Lending Standards, Bank Risk-Taking, and Monetary Policy

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This paper contributes to the burgeoning literature on the so-called "risk-taking channel" of monetary policy by constructing a partial equilibrium model of the commercial loan market, where conventional monetary policy influences bank lending standards and, by extension, the risk of bank default. In the model, borrower heterogeneity in terms of entrepreneurial abilities and project revenue results in optimal commercial bank lending standards. These standards take the form of a minimum ability requirement: only borrowers with abilities greater than or equal to the minimum threshold receive a loan; the rest are completely excluded from the loan market. After loans are made, a negative aggregate shock that lowers all borrowers' project revenue results in unexpected loan losses for the bank that may push it into default.

In the baseline model, lending standards loosen and the probability of bank default rises when the central bank's policy rate falls, a result that suggests that asset quality and lending standards may play an important role in shaping the risk-taking channel of monetary policy. However, the relationship between bank default and the policy rate is sensitive to the extent of bank capitalization. While default risk and the policy rate are negatively correlated at relatively strongly capitalized banks, the risk of default at more weakly capitalized banks increases with the policy rate. Additionally, default risk at strongly capitalized banks is more responsive to changes in the policy rate than at weakly capitalized banks.

This paper contributes to the literature on the risk-taking channel of monetary policy. First, the model enables analysis of the impact of monetary policy on risk-taking in a setting where both bank asset quality and leverage play a role in shaping the risk of bank default. Second, the model's results on the role of bank capitalization imply that the tradeoff between financial stability and monetary policy depends crucially on bank capital ratios. This suggests that recent changes to bank capital requirements embodied in new post-2008 financial crisis regulations may impact the nature and size of the risk-taking channel.

Keywords: Risk-Laking Channel, Lending Standards, Monetary Policy, Bank Default

INTRODUCTION

In the wake of the financial crisis of 2007–2009, the role that monetary policy may have played in stoking the crisis has emerged as a topic of considerable interest and debate.[1] The general narrative dictates that the low interest rate policy of the early 2000s fueled the boom in real estate by flooding financial markets with liquidity. This boom, further exacerbated by the deterioration of mortgage lending standards and the rapid expansion of subprime lending, ultimately left financial institutions more vulnerable to the risk of insolvency and set the stage for the financial crisis that followed.

While multiple factors likely contributed to the loosening of lending standards in the run up to the crisis,[2] this paper investigates the influence of monetary policy by way of the so-called "risk-taking channel" of monetary policy. This part of the transmission mechanism, first formally suggested by Borio and Zhu (2012), centers on the effect of monetary policy on the perception, pricing, and management of risk, and, by extension, the cost and terms of funding and real economic activity.

In particular, I examine the impact of conventional monetary policy on bank lending standards and, by extension, the risk of bank failure. I construct a partial equilibrium model of the commercial loan market, where both borrowers and banks are capable of default. Monetary policy affects bank lending standards—which types of borrowers banks will, and will not, lend to—which influences the risk of bank default.

For the baseline calibration of the model, expansionary monetary policy, modeled as a lower policy rate, loosens lending standards and increases the probability of bank default. However, sensitivity analysis shows that both the sign and magnitude of the correlation between the probability of bank default and the policy rate are sensitive to bank capital ratios. The results suggest that the nature and size of the tradeoff between financial stability and monetary policy depends crucially on bank capital ratios, and that recent changes in bank capital regulation embodied in post-2008 financial crisis regulations (e.g., Basel III) may impact the nature and size of the risk-taking channel of monetary policy.

These results emerge from a partial equilibrium model of the commercial loan market populated by entrepreneurs, financial intermediaries (henceforth, "banks"), and depositors, where both the deposit market and loan market are competitive. Entrepreneurs have access to a non-divisible investment project that yields uncertain returns. Since it is assumed that entrepreneurs have no existing net worth, they must borrow funds from a bank to invest. Additionally, since project revenue is subject to idiosyncratic shocks, entrepreneurs may default on their loan, in which case the bank seizes the entirety of the project's revenue.[3]

While it is assumed in the model that all entrepreneurs seek to borrow the same amount of funds, their publicly known "abilities," which affect their project revenue, do vary. This assumption is similar to how heterogeneous productivity affects firm output in Melitz (2003) or worker productivity, as in Jovanovic (1979) and Strand (2000). All else equal, higher-ability entrepreneurs have higher expected project revenue and are expected to yield a higher return to a bank than low-ability borrowers. This result plays a key role in the determination of lending standards in the model.

In the context of this paper, "lending standards" refer to the internal guidelines used by banks to determine whether a given borrower is creditworthy or whether the borrower's loan application should be denied. In the model, if the expected maximum return on lending to entrepreneurs of a given ability level is less than the required market return on lending, those with that ability level will be denied a loan and excluded from the market.

Thus, lending standards, as interpreted in the framework of the model, refers to a minimum ability level where the expected maximum return on lending to those with the minimum ("cutoff") ability level is just equal to the required market return on lending. Entrepreneurs with ability levels equal to or above the cutoff receive a loan, and those with abilities below the cutoff are denied loans. The entrepreneurs who are denied are willing to pay a higher loan rate; however, no loan rate exists that guarantees the bank can expect to recover the required return on lending. Standards loosen when the cutoff ability level falls and tighten when the cutoff ability level rises.[4]

The return on a bank's loan portfolio is also subject to fluctuations induced by an aggregate shock. This shock is aggregate in the sense that it affects the revenue of all individual projects. When the aggregate shock is negative (below its expected value) all borrowers' project revenues fall, and, as a result, the return realized from lending to low-ability borrowers is lower than expected. If the shock is large enough, the return realized on these loans may fail to cover the cost of deposits and result in a net loss for the bank. The risk of bank default is intrinsically linked to lending standards: when standards are lax, loan losses following a negative aggregate shock are larger due to the lower asset quality. The increase in loan losses implies that the bank cannot withstand large negative aggregate shocks without defaulting, and the probability of bank default increases when standards loosen.

While looser standards unequivocally increase the risk of bank default, conventional monetary policy, modeled as a change in the policy rate, has two opposite effects on the probability of bank default. The "deposits effect" is the impact of the cost of deposits on the risk of bank default: as the policy rate falls, the cost of a given amount of deposits also falls, which, on its own, results in a lower risk of bank default. In contrast, the "standards effect" causes the risk of bank default to increase. All else equal, when the policy rate falls, lending standards loosen. This has the effect of depressing bank profits in the event of a negative aggregate shock and, in isolation, increases the probability ofbank default.

To determine which effect dominates, and the ultimate relationship between the policy rate and the probability of bank default, I calibrate and numerically estimate the model. Under the baseline calibration, the probability of bank default increases when the policy rate falls. Even though the deposits effect pushes the bank further from default, this positive effect is outweighed by the negative effect of looser lending standards. Thus, the net effect of a lower policy rate is a higher probability of bank default.

However, the relationship between the policy rate and the probability of bank default is sensitive to bank capital ratios. When model parameters are set to produce relatively high capital ratios the probability of default and the policy rate are negatively correlated, as in the baseline case. When parameters are set such that equilibrium capital ratios are low this relationship is reversed, and default risk and the policy rate are positively correlated. Additionally, as capital ratios continue to increase beyond the level in the baseline calibration, the relative impact of monetary policy on the risk of bank default grows.

This sensitivity is largely driven by the sensitivity of the deposits effect to bank capital ratios. The amount of deposits raised by a bank—and therefore the deposits effect—is highest when capital ratios are low. Low capital ratios cause the deposits effect to be large relative to the standards effect, and default risk and the policy rate are positively correlated. As bank capital ratios rise, deposits and the deposits effect fall relative to the standards effect. At sufficiently high capital ratios, the standards effect dominates the deposits effect, and default risk and the policy rate are negatively correlated. As capital ratios rise further, the standards effect continues to rise relative to the deposits effect.

This paper adds to the literature on the risk-taking channel of monetary policy by examining the relationship between the policy rate, lending standards, and the risk of bank default in a setting in which both asset quality and leverage play a role in determining the risk of bank insolvency. This is a different approach from the existing literature; most theoretical models exploring the risk-taking channel do not simultaneously incorporate these three factors. For example, Angeloni et al. (2013) and Valencia (2014) explore the relationship between bank default risk and monetary policy where bank leverage is the key determinant of the probability of bank default. In both models, a fall in the policy rate reduces the cost of deposits and incentivizes banks to increase leverage, thereby increasing the risk of bank failure. This model has a similar mechanism regarding the cost of deposits and bank leverage; however, the model described in this work also incorporates a role for asset quality via bank lending standards. While Agur and Demertzis (2012) and Cociuba et al. (2012) investigate the impact of monetary policy on asset quality, unlike my model, those works offer no explicit discussion of the impact of changes in asset quality on bank default risk.

This paper also adds to the growing debate on the impact of capital structure and regulation on risk-taking. Dell'Ariccia et al. (2013) find empirical evidence that the effect of monetary policy on risk-taking is more pronounced for more capitalized banks. DellAriccia et al. (2011) construct a partial equilibrium model, where the relationship between risk-taking, as measured by a bank's optimally chosen risk of default, and monetary policy depends on the bank's capital structure: looser monetary policy increases risk-taking for highly capitalized banks but reduces risk-taking for banks with low capitalization when capital ratios are fixed. When capital ratios are optimally chosen, risk-taking strictly decreases with the policy rate. In contrast, in this paper the risk of bank default is not simply a choice variable for the bank but instead depends directly on the policy rate through the cost of deposits and indirectly through asset quality.

The remainder of the paper is organized as follows: Section 2 develops the model, Section 3 discusses calibration, Section 4 discusses the model's numeric results and significance, and Section 5 offers conclusions and suggests directions for future work.

THE MODEL

Overview and Assumptions

I assume there is a continuum of risk-neutral entrepreneurs and a continuum of risk-neutral banks, each of mass one, that operate in the economy. Each entrepreneur has the opportunity to pursue a non-divisible investment project that requires $1 of initial funds. It is assumed that entrepreneurs have no existing net worth and cannot issue equity, and so must obtain external financing from a bank.[5]

Entrepreneurs are heterogenous in regard to their exogenous abilities, which affect project revenue: entrepreneurs with higher ability realize higher project revenue, all else being equal. Project revenue is also impacted by a random idiosyncratic component that may induce borrower default and a shared shock to the aggregate return to capital. The existence of heterogenous abilities implies that the maximum return on lending is increasing in entrepreneurial ability—this result is key for determining lending standards in the model. I assume that ability is public information known by both entrepreneur and bank.

In relation to commercial banking, the term "lending standards" refers to the internal guidelines used by banks to determine the creditworthiness of potential borrowers. In the context of the model, lending standards take the form of a minimum ability requirement, much akin to minimum credit score requirements lenders use in practice. Only borrowers that can credibly commit to generating the bank an expected maximum return greater than or equal to the market return on lending receive a loan. However, for lower-ability borrowers, the expected maximum return on lending may fall below the market return on lending; these borrowers are denied loans and effectively excluded from the loan market.

The minimum ability threshold and market return on lending are jointly determined in a competitive loan market. I assume that banks are identical[6]; thus, the market supply curve can be derived from the representative bank's need to choose lending standards to maximize expected profits, taking as given the market return of lending.[7]

The question of what demand concept to use in determining the market equilibrium is a more nuanced issue. Since entrepreneurs have no existing wealth and enjoy limited liability, the willingness to pay for a loan is infinite. Therefore, if willingness to pay is used to determine equilibrium, as is standard in most supply and demand frameworks, the equilibrium return on lending is bid up to the point that all entrepreneurs receive loans. However, since the expected maximum return on lending to any given ability group is finite, not all entrepreneurs can credibly commit ex-ante to generating a return that is greater than or equal to the equilibrium market return.

Given this, the relevant demand concept for determining the loan market equilibrium is not willingness to pay, but ability to pay. In the model, the ability to pay is measured by the expected maximum return on lending to borrowers of a given ability level. This expected maximum ability to pay increases with entrepreneurial ability, which implies that when the market return on lending is relatively high, few entrepreneurs have high enough abilities to generate a high enough return to make them attractive borrowers, and so lending standards are relatively tight. As the market return on lending falls, more borrowers are able to generate the returns necessary to incentivize lending, and standards loosen. In equilibrium, the market return on lending and lending standards are determined by borrowers' expected maximum ability to pay and the bank's choice of optimal lending standards.

Entrepreneurs are not the only agents capable of default—banks may also default. While banks can diversify away the risk embodied in borrowers' idiosyncratic shocks, they cannot diversify away all of the risk embodied in the shock to the aggregate return to capital. I assume that, when possible, the loan payment is adjusted after the aggregate shock, so the ex-post return on lending is equal to the market return on lending. However, in the case of a negative shock this adjustment will not be possible for all borrowers. For lower-ability entrepreneurs, the aggregate shock may be "bad" enough that the maximum ex-post return on lending falls below the market return on lending. For these loans, the loan payment is adjusted to maximize the ex-post return on lending; however, since this return is less than expected, banks incur unexpected loan losses. If the realized return on lending is low enough, and unexpected loan losses high enough, banks may become insolvent and default on their deposit liabilities.

I assume that bank loans are financed with a combination of deposits and a fixed amount of exogenous bank capital. For simplicity, I assume that the supply of bank deposits is infinitely elastic, that bank deposits are fully insured, and that the cost of deposits to the bank is equal to the policy rate chosen by monetary authorities. I also assume that banks are unable to raise additional funds (deposits or capital) after an aggregate shock occurs, which implies that banks cannot raise further funds when faced with insolvency.

The timing of events is as follows. After the announcement of the policy rate, but before the realization of idiosyncratic and aggregate shocks, banks make loans to entrepreneurs and contingent loan repayments are stipulated. After entrepreneurs receive loans, all markets close, shocks are realized, and liabilities, less defaults, are repaid. To aid the reader, the relevant notation used throughout the model is summarized in Table 1.

Expected Maximum Return on Lending and Borrower Ability to Pay

Entrepreneurial ability, a, is distributed according to the cumulative density function (cdf) $H(a)$and associated probability density function (pdf) $h(a)$ on a non-negative support. Abilities affect the project revenue $a\omega R^K$; ω is a random idiosyncratic component of the project revenue assumed to be drawn from the common distribution $F(\omega)$ on a non-negative support. R^K is the aggregate return to capital and follows the process below:

$$R^K = \epsilon \bar{R}^K, \quad E(\epsilon) = 1 \tag{1}$$

where $\bar{R}^K > 1$ is the expected gross return to capital, and ϵ is an aggregate shock with mean equal to one and is drawn from a non-negative support.

The revenue of any project is uncertain, and an entrepreneur may default on their debt if it is too low. I define a threshold, $\tilde{\omega}_a$, such that an entrepreneur with ability a and idiosyncratic revenue component $\tilde{\omega}_a$ is just able to make their debt payment:

$$\tilde{\omega}_a = \frac{D_a}{aR^K} \tag{2}$$

where D_a is the contracted loan repayment for a borrower with ability a[8].

The probability the entrepreneur's idiosyncratic return from the project is too low and the entrepreneur defaults is $F(\tilde{\omega}_a)$. Upon default, the project's revenue is seized by the bank and the entrepreneur receives nothing. With probability $1 - F(\tilde{\omega}_a)$ the borrower does not default, the bank receives the debt payment D_a, and the entrepreneur keeps the project's revenue net of the debt payment.

Access to a bank loan ultimately depends on a given borrower's ability to generate a sufficiently high return for the bank, which depends on both the loan payment and the borrower's ability level. The return on lending to an entrepreneur with ability a is

$$(r_l|a) = a\epsilon \bar{R}^K \, \Gamma(\tilde{\omega}_a) \tag{3}$$

where $\Gamma(\tilde{\omega}_a) \in [0, 1]$ is the share of the project revenue that accrues to the bank:

$$\Gamma(\tilde{\omega}_a) = [1 - F(\tilde{\omega}_a)]\tilde{\omega}_a + \int_0^{\varpi_a} \omega dF(\omega) \tag{4}$$

The choice of $\tilde{\omega}_a$ determines how the project revenue is split between the lender and borrower. The first term on the right-hand side of Equation 4 represents the bank's payoff when the borrower does not default and is instead able to repay their contracted payment. The second term represents the payoff when the borrower defaults and the project revenue is seized by the bank.

The value of $\widetilde{\omega}_a$ that maximizes the return on lending to borrowers with ability level a solves the following:

$$1 - F(\widetilde{\omega}_a) = 0 \tag{5}$$

The solution to Equation 5 implies that the maximizing value of $\widetilde{\omega}_a$ is equal to infinity for all ability levels, and the maximizing value of $\Gamma(\widetilde{\omega}_a)$ is equal to one. Combining this with Equation 3 implies that the realized maximum return on lending and expected maximum return on lending to borrowers of ability a are equal to, respectively,

$$(r_l^* | a | \epsilon) = a\epsilon\bar{R}^K \tag{6}$$

$$E(r_l^* | a) = a\bar{R}^K \tag{7}$$

The realized return on lending is maximized when the bank receives 100% of the project revenue $a\epsilon\bar{R}^K$; the expected maximum return on lending, Equation 7, follows, since the expected value of ϵ is equal to one.[9]

Two results follow immediately from Equations 6 and 7:

Result 1. *The expected and realized maximum return on lending increases with entrepreneurial ability.*

Result 2. *For a given ability level, the realized maximum return on lending is increasing and linear with respect to aggregate shock.*

Proof: Result 1 follows directly from the partial derivative of Equations 6 and 7 with respect to a. Result 2 follows directly from the first and second partial derivatives of Equation 6 with respect to ϵ.

At equilibrium, only ability levels that generate an expected maximum return of at least the equilibrium market return on lending for the bank, R^L, receive loans. Given result 1, low-ability entrepreneurs will be unable to generate a sufficiently high return for the bank and are excluded from the loan market. However, high-ability entrepreneurs generate high enough returns to be attractive prospects for the bank and receive loans. In this sense, lending standards take the form of a minimum ability requi≥ement: given R^L, there exists an ability-level cutoff $\tilde{a}$ such that borrowers with abilities $a \geq \tilde{a}$ receive a loan, and borrowers with abilities $a < \tilde{a}$ are denied loans and excluded from the market. The threshold $\tilde{a}$ is determined by the following condition:

$$\tilde{a}\bar{R}^K = R^L \tag{8}$$

The lowest ability group that is not excluded from the loan market, $\tilde{a}$, can provide the bank with an expected maximum return just equal to R^L. However, since the expected maximum return on lending decreases as ability decreases, all borrowers with $a < \tilde{a}$ cannot expect to create a return of at least R^L, even when all of the project's revenue goes to the lender.

Equation 8 is illustrated in Figure 1. This function shows the expected maximum return on lending as a function of borrower ability. When taken together with the equilibrium return on lending, R^L, this curve shows the lowest ability level that can still generate an expected return of at least R^L. Entrepreneurs with ability levels above or equal to the threshold can credibly commit to generating an expected maximum return on lending of at least R^L. These borrowers receive loans. Conversely, entrepreneurs with ability levels below the threshold cannot commit to generating a return of R^L and are excluded from the market. As R^L rises fewer entrepreneurs are able to generate a large enough expected return and the threshold $\tilde{a}$ rises.

This description of how lending standards are determined is closely related to how a good in a perfectly competitive market is distributed among consumers with varying willingness to pay. Given the market price, only consumers with a maximum willingness to pay that is greater than or equal to the price of the good end up purchasing it. Those with a willingness to pay that is less than the price do not purchase it. Similarly, in the model only borrowers who can generate an expected maximum return of at least the equilibrium market return receive a loan. The key difference here is that, unlike in the analogy above, borrowers who do not receive a loan do not self-select out of the market but are instead excluded because banks expect their projects will fail to generate sufficient revenue to make the loan worthwhile. Thus, the relevant demand concept to use in the determination of equilibrium in the lending market is the maximum ability to pay, as measured by the expected maximum return on lending, Equation 7.

While all non-excluded borrowers have an *expected* maximum return on lending of at least R^L, an ex-post return of at least R^L is not guaranteed. Given result 2, that the *realized* maximum return is linear and increases with respect to the aggregate shock, all borrowers are able to generate a return of R^L if the aggregate shock ϵ is at least one (its expected value). However, in the event that ϵ is less than its expected value of one (a "negative aggregate shock"), not all borrowers will be able to generate a return of R^L. For example, consider the return on lending to an entrepreneur with the threshold ability $\tilde{a}$. Ex-ante, the expected maximum return on lending to the threshold ability is exactly equal to the equilibrium market return on lending. If the aggregate shock is smaller than expected, then the ex-post maximum return for the threshold ability group is smaller than expected and, therefore, falls below R^L.

If the aggregate shock is negative, at the very least, lending to borrowers with the threshold ability $\tilde{a}$ fails to generate a return of R^L. As the negative shock becomes worse and ϵ falls, the number of borrowers that are unable to generate an ex-post return of R^L increases. I assume that in such a situation $\widetilde{\omega}_a$ is set to maximize the ex-post return on lending, yielding a return of $a\epsilon\bar{R}^K < R^L$. For borrowers with an ex-post maximum return of at least R^L, $\widetilde{\omega}_a$ is set such that the realized return to the bank is equal to R^L: $a\epsilon\bar{R}^K\Gamma(\widetilde{\omega}_a) = R^L$.

Given a shock $\epsilon(0, 1)$, which borrowers will be able to generate an ex-post return of at least R^L? I define an ability level a^X such that, given a negative aggregate shock, the maximum ex-post return on lending to this group is exactly equal to R^L; that is, $a^X\epsilon\bar{R}^K = R^L$, which, with Equation 8, implies that

$$a^X = \frac{\tilde{a}}{\epsilon} \tag{9}$$

Since the maximum return on lending increases with entrepreneurial ability, borrowers with ability greater than or equal to a^X are able to generate an ex-post return of at least R^L; however, groups with abilities $a \in [\tilde{a}, a^X)$ are unable to do so. Loans to borrowers of these ability levels yield $a\epsilon\bar{R}^K < R^L$. Because a^X clearly increases as ϵ falls, the number of loans that fail to return at least R^L rises as the aggregate shock worsens.

Figure 2 illustrates how a^X is determined. Before the shock, borrowers with abilities $a \geq \tilde{a}$ receive loans, since the expected return on lending to these groups is at least as large as the market return on lending. However, a negative shock causes the ex-post maximum return schedule to shift below the expected maximum return schedule because the negative shock causes the maximum return on lending for all borrowers to fall. Despite the negative shock, borrowers with $a \geq a^X$ are still able to generate a realized return of at least R^L. However, groups with abilities $a \in [\tilde{a}, a^X)$ are unable to generate a return of at least R^L. Thus, while the expected maximum return on lending was at least R^L for all non-rationed borrowers, the realized return on lending as a function of borrower ability is given by the bold, black curve in Figure 2. In the event of a positive shock ($\epsilon > 1$), the ex-post maximum return schedule will shift up, above the expected maximum return schedule, and all borrowers will be able to generate an ex-post return of at least R^L.

Bank Default and the Bank's Problem

Banks choose their lending standards $\tilde{a}$ to maximize their expected profits while taking the market return on lending, R^L, as given. The bank's choice of which entrepreneurs to lend to determines both the extent of the bank's leverage and the riskiness of its assets. Both of these factors influence the risk that realized bank profits will fall below zero and the bank will default on their deposit liabilities. Given that the distribution of entrepreneurs over ability levels is given by the cdf $H(a)$ with pdf $h(a)$, bank profits realized when the aggregate shock is greater than or equal to one is given by the following:

$$(\pi_b|\epsilon \geq 1) = R^L[1 - H(\tilde{a})] - R[1 - H(\tilde{a}) - K] \tag{10}$$

When the aggregate shock is at least as large as its expected value of one, each of the bank's $1-H(\tilde{a})$ loans is capable of generating a return for the bank of R^L. This loan revenue, less the cost of the bank's $1-H(\tilde{a})-K$ deposits gives the bank's total profits.

When the aggregate shock is negative (ϵ less than one), the bank profits realized are equal to

$$(\pi_b|\epsilon < 1) = R^L[1 - H\,(a^X)] + \int_{\tilde{a}}^{a^X} a\epsilon\bar{R}^K \; dH(a) - R(1 - H(\tilde{a}) - K] \tag{11}$$

The first term on the right-hand side of equation 11 represents loan revenue from borrowers with ability levels of at least $a^X = \tilde{a}/\epsilon$; that is, it shows the return from borrowers that are able to generate an ex-post return of R^L. The second term represents loan revenue from ability levels that, due to the shock, cannot generate an ex-post return of R^L, but instead yield the bank $a\epsilon\bar{R}^K < R^L$. The third term represents the cost of deposits.

In the case of a negative shock, the realization of ϵ has a positive impact on realized profits, since the partial derivative of Equation 11 is positive:

$$\frac{\partial(\pi_b|\epsilon<1)}{\partial\epsilon} = [\tilde{a}\bar{R}^K - R^L]h(\alpha^X)\left(-\frac{\tilde{a}}{\epsilon^2}\right) + \int_{\tilde{a}}^{a^X} a\bar{R}^K \, dH(a) = \int_{\tilde{a}}^{a^X} a\bar{R}^K \; dH(a) > 0 \tag{12}$$

where the second equality follows from the definition of $\tilde{a}$ (Equation 8).

Given that profits in the case of a negative shock are increasing with respect to ϵ, realized bank profits may be positive, negative or zero depending on the severity of the negative aggregate shock. I define a threshold shock level $\hat{\epsilon}$ such that profits realized when the aggregate shock is $\hat{\epsilon}$ are exactly equal to zero.[10] Given this, the shock threshold is implicitly defined by the following condition:

$$(\pi_b|\epsilon = \hat{\epsilon}) = R^L[1 - H(\hat{a}^X)] + \int_{\tilde{a}}^{\hat{a}^X} a\hat{\epsilon}\bar{R}^K \, dH(a) - R[1 - H(\tilde{a}) - K] = 0 \tag{13}$$

Where $\hat{a}^X = \dfrac{\bar{a}}{\hat{\epsilon}}$

The $\hat{\epsilon}$ implied by Equation 13 pins down the default threshold, and hence the bank's risk of default. Since bank profits in the case of a negative aggregate shock are increasing with respect to ϵ and the bank breaks even when $\epsilon = \hat{\epsilon}$, if the shock falls below $\hat{\epsilon}$, the return from the bank's loan portfolio is not large enough to repay their deposit liabilities, and the bank defaults. For $\epsilon > \hat{\epsilon}$ the bank is able to repay its deposits and remains solvent.

Realized bank profits, as a function of the aggregate shock, are illustrated in Figure 3. When the shock is positive, all loans return R^L. In this scenario, bank profits are independent of the size of the shock and described by Equation 10. Conversely, if the shock falls below the default threshold $\hat{\epsilon}$, the bank defaults and, due to limited liability, earns zero profits. When the shock falls in the interval $(\hat{\epsilon},1)$, bank profits are positive and increasing with respect to the shock.

Of key interest in this paper is how the probability of bank default changes with lending standards. Ultimately, this depends on the effect of $\tilde{a}$ on the default threshold $\hat{\epsilon}$: if $\hat{\epsilon}$ rises when $\tilde{a}$ falls then, all else equal, the probability of bank default increases when lending standards loosen, and vice versa. However, if $\hat{\epsilon}$ falls when $\tilde{a}$ decreases, then the probability of bank default falls when standards loosen.

Result 3. *All else equal, the probability of bank default decreases when lending standards tighten.*

Proof: To eke out the relationship between default and standards, I take the derivative of Equation 13 with respect to $\tilde{a}$, keeping in mind that $\hat{\epsilon}$ is an implicit function of $\tilde{a}$:

$$0 = (R^L - \tilde{a}\bar{R}^K)h(\hat{a}^X)\left(\frac{1}{\hat{\epsilon}} - \frac{\tilde{a}}{\hat{\epsilon}^2}\frac{\partial\hat{\epsilon}}{\partial\tilde{a}}\right) + (R - \tilde{a}\hat{\epsilon}\bar{R}^K)h(\tilde{a}) + \int_{\tilde{a}}^{\hat{a}^X} a\bar{R}^K \frac{\partial\hat{\epsilon}}{\partial\tilde{a}} dH(a)$$

which, using Equation 8, becomes

$$0 = \left(R - \tilde{a}\hat{\epsilon}\bar{R}^K\right)h(\tilde{a}) + \int_{\tilde{a}}^{\hat{a}^X} a\bar{R}^K \frac{\partial\hat{\epsilon}}{\partial\tilde{a}} dH(a) \qquad (14)$$

The first bracketed term on the right-hand side of equation 14 represents the net loss on loans to the borrowers of lowest ability, conditional upon realized bank profits being equal to zero. Since the return on lending to these borrowers is always the lowest among the different (non-rationed) ability levels, the realized return on lending to the threshold group must be less than the cost of deposits if losses are large enough to push profits to zero.

This implies that the first-term in 14 must be positive. Thus, $\frac{\partial\hat{\epsilon}}{\partial\tilde{a}}$ is necessarily negative: all else equal, when lending standards are eased ($\tilde{a}$ falls) the probability of bank insolvency rises ($\hat{\epsilon}$ rises).

Intuitively, given $\hat{\epsilon}$, loan losses from lending to the threshold ability group increase as standards loosen and minimum ability $\tilde{a}$ falls. This implies that the bank must earn a higher return on loans that do not yield R^L and, thus are less robust to large negative shocks. Therefore, the probability of bank default rises when lending standards loosen and falls when standards tighten.

Although bank capital, *K*, is fixed in this model, it is also useful to see how the probability of bank default is affected by the level of bank capital.

Result 4. *All else equal, the probability of bank default decreases when the bank's capital increases.*

Proof: Differentiating Equation 13 with respect to *K*, keeping in mind that $\hat{\epsilon}$ is an implicit function of K, yields the following

$$0 = (R^L - \tilde{a}\bar{R}^K)h(\hat{a}^X)\left(\frac{1}{\hat{\epsilon}}\frac{\partial\tilde{a}}{\partial K} - \frac{\tilde{a}}{\hat{\epsilon}^2}\frac{\partial\hat{\epsilon}}{\partial K}\right) + R + \int_{\tilde{a}}^{\hat{a}^X} a\bar{R}^K \frac{\partial\hat{\epsilon}}{\partial\tilde{a}} dH(a)$$

Using Equation 8, this becomes

$$0 = R + \int_{\tilde{a}}^{\hat{a}^X} a\bar{R}^K \frac{\partial\hat{\epsilon}}{\partial K} dH(a) \qquad (15)$$

which clearly implies that $\frac{\partial \hat{\epsilon}}{\partial K} < 0$. All else equal, higher levels of capital lower a bank's default threshold $\hat{\epsilon}$ and lower its probability of default.

Intuitively, when a bank has more capital it requires fewer deposits to finance a given amount of loans. The cost-savings associated with this are captured in the first term on the right-hand side of Equation 15. These cost-savings imply that the bank can earn lower total loan revenues and still break even, which results in a lower default threshold $\hat{\epsilon}$.

The Bank's Problem

The bank chooses lending standards $\tilde{a}$ (and by extension, total lending) to maximize expected profits. These expected profits take the following form:

$$E(\pi_b) = \int_1^{\infty} \left[R^L[1 - H(\tilde{a})] - R[1 - H(\tilde{a}) - K]\right] d\Phi(\epsilon) + \cdots$$
$$\cdots + \int_{\hat{\epsilon}}^{1} \left[R^L[1 - H(a^X)] + \int_{\tilde{a}}^{a^X} a\epsilon \bar{R}^K dH(a) - R[1 - H(\tilde{a}) - K]\right] d\Phi(\epsilon) \quad (16)$$

where $\Phi(\epsilon)$ is the cdf for ϵ with associated pdf $\phi(\epsilon)$. The first line in Equation 16 represents the bank profits given a positive aggregate shock. The second line represents the bank profits given a negative, but not default inducing, aggregate shock.

The bank's choice of $\tilde{a}$ affects expected profits both through the size of the bank's balance sheet as well as the default threshold $\hat{\epsilon}$

The bank's problem is to choose a value of $\tilde{a}$ to maximize Equation 16, which yields the following first-order condition:

$$0 = -\int_1^{\infty} (R^L - R)h(\tilde{a}) d\Phi(\epsilon) + \cdots$$
$$\cdots + \int_{\hat{\epsilon}}^{1} \left[\left((\tilde{a}\bar{R}^K - R^L)h(a^X)\right)\frac{1}{\epsilon} - (\tilde{a}\epsilon\bar{R}^K - R)h(\tilde{a})\right] d\Phi(\epsilon) - \cdots$$
$$\cdots - \left[R^L[1 - H(\hat{a}^X)] + \int_{\tilde{a}}^{\hat{a}^X} a\hat{\epsilon}\bar{R}^K \, dH(a) - R[1 - H(\tilde{a}) - K]\right] \phi(\hat{\epsilon}) \frac{\partial \hat{\epsilon}}{\partial \tilde{a}} \quad (17)$$

Notice that the large bracketed term in the last line of Equation 17 is equal to bank profits given the shock is equal to $\hat{\epsilon}$ and, by Equation 13, is equal to zero. Using this, and Equation 8, the first-order condition simplifies to

$$0 = -[1 - \Phi(1)](R^L - R) + \int_{\hat{\epsilon}}^{1} (R - \tilde{a}\epsilon\bar{R}^K) \, d\Phi(\epsilon) \quad (18)$$

The first term on the right-hand side of Equation 18 represents the marginal cost of tighter lending standards. When banks tighten their standards and reduce lending, they lose out on potential profits R^L-R when the aggregate shock is positive. The second term in Equation 18 represents the marginal benefit of tighter standards. Since $R > \tilde{a}\epsilon \bar{R}^K$ in the case of a negative aggregate shock, banks reduce their expected loan losses in the case of a negative aggregate shock by tightening lending standards. If $R^L < R$, the bank makes no loans; however, if $R^L > R$, optimal lending standards are given by Equation 18, where the marginal cost and marginal benefit of tighter standards are equal.

The second order condition necessary for Equation 18 to define a maximum is

$$-\left(R - \tilde{a}\hat{\epsilon}\,\overline{R}^K\right)\phi(\hat{\epsilon})\frac{\partial \hat{\epsilon}}{\partial \tilde{a}} - \int_{\hat{\epsilon}}^{1} \epsilon\overline{R}^K d\Phi(\epsilon) < 0 \quad (19)$$

Changes in $a^{\sim}$ have two counteracting effects on the marginal benefit of tighter standards. Given that $\frac{\partial\hat{\epsilon}}{\partial\tilde{a}} < 0$ by result 3, the first term in Equation 19 is positive and reflects the fact that the marginal benefit of tighter standards increases with $\tilde{a}$ because tighter standards reduce the risk of default. For a given amount of losses in the threshold group, the probability the bank experiences loan losses but not does default is higher which increases the marginal benefit of tighter standards. However, the second term in Equation 19 is negative and reflects the fact that, holding the probability of default constant, tighter standards reduce loan losses to the threshold group by virtue of increasing the quality of the threshold group, which makes this marginal benefit of tighter standards decrease with respect to standards.

For the bank's first-order conditions to represent a maximum, the marginal benefit of standards must decrease with respect to standards. However, given Equation 19, it is not readily apparent that the second-order condition holds in general. I assume for now that 19 is satisfied and verify in the numeric simulations that it does indeed hold, as well as examine the sensitivity of the result to other parameters of the model.

Before proceeding to the analysis of equilibrium in the loan market, it is useful to first examine how the bank's optimal choice of $\tilde{a}$ varies with other key determinants.

Result 5. *Optimal bank lending standards decrease when the market return on lending increases.*

Proof: Differentiating the bank's first order condition, Equation 18, with respect to R^L yields the following condition:

$$0 = -[1-\Phi(1)] + \frac{\partial\tilde{a}}{\partial R^L}\left[-(R-\tilde{a}\hat{\epsilon}\bar{R}^K)\phi(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial\tilde{a}} - \int_{\hat{\epsilon}}^{1}\epsilon\,\bar{R}^K\,d\Phi(\epsilon_K)\right] \tag{20}$$

Notice that the bracketed term on the right-hand side of Equation 20 is the second-order condition for the bank's maximization problem, and is assumed to be negative to ensure the bank's first-order condition defines a maximum. Since the first term on the right-hand side of Equation 20 is also negative, it must be that $\frac{\partial\tilde{a}}{\partial R^L} < 0$. This implies that lending standards loosen when the equilibrium market lending rate increases: when R^L rises, the marginal cost of tighter standards also rises, as the bank is foregoing a higher amount of loan revenue when a positive aggregate shock is realized. As a result, banks find it optimal to loosen standards and lower $\tilde{a}$ when the equilibrium market return to lending rises. Conversely, lending standards are tightened when R^L falls.

Optimal lending standards are also affected by the cost of deposits, which is assumed to be equal to the policy rate (*R)*.

Result 6. *Optimal bank lending standards increase when the policy rate increases.*

Proof: Differentiating the bank's first order condition, Equation 18, with respect to R^L yields

$$0 = [1-\Phi(\hat{\epsilon})] + \frac{\partial\tilde{a}}{\partial R}\left[-(R-\tilde{a}\hat{\epsilon}\bar{R}^K)\phi(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial\tilde{a}} - \int_{\hat{\epsilon}}^{1}\epsilon\,\bar{R}^K\,d\Phi(\epsilon_K)\right] \tag{21}$$

Given that the bracketed term on the right-hand side of Equation 21 is negative by the second- order condition and the first term is positive, it follows that $\frac{\partial\tilde{a}}{\partial R} > 0$. Thus, when the policy rate, and by extension, the cost of deposits, falls, the marginal benefit of tighter standards also falls—the bank now expects lower loan losses from the threshold group in the event of a negative aggregate shock. The fall in the marginal benefit of standards prompts to bank to loosen standards in response, lowering their optimal $\tilde{a}$.

Result 7. *Optimal bank lending standards increase when bank capital increases.*

Proof: Differentiating the bank's first order condition, Equation 18, with respect to K yields the following condition:

$$0 = -(R - \tilde{a}\hat{\epsilon}\bar{R}^K)\phi(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial K} + \frac{\partial\tilde{a}}{\partial K}\left[-(R - \tilde{a}\hat{\epsilon}\bar{R}^K)\phi(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial\tilde{a}} - \int_{\hat{\epsilon}}^{1} \epsilon\, \bar{R}^K\, d\Phi(\epsilon_K)\right] \tag{22}$$

Given result 4, the first term on the right-hand side of Equation 22 is positive. Since the second term in brackets is the bank's second-order condition and negative, this implies that $\frac{\partial\tilde{a}}{\partial K}$ is positive: lending standards tighten when banks have more capital, and loosen when they have less.

Intuitively, a higher level of bank capital, all else equal, reduces the default shock threshold and the probability that banks will default. This increases the marginal benefit of tighter standards at any given level of $\tilde{a}$, because the bank is now more likely to survive to enjoy the lower amount of loan losses from the threshold group that results from tighter standards. This increase in the benefit of standards prompts the bank to tighten its lending standards, increasing $\tilde{a}$.

Equilibrium and the Effect of Monetary Policy

Equilibrium in the loan market is ultimately determined by the bank's optimal lending standards (Equation 18), and the ability cut-off level (Equation 8). Combining these two equations pins down equilibrium lending standards, $\tilde{a}*$, given by the following expression:

$$\tilde{a}^* = g(\hat{\epsilon}^*)\frac{R}{\bar{R}^K} \tag{23}$$

where $\tilde{a}*$ is the equilibrium lending standards, $\hat{\epsilon}^*$ is the equilibrium bank default threshold, and the function $g(\hat{\epsilon}^*)$ captures the effect of the default threshold on equilibrium standards:

$$g(\hat{\epsilon}^*) = \frac{1-\Phi(\hat{\epsilon}^*)}{1-\Phi(1)+\int_{\hat{\epsilon}^*}^{1} \epsilon d\Phi(\epsilon)} \tag{24}$$

Figure 4 illustrates the determination of equilibrium. Since the bank's optimal $\tilde{a}$ is decreasing in R^L and the ability cut-off threshold is increasing in R^L, $\tilde{a}*$ is the unique point of equilibrium in the loan market.

Equilibrium lending standards depend, in part, on the default shock threshold $\hat{\epsilon}^*$. Taking the partial derivative of Equation 23 with respect to $\hat{\epsilon}^*$ yields the following:

$$\frac{\partial\tilde{a}^*}{\partial\hat{\epsilon}^*} = g'(\hat{\epsilon}^*)\frac{R}{\bar{R}^K} = -(R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)\frac{\tilde{a}^*\phi(\hat{\epsilon}^*)}{[1-\Phi(\hat{\epsilon}^*)]R} < 0 \tag{25}$$

Since the return to the threshold ability group must be less than the policy rate in the event of zero profits, $\frac{\partial\tilde{a}^*}{\partial\hat{\epsilon}^*} < 0$.

Equilibrium lending standards also depend on the ratio of the central bank's policy rate to the expected return on capital, $\frac{R}{\bar{R}^K}$. It is obvious from Equation 23 that, all else equal, a higher policy rate results in tighter lending standards and a lower policy rate results in looser lending standards:

$$\frac{\partial\tilde{a}^*}{\partial R} = \frac{g(\hat{\epsilon}^*)}{\bar{R}^K} = \frac{\tilde{a}^*}{R} > 0 \tag{26}$$

Of course, the total effect of the policy rate on lending standards depends on this partial effect, as well as the effect of the policy rate on the bank's default threshold $\hat{\epsilon}^*$. At equilibrium, the default shock threshold $\hat{\epsilon}^*$ is implicitly defined by the following equation:

$$\tilde{a}^*\bar{R}^K[1-H(\hat{a}^{X*})]+\int_{\tilde{a}^*}^{\hat{a}^{X*}} a\hat{\epsilon}^*\bar{R}^K dH(a)-R[1-H(\tilde{a}^*)-K]=0 \quad (27)$$

where $\hat{a}^{X*}=\frac{\tilde{a}^*}{\hat{\epsilon}^*}$.

Equation 27 represents bank profits in the event of a negative shock (equation 13) where the R^L has been replaced by the equilibrium market return on lending (equation 8). Setting this equal to zero yields the equilibrium bank default threshold $\hat{\epsilon}^*$.

The relationship between the policy rate and the risk of bank default can be found by taking the total derivative of equation 27 with respect to *R*. To help build the intuition behind this key relationship, let equation 27 be represented by the following:

$$\pi_b(\tilde{a}^*,\ \hat{\epsilon}^*, R)=0 \quad (28)$$

Total differentiation of Equation 28 with respect to *R* yields

$$\frac{\partial\pi_b}{\partial\hat{\epsilon}^*}\frac{d\hat{\epsilon}^*}{dR}+\frac{\partial\pi_b}{\partial\tilde{a}^*}\frac{d\tilde{a}^*}{dR}+\frac{\partial\pi_b}{\partial R}=0 \quad (29)$$

However, given the dependence of $\tilde{a}^*$ on both *R* and $\hat{\epsilon}^*$ (Equation 23), this expression can be rewritten as

$$\frac{\partial\pi_b}{\partial\hat{\epsilon}^*}\frac{d\hat{\epsilon}^*}{dR}+\frac{\partial\pi_b}{\partial\tilde{a}^*}\left[\frac{\partial\tilde{a}^*}{\partial R}+\frac{\partial\tilde{a}^*}{\partial\hat{\epsilon}^*}\frac{d\hat{\epsilon}^*}{dR}\right]+\frac{\partial\pi_b}{\partial R}=0 \quad (30)$$

Grouping $\frac{d\hat{\epsilon}^*}{dR}$ together yields

$$\frac{d\hat{\epsilon}^*}{dR}\left[\frac{\partial\pi_b}{\partial\hat{\epsilon}^*}+\frac{\partial\pi_b}{\partial\tilde{a}^*}\frac{\partial\tilde{a}^*}{\partial\hat{\epsilon}^*}\right]+\frac{\partial\pi_b}{\partial\tilde{a}^*}\frac{\partial\tilde{a}^*}{\partial R}+\frac{\partial\pi_b}{\partial R}=0 \quad (31)$$

Ultimately, the relationship between the policy rate and risk of bank default depends on the net effect of a change in *R* on equilibrium bank profits in the face of a negative aggregate shock ("bad state profits"). A change in the policy rate affects profits through three avenues:

1. The deposit effect: $\frac{\partial\pi_b}{\partial R}$ A higher policy rate increases the cost of bank deposits, which decreases bad state profits.
2. The standards effect: $\frac{\partial\pi_b}{\partial\tilde{a}^*}\frac{\partial\tilde{a}^*}{\partial R}$A higher policy rate tightens equilibrium lending standards, which increases bad state profits.
3. The effect of the shock threshold on bad state profits both directly through the term $\frac{\partial\pi_b}{\partial\hat{\epsilon}^*}$ and indirectly through equilibrium standards and the term $\frac{\partial\pi_b}{\partial\tilde{a}^*}\frac{\partial\tilde{a}^*}{\partial\hat{\epsilon}^*}$.

Item 3, the effect of the shock threshold on bad state profits, comprises two opposing terms. The direct effect of the default threshold on bad state profits, $\frac{\partial \pi_b}{\partial \hat{\epsilon}^*} > 0$, has a positive effect on profits: on its own, a higher aggregate shock increases the return on lending for loans that fail to pay the full market return on lending, and thereby increases profits. However, the indirect effect of the default threshold on bad state profits through its effect on equilibrium lending standards, $\frac{\partial \pi_b}{\partial \tilde{a}^*}\frac{\partial \tilde{a}^*}{\partial \hat{\epsilon}^*} < 0$, has a negative effect on bad state profits since an increase in the shock threshold decreases lending standards. This, in turn, reduces bad state profits by lowering the equilibrium market return on lending and increases loan losses by decreasing asset quality.

Given the two opposing effects, what is the net effect of the shock threshold on profits? This result in formalized in the following result.

Result 8. All else equal, profits in the event of a negative aggregate shock increase when the equilibrium default shock threshold increases.

Proof: Using Equations 23 and 27, the effect of the shock on bad state profits $\left(\frac{\partial \pi}{\partial \hat{\epsilon}^*} + \frac{\partial \pi}{\partial \tilde{a}^*}\frac{\partial \tilde{a}^*}{\partial \hat{\epsilon}^*}\right)$ is given by the following:

$$\int_{\partial \tilde{a}^*}^{\hat{a}^{X*}} a\bar{R}^K dH(a) - (R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)\left[[1 - H(\hat{a}^{X*})]\bar{R}^K + h(\tilde{a}^*)(R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)\right]\frac{\phi(\hat{\epsilon}^*)\tilde{a}^*}{[1-\Phi(\hat{\epsilon}^*)]R} \tag{32}$$

The bank's second-order condition in equilibrium is

$$-(R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)\phi(\hat{\epsilon}^*)\frac{\partial \hat{\epsilon}^*}{\partial \tilde{a}^*} - \int_{\hat{\epsilon}^*}^{1} \epsilon \bar{R}^K d\Phi(\epsilon) < 0 \tag{33}$$

where $\frac{\partial \hat{\epsilon}}{\partial \tilde{a}}$ can be found by differentiating Equation 27 with respect to $\tilde{a}$ *:

$$\frac{\partial \hat{\epsilon}^*}{\partial \tilde{a}^*} = -\frac{[1-H(\hat{a}^{X*})]\bar{R}^K + h(\tilde{a}^*)(R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)}{\int_{\tilde{a}^*}^{\hat{a}^{X*}} a\bar{R}^K dH(a)} \tag{34}$$

Using Equations 32, 33, and 34 it can be shown that Equation 32 is strictly positive if

$$\tilde{a}^* < \frac{[1-\Phi(\hat{\epsilon}^*)]R}{\bar{R}^K \int_{\hat{\epsilon}^*}^{1} \epsilon d\Phi(\epsilon)} \tag{35}$$

which is true, since $1 - \Phi(1) > 0$ (Equations 23 and 24).

Using Equations 26 and 27, Equation 31 can be rewritten to obtain an explicit function for: $\frac{d\hat{\epsilon}^*}{dR}$

$$\frac{d\hat{\epsilon}^*}{dR} = \frac{d\pi_b}{d\hat{\epsilon}^*}^{-1}\left[[1 - H(\tilde{a}^*) - K] \underbrace{- \frac{\tilde{a}^*}{R}\left[[1 - H(a^{X*})]\right.}_{\text{Deposit Effect} = \frac{\partial \pi_b}{\partial R}} \underbrace{\left.\bar{R}^K + h(\tilde{a}^*)(R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)\right]}_{\text{Standard Effect} = \frac{\partial \pi_b}{\partial \tilde{a}^*}\frac{\partial \tilde{a}^*}{\partial R}}\right] \tag{36}$$

where $\frac{d\pi_b}{d\hat{\epsilon}^*}$ is equal to Equation 32, and strictly positive given result 8.

The sign of $\frac{d\hat{\epsilon}^*}{dR}$ is the same as the sign of the term in brackets on the right-hand side of Equation 36. If this is positive, then the probability of bank default increases when the policy rate increases, and vice versa.

Conversely, if this is negative, the probability of bank default decreases when the policy rate increases and rises when the policy rate falls.

The sign of the relationship between the policy rate and the probability of bank default depends on the deposits and standards effects. The first term in the large brackets is the deposits effect, which captures the negative effect of a higher R on a bank's profits. Since the cost of deposits is equal to the policy rate, as R rises the bank's total cost for a given number of deposits also rises, and bad state profits fall. Thus, the bank must earn more revenue from loans to avoid default, implying that the aggregate shock threshold $\hat{\epsilon}^*$ must rise and, all else equal, the probability of bank default increases as a result of a higher R.

A change in the policy rate also affects the risk of bank default through its effect on equilibrium lending standards. When the policy rate increases, equilibrium lending standards tighten, which increases bank profits in the event of a negative aggregate shock. The second term in the large brackets on the right-hand side of Equation 36 represents this standards effect. It measures the change in bad state bank profits induced by a change in lending standards. Two factors drive the relationship between bad state profits and lending standards. First, as standards tighten, the equilibrium market return on lending increases. This increases revenue from borrowers who can fully repay the market return ex-post. Second, tighter standards reduce loan losses in the event of a negative aggregate shock by virtue of increasing $\tilde{a}^*\hat{\epsilon}^*\bar{R}^K$, and thus also contribute to higher profits.

The standards effect, in isolation, will drive $\hat{\epsilon}^*$, and the risk of bank default, down as R increases. Since tighter standards, induced by a higher policy rate, increase bank profits in the event of a negative aggregate shock, the bank can withstand a worse aggregate shock before profits become negative.

The sign of $\frac{d\hat{\epsilon}^*}{dR}$ depends on which effect dominates. If the deposits effect is large relative to the standards effect, the probability of bank default rises with R, but if the deposits effect is small relative to the standards effect, the risk of default decreases with respect to the policy rate. Unfortunately, it cannot be determined analytically which effect will dominate. However, the model can be numerically estimated, and the sign of this key partial derivative determined.

CALIBRATION

Table 2 summarizes the model parameters that require calibration. There are two distributions (plus associated parameters) and two remaining parameters to select. The distributions in the model include the distribution for the shock to the aggregate return to capital, $\Phi(\epsilon)$, and the distribution of abilities across entrepreneurs, $H(a)$.

The distribution of abilities across entrepreneurs is assumed to be log-normally distributed on interval $(0,\infty)$ with an expected value of one. The choice of variance is slightly complicated by the fact that entrepreneurial ability is not actually directly observable; however, I look to the distribution of consumer credit scores to inform the choice of σ_a^2. Credit scores are commonly used by lenders to assess the creditworthiness of potential borrowers. Since entrepreneurial ability determines creditworthiness in this model, consumer credit scores seem like a fitting proxy for calibrating the distribution of abilities.

Ideally, data on the mean and variance of individual consumers' credit scores would be used to calibrate the distribution of entrepreneurial abilities. This micro data is compiled by the Federal Reserve Bank of New York's in their Consumer Credit Panel/Equifax (CCP) data series. This data reports individuals' Equifax Risk Score, a common measure of consumer credit scores, for a 5% representative sample of the United States.

While this microdata is not available to the greater public, other work that does use the CCP data reports the needed summary statistics. Di Maggio et al (2017) find an average Equifax Risk Score of 697 and a standard deviation of 109 using data from 2007-2013. Cortés, Glover, and Tasci (2018) find an average score of 693 with standard deviation of 107 using data from 2005-2014. I split the difference between the two and use a mean credit score of 695 and standard deviation of 108. Using these data, I calculate σ_a^2 by squaring the standard deviation and normalizing the resulting number by the mean score. This results in a variance of 16.8 for the distribution of entrepreneurial abilities.

I assume that ϵ is log-normally distributed on interval $(0,\infty)$ with an expected value of one. The shock's variance, σ_{ϵ}^{2} is jointly chosen with capital, K, to match two empirical observations from financial market data: the average bank failure rate and the average bank capital ratio. For the model's bank default measure, I calculate the ratio of the average probability of bank default to the average policy rate considered in simulations. Similarly, for the second measure I calculate the ratio of the average capital ratio to the average policy rate for all policy rates considered. These empirical measures are calculated using data for 1993-2007.

The policy rate in the model is proxied by the effective federal funds rate. This data is published in the Federal Reserve's Selected Interest Rates – H.15 publication and is accessed through the Federal Reserve Economic Database. This produces an average policy rate of 4.14%.

Data on bank capital ratios come from the Federal Deposit Insurance Corporation (FDIC) historical bank data. This data provides the total number of institutions, total assets, and total bank equity for commercial banks and saving institutions in the United States at an annual frequency. I use data from both commercial banks and savings institutions; therefore, I first combine the asset and equity data for commercial banks and lending institutions and then use this merged data to construct annual simple capital ratios (total bank equity divided by total assets) for each year from 1993-2007. These annual capital ratios are averaged over time to produce an average annual capital ratio of approximately 8.978%.

Data on the rate of bank failure are drawn from the FDIC's historical data on bank failures and assistance. This data provides the total number of bank failures each year in the United States and includes both commercial banks and savings institutions. The data is roughly divided into three categories based on what happens to the failing bank: the bank charter survives ("assistance transactions"), the bank charter is terminated (deposits are transferred to a successor charter), and payouts (depositor are directly paid out and bank assets are placed into a liquidating receivership). I include data for both commercial banks and savings institutions and exclude cases where the bank charter survives. The total number of banks is calculated by adding the total number of commercial banks and the total number of savings institutions from the FDIC's historical bank data used to construct capital ratios. Failure rates are constructed at an annual frequency by dividing the total number of failures in a given year by the total number of institutions for the same year. These annual failure rates are then averaged across time to produce an average failure rate of approximately 0.0716%.

Given the average annual effective federal funds rate, average bank capital ratio, and average annual bank failure rate imply by the data, the empirical counterparts I attempt to match via my choice of K and σ^2 are an average bank default rate of 0.07219% and an average capital ratio of 9.052%. Given the other parameters of the model, this corresponds to $K = 0.01749$ and $\sigma_{\epsilon}^{2} = 0.06015$. I have also verified that, given the two empirical targets I seek to match, this choice of K and σ_{ϵ}^{2} are unique: there is no other combination of these two parameters that produces the desired average failure rate and average capital ratio.

The choice of time frame is important, since the empirical average default rate is quite sensitive to the time frame used. In the past 30 years, the U.S. has experienced two episodes of unusual financial market distress: the savings and loans crisis of the late 1980s and the financial crisis of 2008. Unsurprisingly, including these periods in our time frame greatly increases the average annual failure rate from 0.0716% to 0.3533%. Since these crises are extreme events and not indicative of "normal" business conditions, it would be inappropriate to include these periods in the sample. During a crisis, general equilibrium effects can have significant effects on outcomes, as the fall in aggregate returns associated with crisis periods has significant multiplier effects via aggregate demand. These forces, however, are not well-captured by the partial equilibrium framework of this model. Guided by this reasoning, I restrict my time period from to 1993–2007—essentially, the period after the end of the savings and loan crisis but before the onset of the more recent financial crisis. As a robustness check, I recalibrate the model using the average capital ratio, fed funds rate, and bank failure rate from 1993–2014. Doing so increases σ_{ϵ}^{2} to 0.09851 and increases K to 0.01824 however, the alternative calibration does not change the baseline results.

Lastly, I set the long-run aggregate return to capital equal to the average annual return on the S&P 500 from 1950–2014. Data on the annual index and dividend data compiled by Damodaran (2019) is used to calculate the average annual return, including dividend payments, which yields a value of 1.1274 for R. I let the policy rate, R, vary from 1.00 to 1.10 while investigating the relationship between the policy rate and the model's equilibrium probability of bank default. After solving the model numerically, I verify that the second-order conditions hold and are reasonably robust to model parameters.

NUMERIC RESULTS

I calibrate the model and obtain numeric results using MATLAB version 2019a. A function was coded to represent the equations for equilibrium lending standards (Equations 23 and 24) and the equilibrium bank default threshold (Equation 27) as functions of lending standards, the default threshold, the policy rate, and other parameters. This system of equations is then solved for equilibrium lending standards and the equilibrium default threshold, given the policy rate. This process is repeated for all policy rates considered.

The relationship between the model's implied probability of default and policy rate under the baseline calibration is depicted in Figure 5. As the figure shows, a decrease in the policy rate results in a higher probability of bank default under the baseline calibration. As R falls, the negative effect of looser lending standards outweighs the positive effect associated with cheaper deposits, and the probability of bank default rises when the policy rate falls.

The baseline result—expansionary monetary policy increases the risk of bank default—is in line with the empirical literature on the risk-taking channel of monetary policy. Specifically, the baseline results of this work mirror the overall results from the existing empirical literature which support the idea that 1) looser monetary policy leads to a higher probability of bank default, and 2) looser monetary policy results in looser lending standards.[11]

While the baseline results of this model indicate that the probability of bank default falls when the policy rate rises, this relationship is sensitive to the amount of bank capital (K) and the variance of the distribution of entrepreneurial abilities (σ_a^2). Relative to the baseline level of bank capital, at low levels of K the probability of bank default increases with respect to R. At intermediate levels of K the probability of bank default initially increases, but then begins to fall as R rises.[12] This sensitivity is illustrated in Figure 6.

Similarly, when σ_a^2 is low relative to the baseline, the probability of bank default increases with respect to R, and at intermediate levels, the probability of bank default is at first increasing and then decreasing with respect to R. This sensitivity is illustrated in Figure 7.

Lowering K and σ_a^2 also has the effect of increasing the average probability of bank default and lowering the average bank capital ratio. For the alternative levels of bank capital considered in Figure 6, if the remaining parameters are adjusted such that the resulting average probability of bank default and average bank capital ratio are equal to the targets of, respectively, 0.07219% and 9.052% discussed in the calibration section, the baseline result persists: the probability of bank default strictly falls when the policy rate rises. A similar pattern holds for σ_a^2: when the other parameters are adjusted to achieve the target average default probability and average capital ratio, the probability of bank default decreases with respect to R.

This suggests that the sensitivity displayed in Figures 6 and 7 is driven by differences in equilibrium capital ratios. When banks are weakly capitalized and have low capital ratios, a lower policy rate decreases the probability of default. However, if banks have high enough capital ratios, lower policy rates increase the risk of default.

Why is the relationship between the risk of bank default and the policy rate sensitive to bank capitalization? The relationship between bank default probability and policy rate ultimately depends on the relative sizes of the deposits and standards effects. In the baseline results, the standards effect dominates the deposits effect, and the risk of bank default falls as R rises ($\frac{d\hat{\epsilon}^*}{dR} < 0$). However, as the sensitivity results suggest, when capital or the variance of entrepreneurial abilities is sufficiently low, bank capital ratios are low, and the deposits effect dominates the standards effect, causing the probability of bank default to increase with respect to the policy rate. Equations 37 and 38 restate the deposit and standards effects, and Figure 8 plots their average values (averaged across policy rates) for varying levels of bank capital.

$$\text{Deposits effect: } D_X = [1 - H(\tilde{a}^*) - K] \tag{37}$$

$$\text{Standards effect: } S_X = \frac{\tilde{a}^*}{R}\left[[1 - H(\hat{a}^{X*})]\bar{R}^K + (R - \tilde{a}^*\hat{\epsilon}^*\bar{R}^K)h(\tilde{a}^*)\right] \tag{38}$$

$$\frac{d\hat{\epsilon}^*}{dR} \propto D_X - S_X \tag{39}$$

The deposits effect is simply equal to the amount of deposits the bank holds. Increasing the amount of capital the bank holds decreases the equilibrium quantity of deposits and reduces the deposits effect. Since deposits are equal to total lending less capital, a higher *K* directly decreases deposits, given the amount of total lending. Additionally, since a higher amount of bank capital results in tighter standards and lower lending (result 7), banks with more capital require even fewer deposits. Thus, as *K* rises, bank leverage (loans to capital) decreases, resulting in a lower level of deposits and a smaller deposits effect.

Since bank capital affects the equilibrium level of standards, changing *K* also influences the standards effect, which measures the effect of changing standards on bank profits. The standards effect has three components (Equation 38): the sensitivity of equilibrium lending standards to the policy rate ($\frac{\tilde{a}^*}{R}$), the number of loans that can repay their loan in full (first term inside brackets), and loan losses from the threshold group $\tilde{a}^*$ (second term inside brackets).

On one hand, increasing capital and, in effect, tightening lending standards implies a smaller number of loans must repay the full equilibrium market return on lending for the bank to still break even ($\tilde{a}^{X}*$ increases). Thus, when the market return on lending rises due to higher standards, the effect on profits is diminished, which causes the standards effect to fall when bank capital increases, all else equal. On the other hand, more capital increases the amount of loan losses from the threshold ability group $\tilde{a}^*$ the bank can withstand: banks with higher levels of capital can experience larger loan losses from this threshold group and still break even compared to those with less capital. The effect of loan losses, in isolation, amplifies the effect of standards on profits and causes the standards effect to increase as bank capital increases. Additionally, by virtue of tightening lending standards, more bank capital makes equilibrium standards more sensitive to changes in the policy rate, which also makes the standards effect larger as *K* increases.

Figure 9 illustrates the contribution to the average standards effect of loan losses to the threshold ability group, the number of borrowers that repay the full market return on lending, and the sensitivity of standards to the policy rate at varying levels of bank capital. As capital increases, the effect of loan losses to the threshold group and the effect of the policy rate on standards rises and the effect of fully paid loans falls. Taken together, the net effect of higher bank capital is to decrease the standards effect, as illustrated in Figure 8.

While both the deposits and standards effects decrease as capital rises, the deposit effect decreases much more relative to the standards effect. At low levels of bank capital, the deposits effect is large relative to the standards effect, and the probability of bank default increases with the policy rate. As the amount of bank capital increases, both effects fall; however, the deposits effect falls much faster than standards effect, and at larger capital levels (including the baseline level) the probability of bank default decreases when the policy rate increases.

As the amount of capital a bank holds rises, the deposits effect decreases faster than the standards effect; that is to say, deposits are much more sensitive to changes in capital than bank profits are to standards. If bank capital rises by \$1, deposits fall by more than \$1. On the one hand, a \$1 increase in capital means that deposits fall by \$1 to continue financing a given amount of loans. However, since more capital implies less lending, deposits decrease by more than \$1 when capital increases by \$1 because of the decrease in total lending. Thus, as capital rises, the deposits effect of R on the risk of bank default decreases relatively quickly.

Conversely, as K rises, bank profits become less sensitive to changes in lending standards and the standards effect decreases slowly relative to the deposits effect. The positive effect of threshold loan losses, and the sensitivity of standards to the policy rate (described above) causes the standards effect to be less responsive to changes in capital than the deposits effect. Put another way, as bank capital increases, the effect of standards on bank profits increases relative to the effect of the cost of deposits on bank profits.

These results have important implications for the relationship between bank default risk and the policy rate. First, it implies that a weakly capitalized bank's risk of default is increasing with respect to the policy rate, while a strongly capitalized bank's risk of default is decreasing with respect to the policy rate.

Second, it also implies that, as bank capitalization continues to increase, the effect of the policy rate on the risk of default rises. This can be seen in Figure 10, which plots the average percentage change in the default threshold caused by a change in the policy rate (averaged over the policy rates considered) for varying levels of bank capital. While higher bank capital reduces the absolute risk of bank default, it strengthens the relative impact of the policy rate on changes in default risk.

A similar pattern emerges upon examination of the sensitivity of the results to changes in σ_a^2, the variance of entrepreneurial abilities. As the variance of entrepreneurial abilities increases, lending standards tighten, and bank capital ratios increase. This causes the deposits effect to decrease relative to the standards effect and, at high enough variances and capital ratios, causes the risk of bank default to be decreasing with respect to the policy rate. The average standards and deposit effects for varying levels of entrepreneurial abilities are illustrated in Figure 11, where both effects are averaged over the policy rates considered. The average percentage change in the default threshold caused by a change in the policy rate (also averaged over the policy rates considered) for varying levels of entrepreneurial ability is shown in Figure 12.

The numeric results from the model have important implications for policymakers. First and foremost, the results suggest that both the direction and magnitude of the risk-taking channel of monetary policy depends crucially on the extent of bank capitalization. This implies that if central banks incorporate financial stability into their policy objectives, the nature of the tradeoff that policy makers will face with respect to full employment, stable inflation, and financial stability depends in large part on bank capital ratios. For weakly capitalized banks, the risk of bank default and the policy rate are positively correlated. The tradeoff of achieving lower inflation (i.e., the cost of a higher policy rate) is driven by a widening output gap and higher probabilities of bank default. However, as the baseline results suggest, when the policy rate and risk of bank default are negatively correlated, higher bank default risk is a cost of reducing the output gap.

Second, the results suggest that the magnitude of the tradeoff between employment, inflation, and financial stability also depends on the extent of bank capitalization: as the model parameters increase relative to the baseline level, equilibrium capital ratios rise and changes in the policy rate have larger relative effects on the risk of bank default. This relationship may introduce an interesting interplay between minimum capital requirements and optimal monetary policy. If bank capital ratios rise in response to stricter requirements, as they have done in the wake of the financial crisis, changes in the policy rate have a larger relative impact on the risk of bank default and, by extension, the stability of the financial system as a whole.[13] This increases the tradeoff policymakers face between achieving full employment and financial stability. The result that higher capital ratios strengthen the relative effect of the policy rate on the risk of bank default suggests that when financial stability is an objective, optimal monetary policy is affected by capital ratios and, by extension, minimum capital requirements.

This differing effect of monetary policy on bank default risk may also be of significance in determining appropriate policy responses to limit systemic risk. Large banks, which tend to be the most important in terms of systemic risk, also tend to have lower capital ratios;[14] managing risk at these banks may entail different policy responses than required for smaller, better-capitalized banks.

CONCLUSION

This paper constructs a partial equilibrium model of the commercial loan market to investigate the impact of monetary policy on bank lending standards and the risk of bank default. In the model established in this work, a reduction in the policy rate results in an easing of lending standards and, in the baseline calibration, an increase in the risk of bank default. However, the relationship between the policy rate and the probability of bank default depends critically on the extent of bank capitalization. While the relatively high capital ratios in the baseline calibration produce a negative relationship between the policy rate and the probability of bank default, when parameters are changed to produce low equilibrium capital ratios, the risk of bank default increases with the policy rate. Additionally, as equilibrium capital ratios increase relative to baseline levels, changes in the policy rate have a larger impact on the risk of bank default.

While this paper does provide insights into the role lending standards play in shaping the risk-taking channel of monetary policy, several avenues for future research and extensions to this work remain. First, incorporating entrepreneurial net worth and variable loan size would give a more complete picture of the commercial loan market. As is, changes in total lending (and by extension, bank leverage) are driven completely by the "extensive" margin: after a fall in the policy rate, lending standards loosen, thus increasing the number of borrowers and total lending. Introducing entrepreneurial net worth and variable loan size would incorporate the "intensive" margin as well: after a fall in the policy rate, existing borrowers would seek to borrow more, thus increasing total lending and bank leverage even further.

Second, seeing as the results of this model are sensitive to bank capital structure, it would be interesting to endogenize the amount of capital banks optimally prefer to hold. This would allow a more complete analysis of the bank's problem, as well as an avenue to introduce and examine the direct effect of minimum capital requirements on banks' behavior and risk-taking.

Last, embedding the model into a general equilibrium setting could provide additional insights into how changes in lending standards and the quality of bank assets affect the overall economy, as well as the impact of asset prices on lending standards and risk-taking. A general equilibrium model that incorporates bank default will also be valuable for central bankers. Bank default, and the risk thereof, may alter the effect of monetary policy on the real economy and on inflation. In addition, if policy makers aim to achieve financial stability objectives, understanding the tradeoff between bank risk, inflation and output will be vital.

ENDNOTES

1. Jeffrey Sachs, The Price of Civilization, (New York, NY: Random House, 2012); Brad DeLong, "Three or Four Mistakes in American Monetary Policy?", <http://delong.typepad.com/sdj/2009/06/three-or-four-mistakes-in-american-monetary-policy.html>; John Talyor, Getting Off Track, (Stanford, CA: Hooser Institution Press, 2009).
2. See Lo (2012) and Thakor (2015) for an overview of the causal factors of the crisis.
3. One aspect of the model is that banks are willing to lend to borrowers that hold no collateral. While borrowers have no existing assets to pledge as collateral ex-ante, the realized returns of the project serve as a form of collateral ex-post: if the borrower defaults on their loan, the bank seizes the project's revenue, similar to the loan contract in Bernanke et al. (1999).
4. This ability cut-off is similar to the firm productivity cut-off in Melitz (2003); the key difference is that in Melitz (2003), firms self-select out of production and/or exporting. In this model, borrowers do not self-select out of the loan market—they are denied by banks.
5. Throughout this work I use the terms "entrepreneur" and "borrower" interchangeably.
6. The assumption of identical banks is problematic in a general equilibrium setting; in equilibrium, either all banks default or none default. As this model is a partial equilibrium model, I abstract away from such issues.
7. Since loan size is fixed, total lending is determined solely by lending standards. Thus, the bank's problem is the same as choosing total lending to maximize expected bank profits.

8. Since each loan is $1, D_a can also be interpreted as one plus the loan interest rate.
9. That the bank can seize 100% of the project revenue is a significant departure from the costly state verification literature originating with Townsend (1979), where only a fraction of a defaulting project can be successfully collected by the bank. Introducing verification costs to the present model implies that the lender's maximum share of project revenue, net of verification costs, is strictly less than one; however, this does not change the key results.
10. Since I am not examining the real effects of bank default in a general equilibrium setting, I have assumed that banks are identical, which implies that either all banks default or all banks survive. Given this is a partial equilibrium model and I am not interested in the impact of bank default (only the probability) I abstract away from such issues.
11. See Maddaloni and Peydr´o (2011) and Altunbas et al. (2014) for details.
12. Increasing K does not alter the baseline result. Of course, if K is high enough, the probability of bank default is zero and independent of the policy rate.
13. According to data from the FDIC's historical data on banks, the average annual capital ratio at all banks (commercial bank and savings institutions) from 2008–2015 was 10.855% compared to an average of 8.978% from 1993–2007.
14. See Laeven et al. (2014).

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APPENDIX

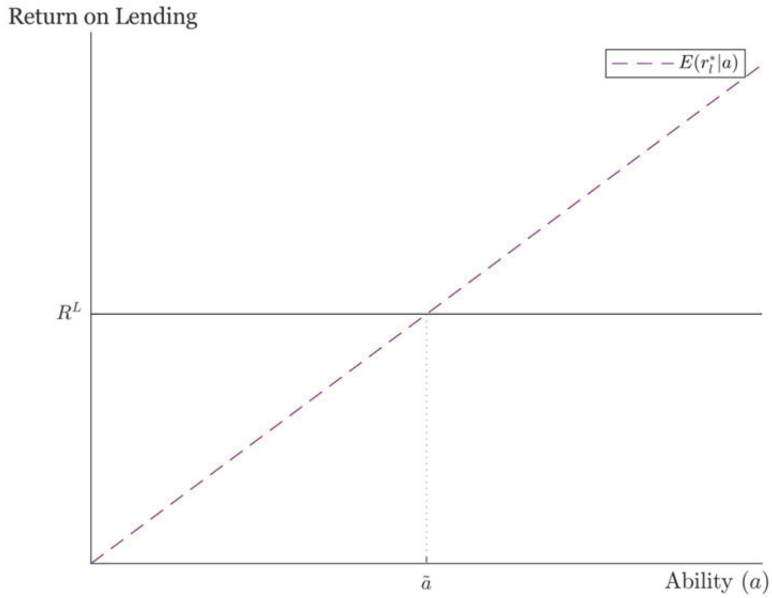
TABLE 1
NOTATION GUIDE

Returns	
Variable	Explanation
R^K	Realized aggregate return to capital. Subject to shocks: $R^K = E\bar{R}^K$.
$\bar{R}^K$	Expected aggregate return to capital.
$E(r_l^{*}\|a) = a\bar{R}^K$	Expected maximum return on lending to a borrower with ability a. Expectations with respect to the aggregate shock.
$(r_l^{*}\|a) = a\epsilon\bar{R}^K$	Realized maximum (ex-post) return on lending to a borrower with ability a given realized aggregate shock ϵ.
R^L	Market return on lending.
Shocks/Distributions	
Variable	Explanation
$\epsilon \sim \Phi(\epsilon)$	Shock to the aggregate return to capital.
$\omega \sim F(\omega)$	Idiosyncratic component of the project return.
$\hat{\epsilon}$	Default shock threshold for bank.
$\hat{\epsilon}^*$	Equilibrium bank default shock threshold.
$\tilde{\omega}_a = \dfrac{D_a}{aR^K}$	Default shock threshold for borrower with ability a. D_a is the contracted loan repayment.
$H(a)$	Distribution of entrepreneurial abilities.
Bank Balance Sheet	
Variable	Explanation
K	Bank capital.
$1 - H(\tilde{a})$	Total deposits.
$1 - H(\tilde{a}) - K$	Total loans.
$1 - H(a^X) = 1 - H(\tilde{a}/\epsilon)$	The total amount of loans that can generate a realized return of at least R^L given an aggregate shock ϵ_K.
Other	
Variable	Explanation
$\tilde{a}$	Lending standards/minimum ability requirement to receive a loan. Satisfies $E(r_l^{*}\|\tilde{a}) = R^L$.
$a^X = \tilde{a}/\epsilon$	Lowest ability group that can repay R^L after an aggregate shock Satisfies $(r_l^{*}\|a^X) = R^L$.
$\tilde{a}^*$	Equilibrium lending standards.
$\tilde{a}^{X_*} = \tilde{a}^*/\hat{\epsilon}^*$	Lowest ability group that is able to repay R^L conditional on the bank earning zero profits and given equilibrium lending standards.

TABLE 2
PARAMETER CALIBRATIONS

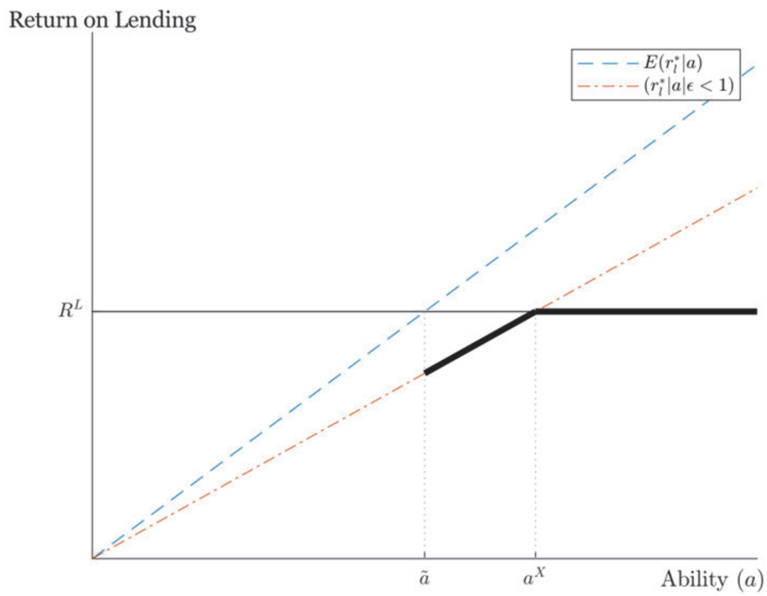
Parameter	**Value**	**Source**
$H(a)$; $h(a)$	Lognormal on $(0, \infty)$; $E(a) = 1$	–
σ_a^2	16.8	Chosen to match the standard deviation of consumer credit scores divided by the average credit score. Estimate is derived from the Consumer Credit Panel/Equifax data as reported in Di Maggio et al (2017) and Cortés, Glover, and Tasci (2018).
$\Phi(\epsilon)$; $\phi(\epsilon)$	Lognormal on $(0, \infty)$; $E(\epsilon_K) = 1$	–
σ_ϵ^2	0.06015	Chosen (with K) to match empirical average annual bank capital ratios and average annual bank default rates. Estimate is derived from the FDIC historical data on banks and bank failures.
K	0.01749	Chosen (with K) to match empirical average annual bank capital ratios and average annual bank default rates. Estimate is derived from the FDIC historical data on banks and bank failures.
$\bar{R}^K$	1.1274	Average annual return (including dividends) of the S&P 500 from 1950–2014. Estimate is derived from data compiled by Damodaran (2019).
R	range $1.00 - 1.10$	–

FIGURE 1
DETERMINATION OF LENDING STANDARDS



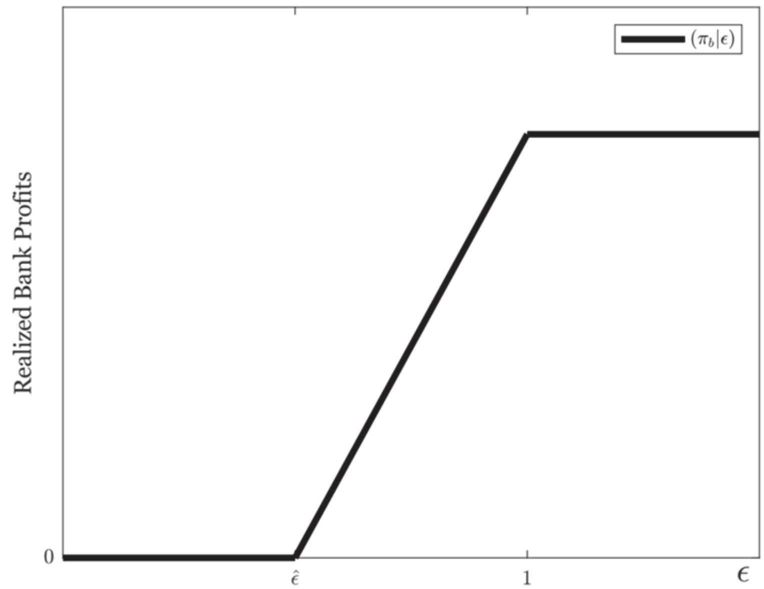
Borrowers with an expected maximum return on lending that is greater than or equal to R^L receive a loan; these are borrowers with abilities greater than or equal to $\tilde{a}$. Those with a maximum expected return on lending below R^L are excluded from the market; these are borrowers with abilities less than $\tilde{a}$.

FIGURE 2
DETERMINATION OF A^X



Given a negative aggregate shock ($\epsilon < 1$), borrowers with abilities below a^X are unable to generate an ex-post return of R^L, while borrowers with an ability of at least a^X are able to generate an ex-post return of at least R^L. The black curve gives the bank's realized return on lending in the event of a negative aggregate shock. In the case of a positive aggregate shock to ($\epsilon \geq 1$), all borrowers return R^L.

FIGURE 3
BANK PROFITS REALIZED AS A FUNCTION OF AGGREGATE SHOCK



If the aggregate shock falls below $\hat{\epsilon}$, the bank defaults. As the shock increases from $\hat{\epsilon}$ to 1, the profits realized increase. For shocks greater than or equal to 1, profits are positive and constant with respect to the shock.

FIGURE 4
EQUILIBRIUM IN THE LOAN MARKET IS DETERMINED BY BOTH THE BANK'S FIRST-ORDER CONDITION AND THE SCHEDULE OF THE MAXIMUM EXPECTED RETURN ON LENDING

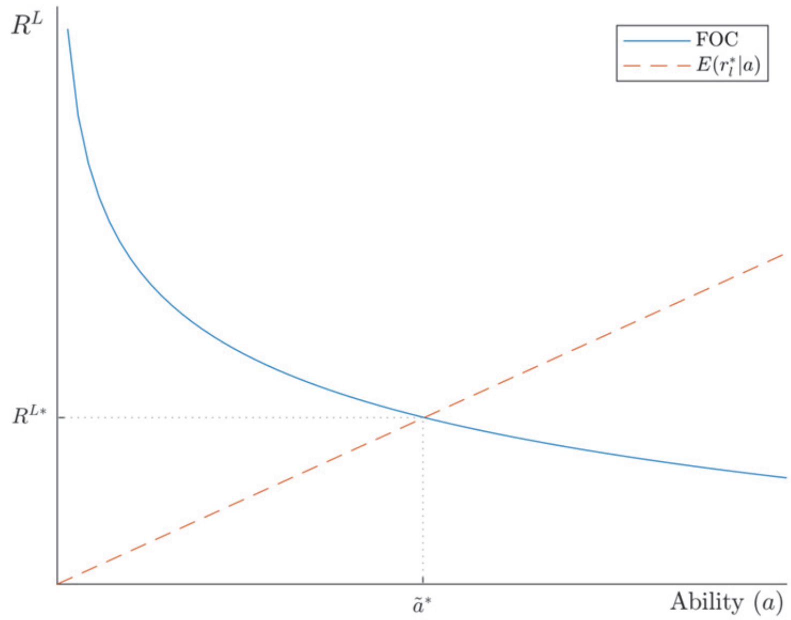
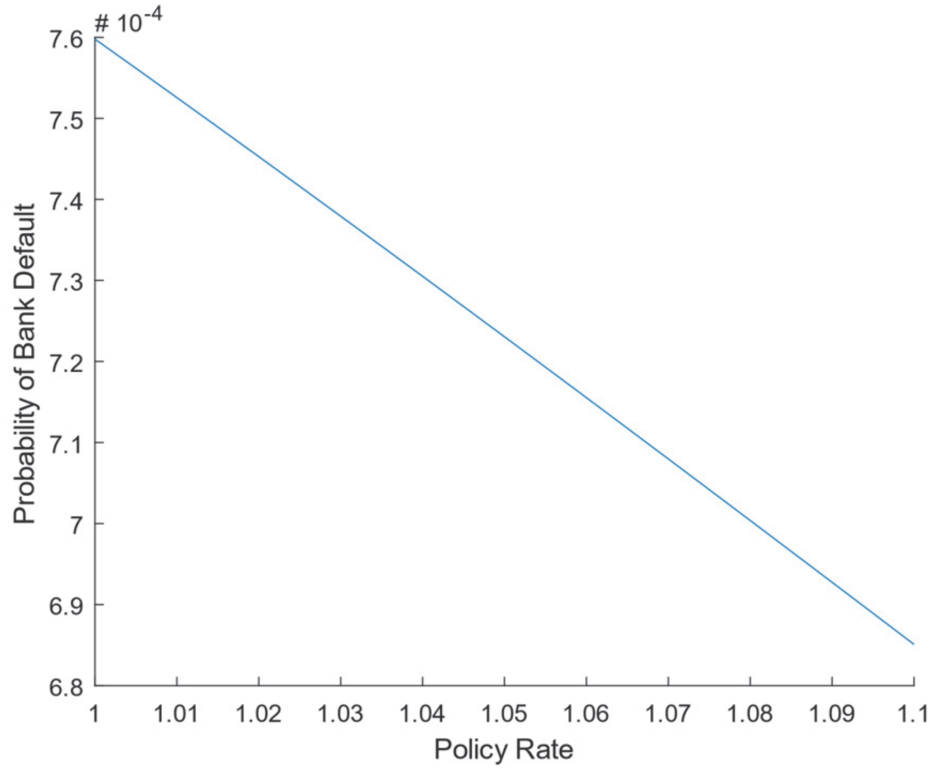
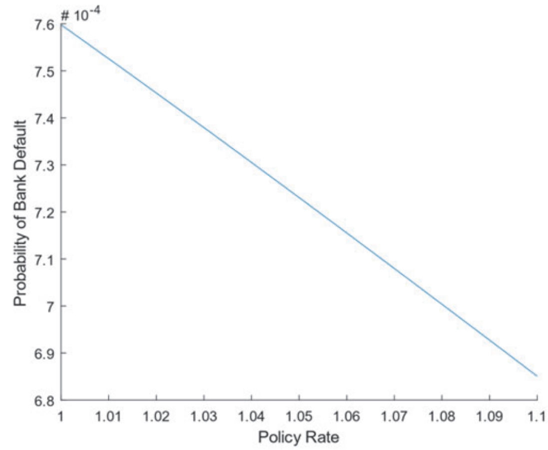


FIGURE 5
RELATIONSHIP BETWEEN THE POLICY RATE AND THE PROBABILITY OF BANK DEFAULT UNDER THE BASELINE CALIBRATION

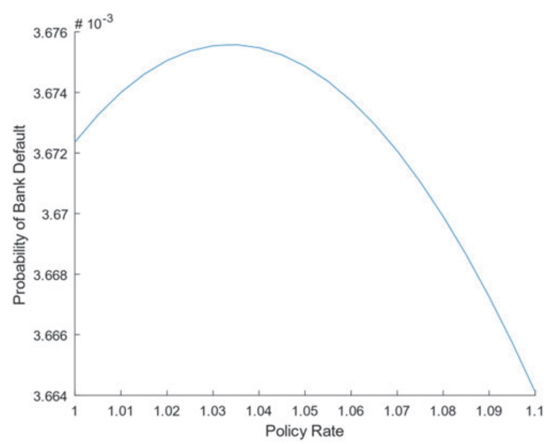


As the policy rate rises, the risk of bank default falls. At lower policy rates, the risk of bank default is higher.

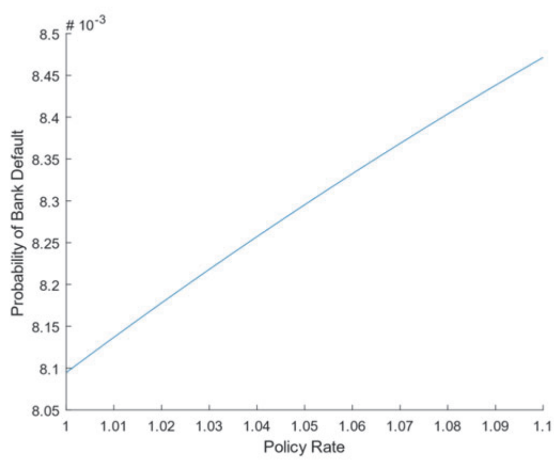
FIGURE 6
RELATIONSHIP BETWEEN THE PROBABILITY OF BANK DEFAULT AND POLICY RATE UNDER VARIOUS PARAMETERIZATIONS FOR BANK CAPITAL, *K*



(a) Baseline calibration, $K = 0.0176$



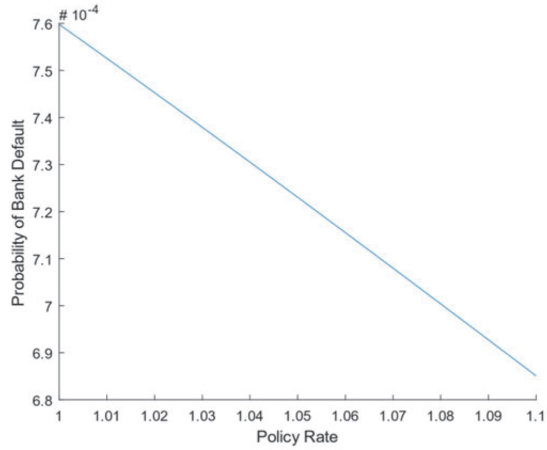
(b) $K = 0.0088$



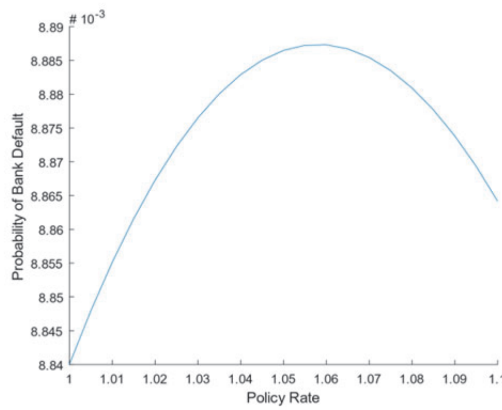
(c) $K = 0.0044$

This relationship is sensitive to bank capital: when banks' capital and capital ratios are low, the risk of bank default increases with the policy rate. At intermediate levels, it initially increases but eventually decreases in the policy rate.

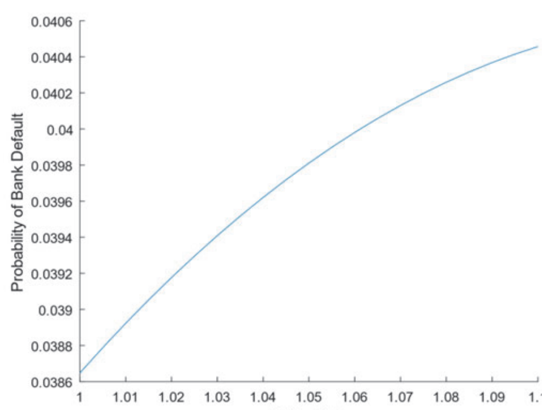
FIGURE 7
RELATIONSHIP BETWEEN THE PROBABILITY OF BANK DEFAULT AND THE POLICY RATE UNDER VARIOUS PARAMETERIZATIONS FOR THE VARIANCE OF THE DISTRIBUTION OF ENTREPRENEURIAL ABILITIES, σ_a^2



(a) Baseline calibration, $\sigma_a^2 = 16.8$.



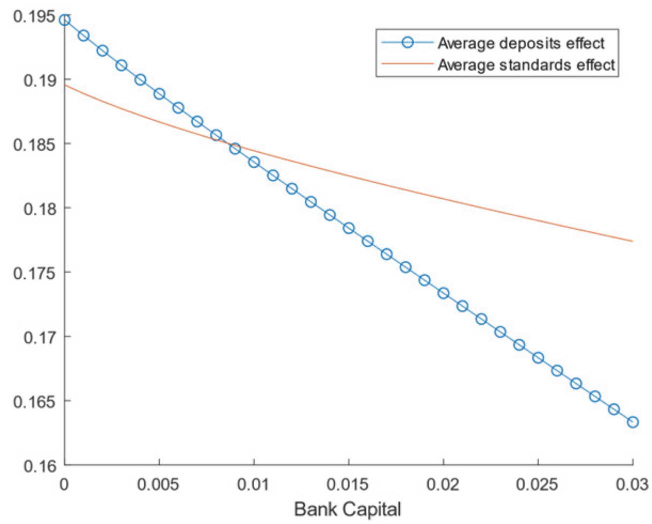
(b) $\sigma_a^2 = 2$



(c) $\sigma_a^2 = 0.5$

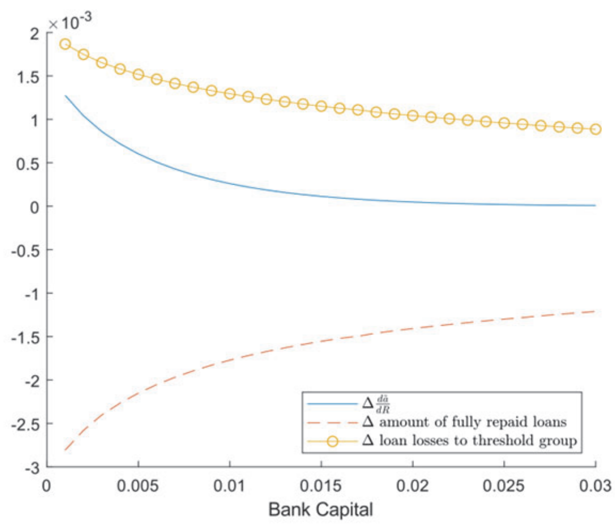
This relationship is sensitive to σ_a^2: at low values of σ_a^2, and low capital ratios, the risk of bank default increases with respect to the policy rate. At intermediate levels, it initially increases, but eventually decreases with respect to the policy rate.

FIGURE 8
AVERAGE EQUILIBRIUM DEPOSITS EFFECT AND STANDARDS EFFECT (AVERAGED OVER ALL POLICY RATES CONSIDERED) AS A FUNCTION OF BANK CAPITAL



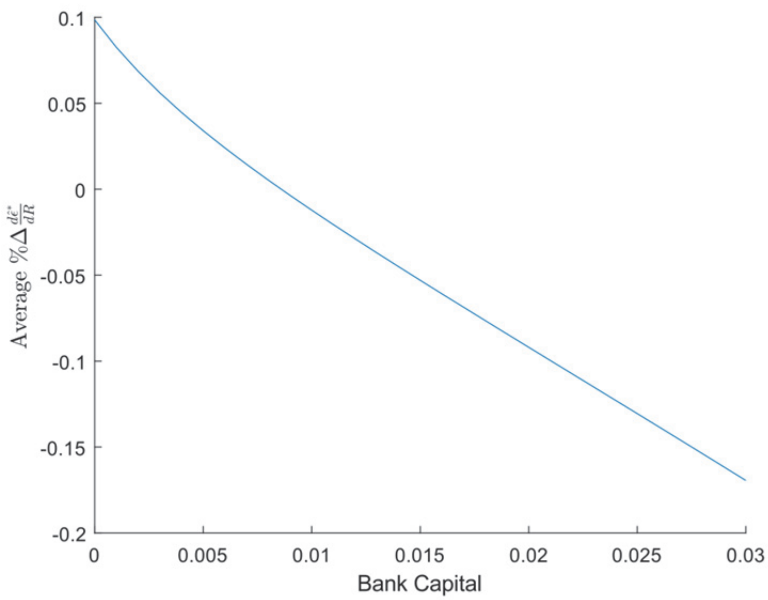
When bank capital is relatively low, the deposits effect is stronger than the standards effect, which causes the risk of bank default to fall when the policy rate falls. However, at higher levels of capital and higher capital ratios, the relative strengths of the deposits and standards effects are reversed, and the probability of bank default increases when the policy rate falls.

FIGURE 9
DECOMPOSITION OF THE EQUILIBRIUM STANDARDS EFFECT (AVERAGED OVER ALL POLICY RATES CONSIDERED) FOR VARYING LEVELS OF CAPITAL



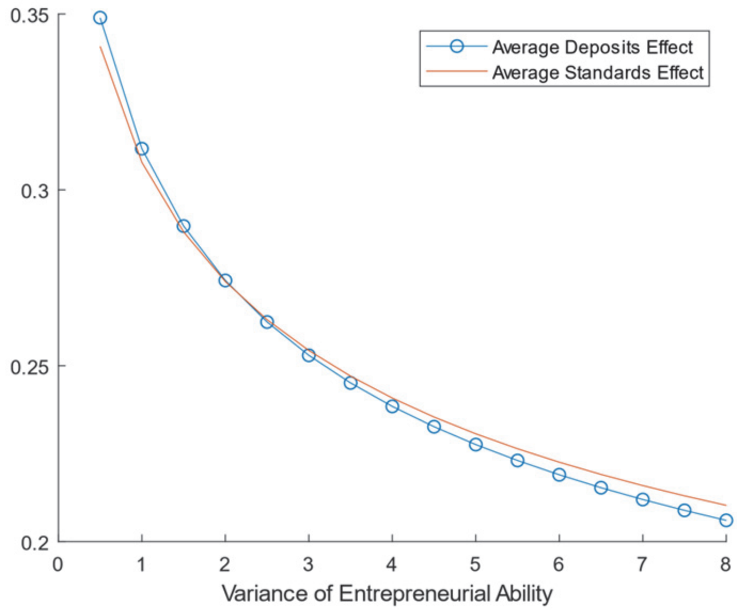
The sensitivity of the baseline results to bank capital is partially due to the relative insensitivity of the standards effect to changes in capital. The standards effect falls when *K* rises due to the lower number of loans that must fully be repaid for bank solvency; however, this effect is diminished by rising loan losses and an increase in the sensitivity of standards to the policy rate. These opposing effects make the standards effect less sensitive to changes in bank capital than the deposits effect.

FIGURE 10
AVERAGE PERCENTAGE CHANGE IN A BANK'S DEFAULT THRESHOLD (AVERAGED OVER ALL POLICY RATES CONSIDERED) AS A FUNCTION OF BANK CAPITAL



As bank capital increases past the baseline level, the policy rate has a larger relative effect on the risk of bank default than lower capital levels.

FIGURE 11
AVERAGE STANDARDS EFFECT AND DEPOSITS EFFECT AT VARYING LEVELS OF THE VARIANCE OF ENTREPRENEURIAL ABILITIES, σ_a^2



When σ_a^2 is relatively low, and capital ratios are low, the deposits effect is stronger than the standards effect, which causes the risk of bank default to fall when the policy rate falls. However, as σ_a^2 and capital ratios rise, the relative strengths of the deposits and standards effects are reversed, and the probability of bank default increases when the policy rate falls.

FIGURE 12
AVERAGE PERCENT CHANGE IN THE BANK'S DEFAULT THRESHOLD (AVERAGED OVER ALL POLICY RATES CONSIDERED) AS A FUNCTION OF THE VARIANCE OF ENTREPRENEURIAL ABILITIES, σ_a^2. AS σ_a^2A, A INCREASES PAST THE BASELINE LEVEL, THE POLICY RATE HAS A LARGER RELATIVE EFFECT ON THE RISK OF BANK DEFAULT THAN LOWER CAPITAL LEVELS