but also of profit. Several explanations have been offered for this emphasis. Before deciding to undertake a new venture, it is important for managers to understand how much output must be produced before the venture yields a profit. Determining the break-even output level using the model, moreover, is an objective process, whereas determining an appropriate profit objective in a situation of demand uncertainty is more subjective. A strategy of preference for loss avoidance often matters, too.

Break-even, of course, is not usually a primary objective of the firm. Algebra which identifies a profit objective (the "target) is similar to that of break-even, the difference being replacement of zero profit by the addition of a positive profit value, followed by focus on determining volume necessary to attain it. A well-known specification is:

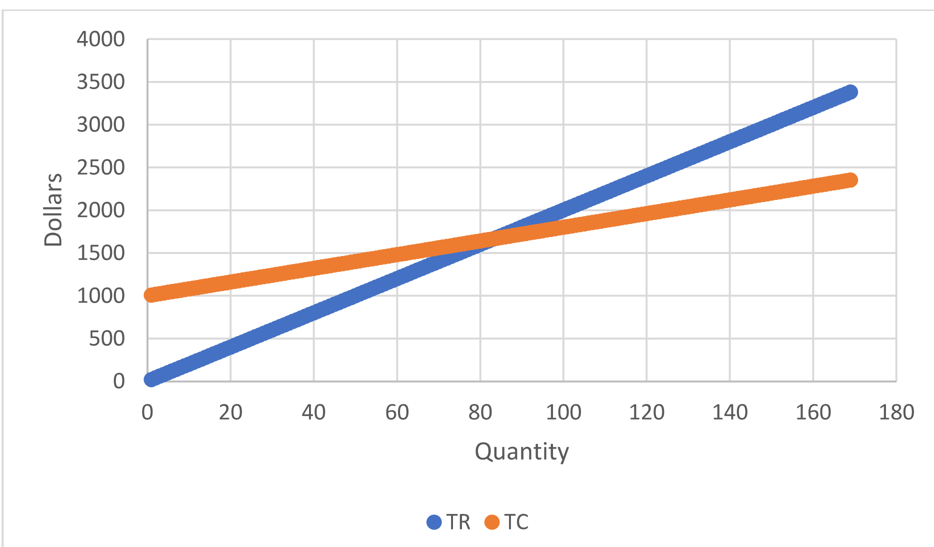
$$Q_\pi = (FC + \pi)/(p - v) \tag{1}$$

where Q_π is output volume resulting in profit level π, FC is fixed cost, p is product selling price, and v is variable cost per unit. The denominator in the expression on the right, the excess of price over variable cost per unit, is known as the "contribution margin," reflecting the fact that an additional unit of output produced and sold reduces the amount of fixed cost yet to be recovered. In the important special case of break-even output (Q_{BE}) determination, π equals zero, and the volume necessary to break-even equals fixed cost divided by contribution margin.

$$Q_{BE} = FC/(p - v) \tag{2}$$

Figure 1 illustrates the interplay of total revenue and total cost in determining break-even quantity (the output level at which these curves intersect) and indicates growing levels of profit (the area between the two curves) as quantity produced and sold increases. In the illustration fixed cost is assumed to be 800, price always equals 20, and variable cost per unit is a constant value of 8.

FIGURE 1
TOTAL REVENUE AND TOTAL COST

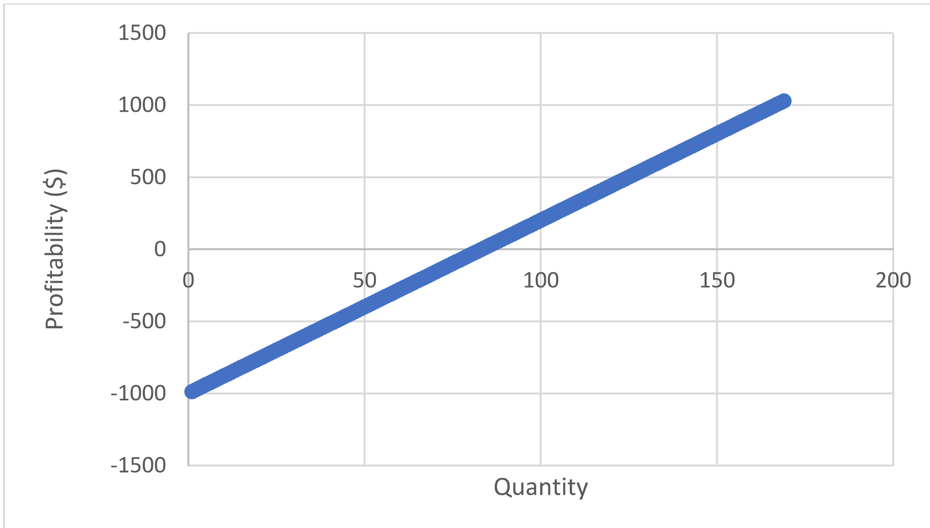


While the visual profit area is descriptive, a direct presentation of linkage between quantity and specific profit can also be useful. To show this, rearrange terms in equation (1) as:

$$\pi = (p - v)\, Q_\pi - FC \tag{3}$$

which now identifies profit as the difference between contribution margin times output minus fixed cost. Using the same price, fixed cost, and variable cost per unit values as before, the relationship is plotted in Figure 2, a graph known as the profitability-volume chart.

FIGURE 2
PROFITABILTY-VOLUME



Decision-makers may have in mind for π a "target profit," a useful rubric with which to determine success. Whether the volume needed to achieve that target can be sold at the given price is unanswered because the CVP model does not specify demand. Using the profitability-volume curve, though, decision-makers can view the volume necessary to meet a particular profit objective, or examine the entire curve to see what profit may emerge from alternate production levels. Higher profitability targets may appear enticing, but demand uncertainty coupled with a particular preference for risk avoidance can temper enthusiasm. Decision-makers must use judgment to determine likelihood of achieving a particular target, and whether the potential return is worth assuming the risk.

Outsourcing

Companies often have the option of purchasing a good at wholesale from a supplier and reselling it at retail. Sometimes called the "make-buy" decision, managerial outsourcing concerns include quality, timeliness of delivery and confidentiality while inhouse production concerns include capacity constraints, alternate uses of resources, lack of specialized knowledge, and fixed costs. The CVP model, however, limits the range of concerns, focusing on cost comparison, the question whether outsourcing is cheaper. While different types of outsourcing agreements are possible, the base case presumes that fixed cost is borne directly only for inhouse production (contractors may have such costs rolled into the buy price). Then if the buy price is lower than inhouse variable cost per unit, outsourcing is always cheaper. If not, fixed cost renders inhouse production expensive at low volume, but higher volume will create a range in which inhouse is the cheaper alternative. Which range a firm anticipates operating in is linked to the target profit objective, as indicated in the equations below.

The output level which divides the ranges is determined by setting the cost of each option to equal one another. Representing the unit buy price of output as v_o we have $v_oQ = FC + vQ$, and the level of output at which outsource and inhouse cost are equal (Q_e) is:

$$Q_e = FC/(v_o - v) \quad (4)$$

At Q_e it matters not whether the company makes or buys output. For other output levels the decision is:

If $Q < FC/(v_o - v)$ outsource (5a)

If $Q > FC/(v_o - v)$ produce inhouse (5b)

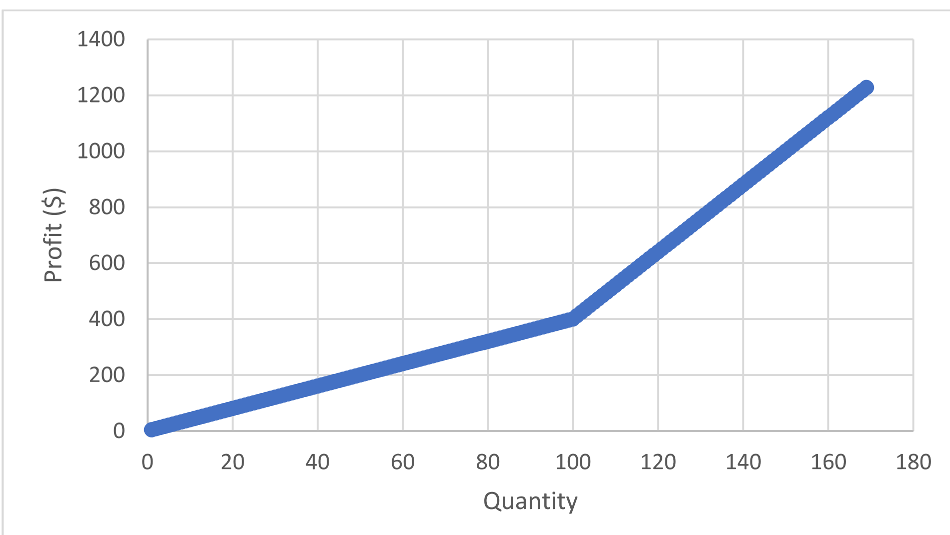
Shifting the focus from cost to profitability, when (5a) holds, so long as the wholesale buy price (v_o) is less than the retail sell price (p), profit will grow with each increase in output. (Case in point - Kutscher's, a legendary New York State resort recognized for quality of home-prepared meals, switched to a catering service when the number of guests declined (Memensha, 2015)). Beyond Q_e, (5b) applies, and profit is defined by equation (3). So the equation regarding profitability when outsourcing is considered a possibility is:

If $Q <= Q_e$, $\pi = (p - v_o) Q$ (6a)

If $Q > Q_e$, $\pi = (p - v) Q – FC$ (6b)

The corresponding profitability-volume curve provides insight as to whether to outsource or produce inhouse. The curve (see Figure 3) exhibits a kink at the switching quantity Q_e. At a lower volume, outsourcing is cheaper and therefore the more profitable alternative, but at higher volume inhouse production is more profitable. By specifying target profit, locating it on the on the vertical axis of the chart, and then identifying where that profit level intersects the curve, both volume and the make-buy choice become apparent. For example, using given hypothetical cost values, a target profit of $300 can be realized at lower volume (75) using outsourcing. A target profit of $700, on the other hand, could be achieved at smaller volume (125) by producing inhouse.

FIGURE 3
PROFITABILITY WITH MAKE-BUY SWITCHING



THE LEARNING EFFECT

The assumption that inhouse variable cost per unit is constant is challenged by empirical studies which found that in successful firms, incremental cost typically falls over time as production continues. Early learning studies focused on knowledge and skill gained through experience (i.e. learning by doing) leading to a reduction in the labor time required to complete subsequent repetitions. Wright's (1936) study of the cost of airframe manufacturing is often credited as the earliest known paper. He observed that with each doubling of output, labor time in airframe production declined at a constant rate. Since labor time involves cost, less labor time per unit of output results in commensurate lower total cost (historically, labor cost has averaged two-thirds or more of total variable cost). In the next several decades learning studies focused on cost savings in a number of manufacturing industries. Papers on service industries in which workers performed repetitive tasks came later. More recently, several have addressed a broad spectrum of service activities and even entire organizations (Boone, Ganeshan, Hicks, 2008), (Labre and Nembhard, 2010). Evidence from technical fields such as medicine also indicates diminishing incremental labor cost through repetition.

Specifics of the informal learning process in industry and elsewhere remain a matter of much discussion. Some of the cost reduction has been attributed to individual workers or teams discovering through repetition how to perform tasks more efficiently. But the efficiency gain can also be due to managerial actions associated with trial and error learning, such as assembly line reorganization, quicker supply chain, lower per unit materials cost from larger purchases, job reassignment, et al. When the overall (multifactor) reduction in variable cost per unit is considered, the term "experience curve" has been used. Whatever the cause(s) or descriptive title, the emphasis here is on learning through experience resulting in diminishing variable cost per unit.

Learning, of course, is not guaranteed, and quicker learners will realize greater cost reductions, while non-learners none. For any positive learning rate, meanwhile, the relationship between repetitions performed and time or cost of the next unit is nonlinear because it is easier to realize larger gains at first when tasks are performed inefficiently. A widely used mathematical specification of the learning equation which captures this nonlinearity is the powerform, though other alternatives such as the exponential, Stanford, and DeJong have been used (Lehto and Burke, 2012). The powerform indicates the extent to which the time needed to complete the task is reduced by a learning factor, derived from the "learning percentage" a term focused on the reduced time required to perform a task number that is the double of the previous number applied to (multiplied by) the earlier task. For example, using an 80% learning percentage, if the first time a task is performed it requires k hours, the next time it is performed it will take .80 k. Doubling the task number, i.e. the fourth task, will require 80% of the time the second task took, which equals (.80)(.80) = .64 k. Doubling the task number again, the time required for the eighth task is (.80)(.64) = .512 k, and so on.

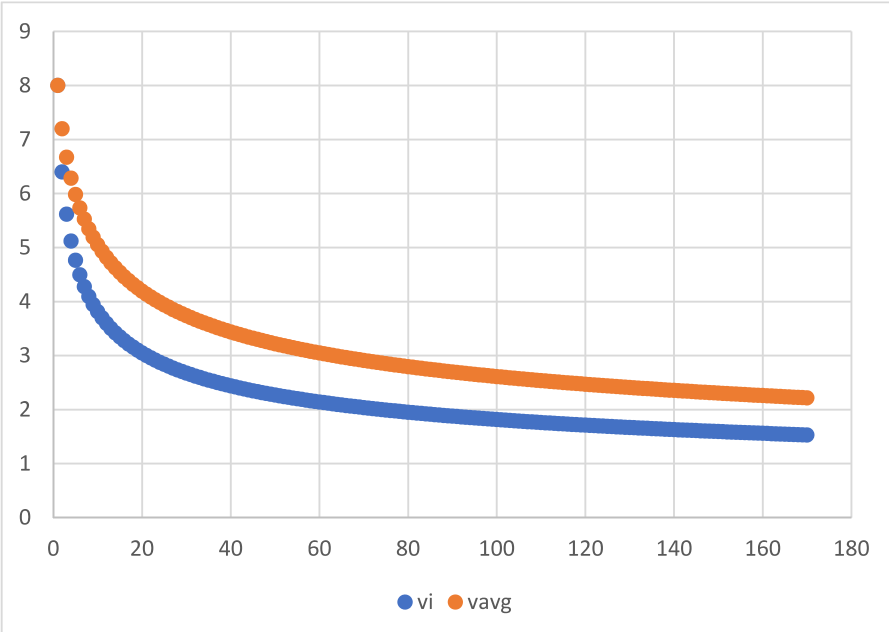
Using this specification, the variable cost for a given repetition can be predicted by the following equation:

$$v_i = kX_i^b \tag{7}$$

in which k is the variable cost of the first unit, X_i is the particular (i^{th}) sequential task number considered, and b is the ratio of the natural logarithm of the learning percentage divided by the natural logarithm of 2. With an 80% learning rate, for example, the exponent b equals ln(.8)/ln(2), or approximately -.322.

Figure 4 illustrates the extent to which predicted variable cost per unit falls when applying the powerform model. The case illustrated is for the 80% learning rate and initial cost k as before at 8. The v_i curve indicates that the biggest cost savings occur early in the repetition cycle, though smaller and diminishing cost savings continue throughout the entire output range. The general shape of the curve (convex to the origin) applies to other learning rates, though slope is altered. Changing the initial cost, meanwhile, resets the vertical value of the first unit of output.

FIGURE 4
WITH LEARNING VARIABLE COST PER UNIT AND AVERAGE VARIABLE COST PER UNIT



To what extent are predictions such as these consistent with actual experience? Some examples of scatterplots of real-world data imply not all actual times are near values predicted from the model. Studies show a typical (consensus) learning rate around 80%, but exhibit movements above and below the predicted learning time for the same organization as the level of output changes. Boone, Ganeshan and Hicks write: "there has been a widespread acceptance of the 80% learning curve … (but) there is a wide variation" (2008, p. 1232). Lehto and Buck (2012) also summarize the evidence in a similar manner. This suggests, perhaps, it may be advisable to regard the average of projected variable costs up to the i^{th} unit as a safer forecast of variable cost of the n^{th} unit. The average variable cost based upon the same learning specification is defined by the formula below.

$$v_{avg} = [\sum_{i=1}^{n} vi]/n = k\ [\Sigma\ X_i^{b}]\ /\ n \quad (8)$$

The behavior of average variable cost per unit assuming an 80% learning rate is also depicted in Figure 4. Underlying values were computed using a spreadsheet as follows. First, the X_i^b term on the right hand side of equation (7) was calculated for repetitions 1 through 170. Then, v_i equals k (the initial variable cost per unit, which equals 8 here) multiplied by values of X_i^b for each repetition. Finally, average variable cost (v_{avg}) for a given repetition (i) is calculated sequentially by taking the average of all units up to and including that unit. Proceeding sequentially, it is easy to calculate this value for each repetition number (1–170) using the spreadsheet sum formula. Comparing incremental variable cost (used in Figure 5 to average variable cost used in Figure 6, after the first unit of output the avc curve lies above the incremental variable cost per unit curve, but the vertical gap between the two becomes smaller as output increases.

The break-even quantity when learning occurs (Q_{BEL}) can be obtained by replacing v in equation (2) with v_{avg}:

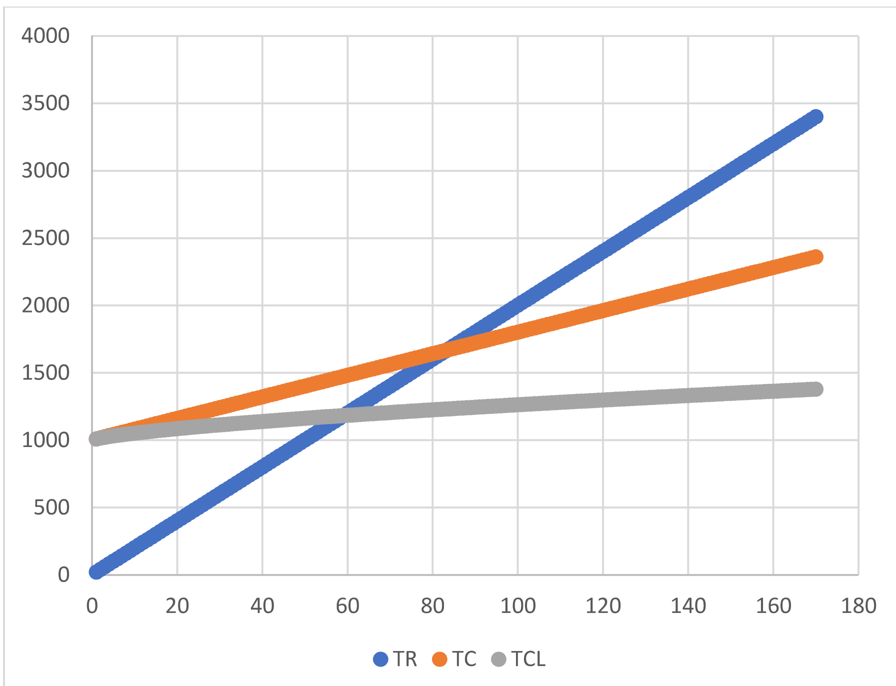
$$Q_{BEL} = FC/(p - v_{avg}) \quad (9)$$

The profitability formula given in equation (3), too, can be similarly modified to reflect the effect of learning. This new formula in which π' represents profit when learning occurs is:

$$\pi' = [(p - v_{avg})\, Q] - FC \qquad (10)$$

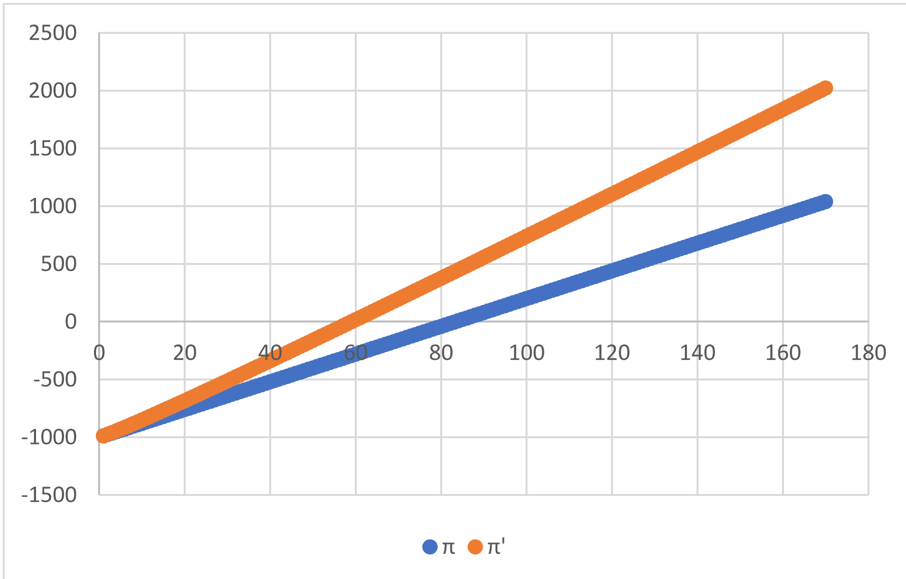
The effect of learning on the break-even quantity is illustrated in Figure 5, which is simply Figure 1 modified by the addition of a second total cost curve which includes the effect of learning, that is, $TC_L = FC + [(v_{avg})\, Q]$. Where each total cost curve crosses the total revenue line identifies the break-even level of output for that assumed learning behavior. For the linear cost curve (no learning), the output level at which the curves cross is as defined by equation (2), equal to the fixed cost divided by contribution margin. For the total cost incorporating learning/experience, break-even quantity is defined by equation (9), and the graph shows the intersection of the curves occurs at a substantially lower quantity of output. Faster learning would reduce the break-even quantity more, slower learning less, so a direct payoff from better teamwork and/or faster individual learning is that profitability occurs sooner.

FIGURE 5
BREAK-EVEN OUTPUT WITH AND WITHOUT LEARNING



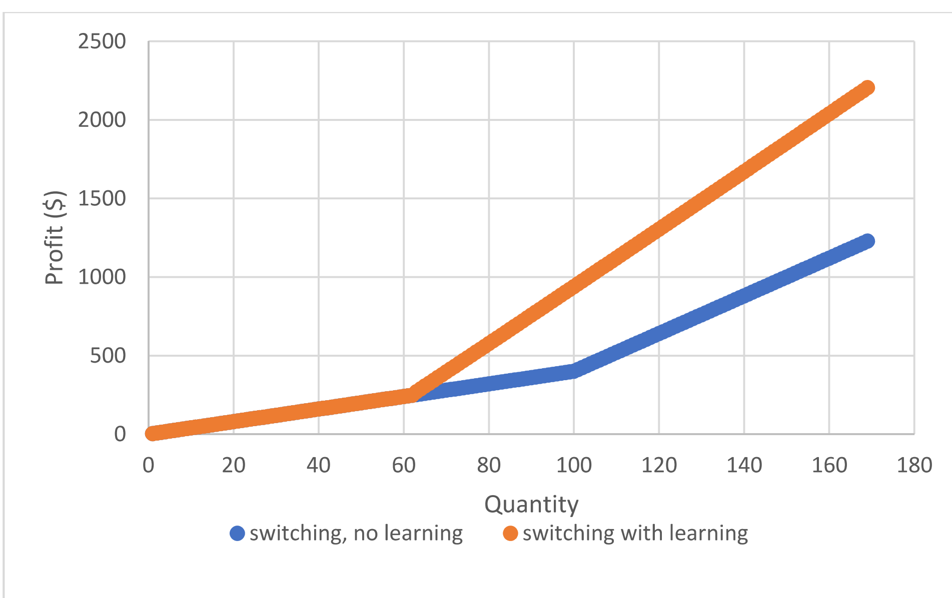
The implication for realizing a target profit is similar. Figure 6 shows profitability-volume curves with and without learning. For any volume beyond the first unit, profitability is higher when learning takes place. As volume increases, the profit gap between the two curves grows. Alternately, the diagram indicates a specified target can be achieved at lower production levels when learning occurs.

FIGURE 6
PROFITABILITY-VOLUME WITH AND WITHOUT LEARNING



In a similar manner, when outsource and inhouse production sources are available options, learning will affect the shape of the combined profitability curve. Learning can affect both contractor and inhouse production costs, and will do so unequally if different learning rates are present. The situation in which learning cost reductions are not passed along by the contractor (i.e. v_o remains unchanged) is illustrated in Figure 7. Compared to the profitability-volume curve with no learning (Figure 3), the kink occurs a lower volume, and profitability after the new kink is increasingly greater compared to the no-learning situation. This implies that once volume is beyond the kink, a given target profit can be achieved at reduced volume, or a greater profit realized at the previous volume when learning occurs.

FIGURE 7
PROFITABILITY, LEARNING AND MAKE-BUY SWITCHING



CONCLUDING REMARKS

Including individual, team or organizational learning into the Cost-Volume-Profit model can have a powerful consequence in terms of reducing the volume necessary to break-even or achieve a target profit. Some suggestion of the possible quantitative magnitude of this reduction appears in the simulation above using a typical but not universal learning rate of 80 percent. For the chosen fixed cost and initial variable cost per unit parameter values, the reduction in break-even quantity was over 30%. Faster learning would enlarge this difference, slower learning, reduce it. But this percentage differential is also subject to specific values of parameters of initial variable cost per unit, price and fixed cost.

A similar percentage change occurred with regard to the make-buy decision. In the simulation shown, the volume at which inhouse production became the less expensive alternative fell by over 40% when experiential learning occurred. This percentage is again sensitive to parameter values, including rate of learning, and to the buy price of outsourced output.

When implementing the learning-modified instead of traditional CVP model, the only additional piece of information needed is the learning rate. This can be estimated from actual information on time required when such data exist (the powerform model is linear in logarithms and can be estimated using regression analysis), but the "typical" 80% rate may still be useful as a first approximation.

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