Is It a Curse or a Blessing for a Distressed Public Company to Go Private Voluntarily?

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We establish a search based model to investigate the impact of the delisting risk on the distressed public company's going private decision. We distinguish between two types of going private: voluntary delisting and involuntary delisting. Our simulation results show that there exists a tight link between them. Our model indicates that for any distressed public company, selling its entire equity voluntarily to a private equity fund at a price higher than its reservation price should not be considered as a curse, but rather as a blessing when compared to the worse scenario of being delisted involuntarily by the stock exchange.

INTRODUCTION

While going public, also known as initial public offering (IPO), is a dream of most of privately-owned firms, going private is usually considered as a disaster for a publicly-traded company. Compared to the rather rich literature in the field of IPOs, research on going private is far less common. The central reason for this could be due to the crucial function played by IPOs, not only to the involved parties, but also to the field of corporate finance, while the importance of going private has not been fully recognized yet. From the viewpoint of the entire economy, going private actually has played at least a significant role as well, if not one equal to that of the IPO.

Scarcity is a fundamental problem of economics and one of the key features of the market economy is "capital scarcity". Thus it is impossible for the society to sponsor all of the projects which are thirsty for capital or funds. The stock exchange provides a well-organized liquidity platform to link fund providers to fund seekers. Owing to the capacity constraints of the stock exchange, it can only accommodate a limited number of companies traded over the exchange. In other words, public listings are "scarce". Complementing the entry mechanism for the stock exchange (i.e., IPO), an exit mechanism (i.e., going private) is imperative for the healthy operation of the financial market. In this way, the entire financial system is balanced: the seats of low quality and poor performance companies will be replaced by high quality and good performance newcomers. Otherwise, the financial market could not function well and the entire economy could not evolve and make further progress over time. This phenomenon is a vivid finance version of "creative destruction" initially suggested by economist Joseph Schumpeter, who describes the "*process of industrial mutation that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one*" (Schumpeter 1942). The importance of going private cannot be overemphasized, although the process may be full of pain and struggle for the stakeholders.

Either due to the broader harsh operating environment or resulting only from their own poor management, publicly-traded companies can sometimes fall into financial distress. If the hardship is only temporary, some of them may return to normal. However, for other distressed public companies, a longer period of underperformance will not only trigger an alert for their shareholders, but will also draw the attention of the stock exchange where they are listed. Ultimately, there are two possible fates ahead of them. When some of the key financial measures of the distressed public companies touch on the critical points pre-established by the stock exchange, then the stock exchange has the power to delist those companies from the public trading. For example, the New York Stock Exchange (NYSE) will delist a company if the average closing price of an NYSE-traded company's share is less than $1 over a consecutive thirty trading-day period. Strictly speaking, those companies have no way but to go private involuntarily.

Meanwhile, it is also likely for a distressed public company under delisting pressure to actively search for a private equity fund (PE) as a potential buyer of its outstanding shares. This alternative exit mechanism is called going private voluntarily. Thus, instead of only one type of going private - involuntary delisting, there are actually two types of going private for distressed public companies, voluntarily or involuntarily.

Lehn and Poulsen (1989) investigate the relationship between free cash flow and a public company's decision to go private. They find that the source of stockholder gains when firms go private mainly comes from the mitigation of agency problems associated with free cash flow. Boot, Gopalan, and Thakor (2008) study how public-market investor participation affects stock price level, volatility, and the public firm's incentives to go private, which provide a link between investor participation and firm participation in the public market. Valenti and Schneider (2012) utilize the agency model to study how managerial ownership might affect a public company's decision to go private. Ferreira, Manso, and Silva (2014) study how public and private ownership structures could impact firms' incentives to invest in innovative projects. They find that it is optimal to go public if exploiting existing ideas, but at the same time it is optimal to go private if exploring new ideas.

Li, Zhang, and Zhou (2006) find that IPO firms with aggressive earnings management are more likely to delist for performance failure and tend to delist sooner, while IPO firms with conservative earnings management are more likely to be merged or acquired and they earn positive abnormal returns.

Macey, O'Hara, and Pompilio (2008) provide a thorough examination of the law and economics of the involuntary delisting process. They study the economic motivations for delisting, the stock exchange's legal rules to trigger a delisting, and the effects and consequences of delisting. According to their paper, more than 9,000 public companies have been delisted from U.S. stock exchanges since 1995, almost half of which are involuntary. We can infer of course that half of these delistings are voluntary.

In this paper we establish a random search based model to investigate the impact of the potential delisting risk imposed by the stock exchange on a distressed public company's decision to voluntarily go private. Our model makes a distinction between the two types of going private, i.e. *voluntary delisting* and *involuntary delisting*. Voluntary delisting is initialized by a public company itself while involuntary delisting is coerced by the stock exchange upon a public company. While the current literature focuses on involuntary delisting, we consider both types of delisting simultaneously, deviating from the conventional viewpoint that sees voluntary delisting as a curse instead of an acceptable or rational strategy.

More importantly, to the best of our knowledge, our paper is the first one to directly study the interaction between these two types of going private. We find that the potential delisting threat (or risk) imposed by the stock exchange has a profound impact on a distressed public company's decision to go private voluntarily. This potential threat can lower a distressed public company's bargaining power when meeting and negotiating with a private equity fund buyer. However, compared to the worse-case scenario of being delisted involuntarily by the stock exchange, voluntarily selling its entire equity to a private equity fund at a price higher than its reservation price could be a blessing to a distressed public company.

Although the concept of a distressed public firm's reservation price proposed and calculated in this paper belongs to a broad class of "reservation prices/wages" commonly used in auctions and labor markets, we are the first to associate this concept with the public firm's going private decision. Using the concept of the reservation price as a bridge, we find that the higher the probability of being delisted involuntarily by the stock exchange and the more severe the loss of the return per period due to this delisting, the more likely that the distressed public company will go private voluntarily.

In addition, we also study other key factors' impacts on the distressed public company's decision to voluntarily go private. Our simulation results show that the distressed public company's probability of voluntarily going private is positively related to the firm's opportunity for meeting a private equity fund buyer, and the degree of the severity of its financial distress. The company's likelihood of emerging from financial distress has a negative effect on its probability of voluntarily going private.

In this paper, the authors endeavor to apply search theory, one of the most important macroeconomic theories in the field of corporate finance. Diamond (1984), Mortensen and Pissarides (1994) widely apply search theory to explore the matching behavior between workers and firms in the labor market. Duffie, Garleanu, and Pedersen (2002, 2005, and 2007) are pioneers in introducing search theory to dynamic capital asset markets.

The remaining part of this paper is organized as follows: section 2 depicts our model qualitatively; section 3 establishes the model mathematically; section 4 calibrates the model, does the simulation, and analyzes the empirical implications of the model; section 5 concludes the paper. Symbols and notations are summarized in Appendix A. Proofs of propositions are provided in Appendix B.

MODEL SET-UP

In a stylized business world, there are two types of public companies traded over the stock exchange. The total mass of those companies is normalized as "1". The first type is called the "good" public company with high quality and good performance (denoted as m_H) and the second type is called the "distressed" public company with low quality and poor performance (denoted as m_L).

Whether due to their own missteps or due to the overall financial environment, some good public companies can fall into a distressed condition, and some distressed public companies can return to good status as well. Let λ_u represent the jumping-up rate of a distressed public company to good public company, and let λ_d represent the falling-down rate of a good public company to distressed public company.

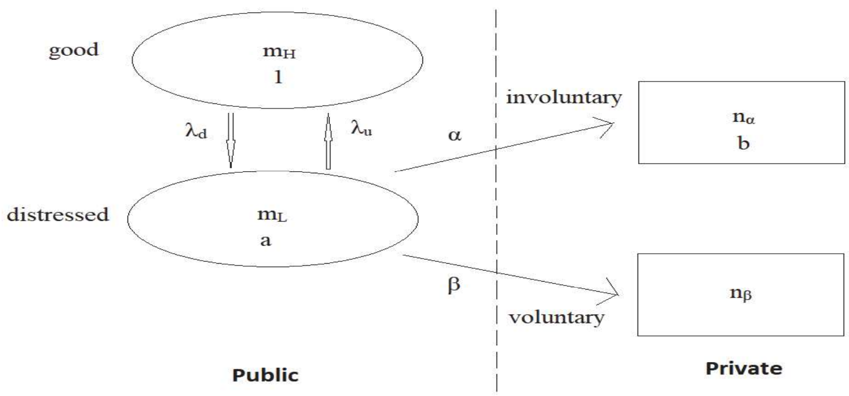
To simplify, the two types of public companies in our model are distinguished only by the dividend payout. During each time period, a good public company will pay one unit of consumption goods to its shareholders, while a distressed public company can only pay "a" units of consumption goods to its shareholders, where $a < 1$.

In the model, a distressed public company faces four possible states or outcomes. First, it could jump up and become a good public company with a rate of λ_u if its financial environment has improved or if its poor management team has been replaced by a more efficient one. Second, it could remain in its current distressed state for an indeterminate period of time without relief. In this state, there is a probability of α that the distressed public company will be sanctioned by the stock exchange and be forced to involuntarily delist. In this case, the distressed public company becomes a privately-owned company (denoted by n_α) which can provide "b" units of consumption goods to its shareholders during each time period. Here we assume that b is less than a, which indicates that going private involuntarily (or involuntary delisting) is a costly process for a distressed public company. Lastly, there is a probability of β that the distressed public company could meet a private equity fund (PE), sell its entire equity to the PE and voluntarily become a privately-owned company (denoted by n_β). The price offered by the PE is P. We assume that P is a random variable which follows a cumulative probability distribution of F(P). We assume that only distressed public companies can go private, either involuntarily or voluntarily, while the good public companies have no such an option.

In addition, we assume that all players in our model are risk neutral and the market on-going (risk-free) discount rate is denoted as r. We further assume for simplicity's sake that there is zero probability of further relisting for a distressed public company if it has already been delisted, either voluntarily or involuntarily.

Below is a schematic diagram showing the private ownership vs. public ownership status evolution.

FIGURE 1
SCHEMATIC DIAGRAM OF THE PRIVATE OWNERSHIP VS. PUBLIC OWNERSHIP STATUS EVOLUTION



According to Figure 1, we can define four market value functions corresponding to the four types of companies in our model as below:

V_{mH}: the market value of a good public company traded over the stock exchange
V_{mL}: the market value of a distressed public company traded over the stock exchange
$V_{n\alpha}$: the market value of a private company involuntarily delisted by the stock exchange
$V_{n\beta}$: the market value of a private company voluntarily acquired by a private equity fund

MATHEMATICAL MODEL

In this section, we first set up two equations to characterize the model's process of a public company's status changes:

$$rV_{mH} = 1+ \lambda_d (V_{mL}-V_{mH}) \tag{1}$$

$$rV_{mL} = a+ \lambda_u (V_{mH}-V_{mL})+\alpha(V_{n\alpha}- V_{mL})+\beta E[\max(P-V_{mL}, 0)] \tag{2}$$

For Equation (1), the left hand side represents the flow value of a public company remaining in good status, which equals the product of the discount rate r and the corresponding market value function V_{mH}; the right hand side shows the sum of dividends (in terms of consumption goods equal to 1 for good companies) and the public company's value change falling from the good status down to the distressed status.

In the same way, the left hand side of Equation (2) represents the flow value of a distressed public company while the right hand side is the sum of the dividends remaining in this status and the three value change options confronting the distressed public company. For the fourth item of the right hand side of Equation (2), E represents expectations. *Only if* the price offered by a private equity fund, P is higher than the distressed public company's current market value V_{mL} will the distressed public company go private voluntarily, i.e., allow itself to be acquired by the private equity fund. Otherwise, the distressed public company will remain in its current status and wait for another opportunity.

Next, a private company's market value functions, $V_{n\alpha}$ and $V_{n\beta}$ must satisfy the two equations below:

$$rV_{n\alpha} = b \tag{3}$$

$$V_{n\beta} = P \tag{4}$$

While Equation (3) is a standard constant-dividend discount model (DDM) in the flow- value format, Equation (4) shows that the private equity fund would like to buy the entire equity of the distressed public company at the price P, which is exactly equal to the intrinsic market value of the private company that voluntarily goes private.

When considering Equation (2), let us focus on its expectation part and define the distressed public company's reservation price P^* as $P^*=V_{mL}$. Thus we have:

$$P-V_{mL} \begin{cases} >0 & \text{when } P > P^* \\ =0 & \text{when } P = P^* \\ <0 & \text{when } P < P^* \end{cases}$$

The distressed public company's decision rule is that if the price offered by the private equity fund is larger than its reservation price, the distressed public company will sell itself to the private fund, i.e., go private voluntarily. Otherwise, the distressed public company will be waiting for the arrival of a better deal. Definition 1 summarizes the discussion above.

Definition 1: *A distressed public company's reservation price P^* is defined as $P^*=V_{mL}$. The distressed public company's decision rule is that when the price offered by a private equity fund , P, is larger than P^*, the distressed public company will go private voluntarily; when the offered price P is less than or equal to P^*, the distressed public company will stay at the current status.*

Combining Equations (1)-(4) together, we can derive a condition which can be used to pin down the distressed public company's reservation price P^* at the market equilibrium summarized in Proposition 1 (See Appendix B for details).

Proposition 1: *If the price P offered by a private equity fund is a random variable with a cumulative probability distribution function (CDF) of F(P) and a support of $[0,\overline{P}]$, a distressed public company's reservation price P^* at the market equilibrium satisfies the following condition:*

$$\left(r+\alpha+\frac{\lambda_u r}{r+\lambda_d}\right)P^* = a+\frac{\lambda_u}{r+\lambda_d}+\alpha\frac{b}{r}+\beta\int_{P^*}^{\overline{P}}[1-F(P)]dP \tag{10}$$

In order to obtain an analytical solution to P* from Equation (10), we need to provide a functional form for the cumulative probability distribution function F(P). For the sake of simplicity, we assume that F(P) follows a continuous uniform distribution over the support of $[0, \overline{P}]$. Then we can derive Proposition 2.

Proposition 2: *If the price P offered by a private equity fund is uniformly distributed over the support of $[0, \overline{P}]$, a distressed public company's reservation price P^* at the market equilibrium satisfies the following quadratic equation:*

$$mX^2-nX+l=0 \tag{15}$$

Here, X represents P^* and three new parameters (m, n, l) are defined as:

$$m=\frac{\beta}{2\overline{P}};\ n=r+\alpha+\beta+\frac{\lambda_u r}{r+\lambda_d};\ l=a+\frac{\lambda_u}{r+\lambda_d}+\alpha\frac{b}{r}+\beta\frac{\overline{P}}{2}$$

Corollary 1: *In order for Equation (15) to have a solution, the three parameters (m, n, l) have to satisfy the following condition:* $n^2 \geq 4ml$.

Corollary 2: *Under the condition* $n^2 \geq 4ml$, *only one solution of Equation (15) is reasonable for our system and its analytical format is:*

$$P^* = X_2 = \frac{n-\sqrt{n^2-4ml}}{2m} \tag{17}$$

EMPIRICAL IMPLICATIONS

In this section, we first calibrate the key parameters showing up in our model based on the market data and the current literature on delisting. Then we do a model simulation to study the impacts of a variety of variables on a distressed public company's reservation price P^* and on its probability of going private voluntarily (denoted as P_r). During the process, we will discuss the empirical implications of our model. More importantly, we will illustrate that there exists a strong interaction between voluntary delisting and involuntary delisting for a distressed public company.

Parameter Calibration

We start our parameter calibration with the choice of the discount rate r. Here we choose the median of monthly 10-year Treasury constant maturity nominal yields from January, 1980 to December, 2017, which is approximately 6.60%, as the discount rate r applied in our model.

According to Table 1 from Macey, O'Hara, and Pompilio (2008), there are 9,273 cases of delisting from U.S. stock exchanges and markets from 1995 to 2005. Thus the average number of delisting for these 11 years is: 9273/11=843≈800. Furthermore, assume that the average number of publicly-traded companies in the U.S. is around 5,000 since we usually use the Wilshire 5000 Total Market Index to measure the overall performance of the U.S. equity market (although the Wilshire 5000 listed 7,562 companies in 1998 and its current number is only 3,492 as of December 31, 2017). If we assume that one of every five public companies would show signs of distress, then there will be around 4,000 good public companies and 1,000 distressed public companies. The ratio of the average number of annual delisting to the number of distressed public companies thus equals 800/1000≈0.8. Still based on the data from Macey, O'Hara, and Pompilio (2008), the number of companies involuntarily delisting is approximately equal to the number of voluntarily delisting. So we choose 0.8/2=0.4 as both the typical value of the probability of a distressed public company being delisted involuntarily by the stock exchange, α, and the typical value of the probability of a distressed public company meeting a potential private equity fund, β, i.e., we assume that $\alpha = \beta = 0.4$.

We further assume that each year the average number of IPOs is roughly the same as the number of delistings in order to keep the system in a stable condition although the actual number of IPOs fluctuates each year. For example, in 2017, there were only 160 IPOs, while during the 1990s, there were more than 400 IPOs each year. Furthermore, in recent decades there has been a trend that the number of IPOs each year is usually less than the number of delistings, which has led to a continuing decline of the total number of companies listed in the Wilshire 5000. Moreover, we assume that it is equally likely for any public company, whether a good company or a distressed one, to go up or down, i.e., the falling-down rate λ_d is equal to the jumping-up rate λ_u. Now consider the balance of the in-flow and out-flow for the mass of distressed public companies: $4000\,\lambda_d = 1000\,\lambda_u + 800$. We can solve for $\lambda_d = \lambda_u \approx 0.25$, which means that during each period, the probability of a good public company falling down to a distressed state or a distressed public company to a good state is 25% in both cases.

In addition, since we normalize the return of a good public company as one unit of consumption goods each period in our model, we assume that a distressed public company can only earn a half of the return of a good public company each period, i.e., a=0.5. Considering both the costs and benefits of involuntary delisting by the stock exchange, we further assume that the return of an involuntarily delisted company each period, b, as 0.4 to indicate that any distressed public company would prefer to stay at the public status rather than be delisted involuntarily by the stock exchange, if it had that option. Our following results will show that when the value chosen for a or b changes, our main conclusion remains valid.

Lastly, in our stylized public ownership versus private ownership transfer world, according to the standard dividend discount model, the maximum market value of a public company could be $1/r=1/0.066\approx15$. Thus we let the upper limit of the price offered by a private equity fund buyer, $\overline{P}$, be 15.

Table 1 summarizes the key parameters and their typical values used in our model's simulation.

TABLE 1
PARAMETERS AND THEIR TYPICAL VALUES USED IN THE MODEL

Parameter	Notation	Typical Value
The (risk-free) discount rate	r	6.60%
The probability of a distressed public company being delisted involuntarily by the stock exchange	α	0.4
The probability of a distressed public company meeting a potential private equity fund buyer	β	0.4
The jumping-up rate of a distressed public company to good public company	λ_u	0.25
The falling-down rate of a good public company to distressed public company	λ_d	0.25
A distressed public company will achieve a units of return per period if staying public	a	0.5
An involuntarily delisted company will achieve b units of return per period	b	0.4
The upper limit of the price offered by a private equity fund buyer	$\overline{P}$	15

Simulation Results

In this sub-section, we study, via model simulations, the influence of different factors on a distressed public company's decision to go private voluntarily. From the viewpoint of our model, the distressed public company's decision to go private voluntarily consists of two main components. The first component is linked to the distressed public company's reservation price $P^*$when facing a random offer provided by a potential private equity fund. Now we investigate which factors and how those factors could affect the value of the distressed company's reservation price P^* quantitatively. The second component is related to the distressed public company's probability of going private voluntarily P_r. According to our model, this probability can be estimated as $\beta[1-F(P^*)]$. Here β is the probability of meeting a private equity fund buyer and $[1-F(P^*)]$ represents the probability that the private equity fund's offered purchasing price P is higher than the distressed public company's reservation price P^*. Only if the offered price P is higher than P^* will the distressed public company go private and delist voluntarily. We explore the effects of those factors on that probability as well.

Before we go to the detailed model simulation results, we implement the point estimation first. When we utilize the typical values of parameters from Table 1 for our model, we find that the distressed public company's typical reservation price P^* equals 8.32 according to Corollary 2, which means that only when the price offered by a private equity fund buyer is higher than 8.32 would the distressed public company like to sell itself to the private equity fund. Table 2 summarizes the typical market values of different types of companies *under ideal conditions, i.e.,* without search or status transfer frictions via the standard discount dividend model. According to our model, the magnitude of the distressed public company's reservation price P^* balances both the downside risk of becoming an involuntarily delisted private company (company value = 6.06) and the upside potential of jumping to good public company status (company value = 15.15). Ultimately, we find that the distressed public company's reservation price P^* (8.32) is about 10% higher than the typical market value of the distressed public company under the ideal condition (7.58), i.e., successful cases of voluntary delisting are all overvalued when using the distressed public company's market value under the ideal condition as the benchmark. Thus we claim that it is not a curse, but rather a blessing for a distressed public company to go private voluntarily.

TABLE 2
MARKET VALUES OF DIFFERENT TYPES OF COMPANIES UNDER IDEAL CONDITIONS

Market Value	Formula	Typical Value
Good public company	1/r	15.15
Distressed public company	a/r	7.58
Involuntarily delisted company	b/r	6.06

The Effect of the Probability of Being Delisted Involuntarily by the Stock Exchange, α, on the Distressed Public Company's Decision to Go Private Voluntarily

One of the key contributions of our "public ownership versus private ownership" transfer model is to establish a tight link between voluntary and involuntary delisting. To the best of our knowledge, our paper is the first to make a clear distinction between voluntary and involuntary delisting and to target the interaction between those two types of delisting.

In this part, we study the influence of the probability of involuntary delisting α on the distressed public company's reservation price P^* and on the probability of the distressed public company to go private voluntarily P_r.

When the probability of involuntary delisting α increases from 20% to 60%, the distressed public company's reservation price P^* decreases from 9.25 to 7.82 and the probability of the distressed public company going private voluntarily increases from 15.33% to 19.15%. Those results are illustrated in Figures 2 and 3, respectively.

Our simulation results show that the potential delisting threat by the stock exchange will have a far-reaching impact on the distressed public company's own decision to go private voluntarily. The higher the probability of being delisted by the stock exchange, the lower the distressed public company's reservation price, thus the more likely the distressed public company will go private voluntarily. Generally speaking, since the delisting decision by the distressed public company is voluntary, it should not be treated as a curse but rather as a blessing for this company, once we observe that a public company goes private voluntarily from the stock exchange in reality.

FIGURE 2
THE EFFECT OF α ON P^*

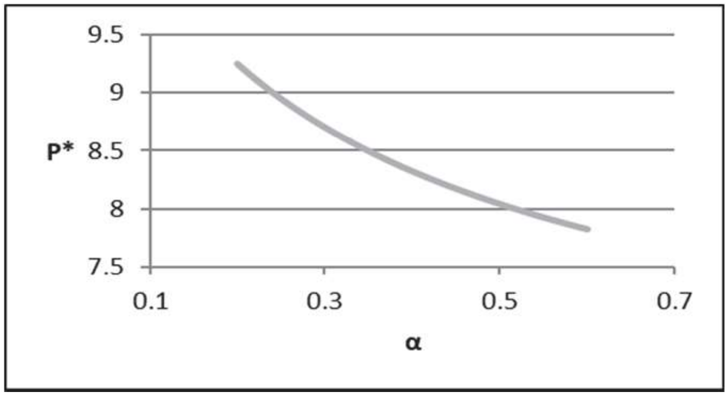
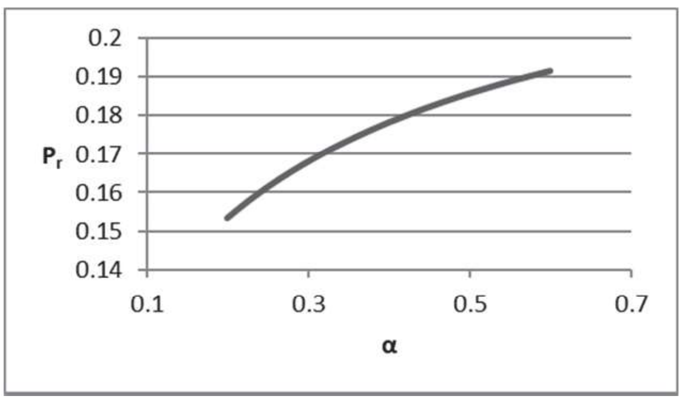


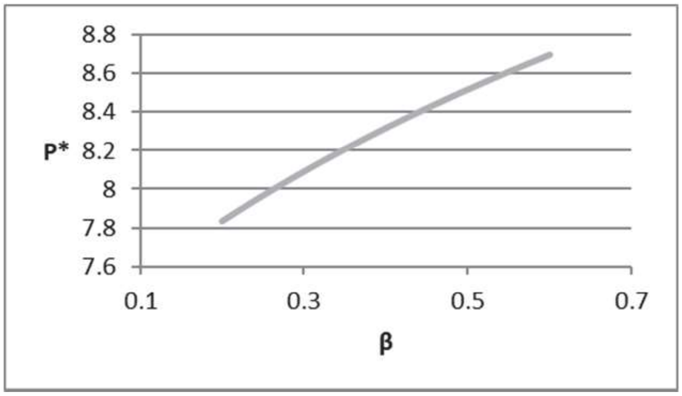
FIGURE 3
THE EFFECT OF α ON P_r



The Effect of the Probability of the Distressed Public Company Meeting a Private Equity Fund Buyer, β, on the Distressed Public Company's Decision to Go Private Voluntarily

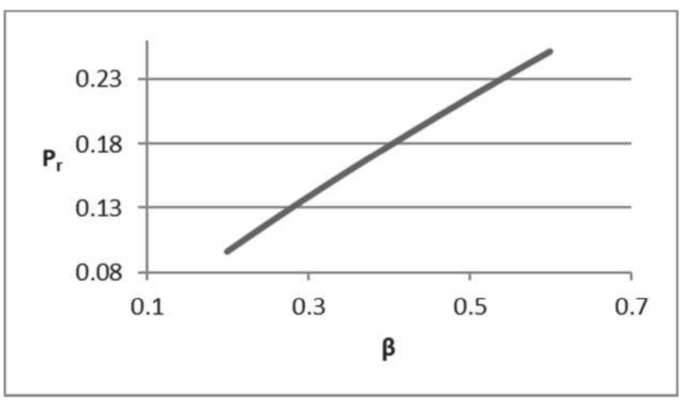
In Figure 4, we illustrate that when the probability of the distressed public company meeting a private equity fund β goes up from 20% to 60%, the distressed public company's reservation price P^* increases from 7.83 to 8.70. The more likely the distressed public company will meet a private equity fund buyer, the higher the bargaining power of the distressed public company. Thus the distressed public company's reservation price will rise as the probability of meeting a private equity fund increases. This result is rather common in the framework of random search theory.

FIGURE 4
THE EFFECT OF β ON P^*



The impact of the probability of the distressed public company meeting a private equity fund (β) on the probability of the distressed public company voluntarily going private is not as straightforward insofar as β has two opposite effects: the first is the "positive direct effect", i.e., when the meeting probability increases, the distressed public company has more chances to meet a private equity fund; the second, however, is the "indirect negative effect", i.e., as the distressed public company's reservation price goes up (shown in Figure 4), this may lead to a higher probability of rejection by the distressed public company for each meeting. Combining those two effects under the typical values of our model's parameters, we find that the first direct positive effect dominates the second indirect negative effect. Finally, when β increases from 20% to 60%, the probability of the distressed public company voluntarily going private increases as well, from 9.60% to 25.20%. This result is shown in Figure 5.

FIGURE 5
THE EFFECT OF β ON P_r



The Effect of the Jumping and Falling Rates (λu and λd) between the Two Types of Public Companies on the Distressed Public Company's Decision to Go Private Voluntarily

In Figures 6 and 7, we only investigate the influence of the jumping-up rate λ_u on the distressed public company's reservation price P^* and on the probability of the distressed public company to go private voluntarily. We don't provide the effect of the falling-down rate λ_d on P^* and on P_r here since the effect is exactly the opposite to that of λ_u.

We find that when the jumping-up rate λ_u increases from 0.1 to 0.4, the reservation price P^* increases from 7.998 to 8.614 and the corresponding probability of the distressed public company voluntarily going private falls from 18.67% to 17.03%. This finding indicates that when the overall economic operating environment improves and more distressed public companies can get out of the financial difficulty, the private equity fund has to offer a higher purchasing price in order to entice the distressed public company to go private voluntarily.

FIGURE 6
THE EFFECT OF λ_u ON P^*

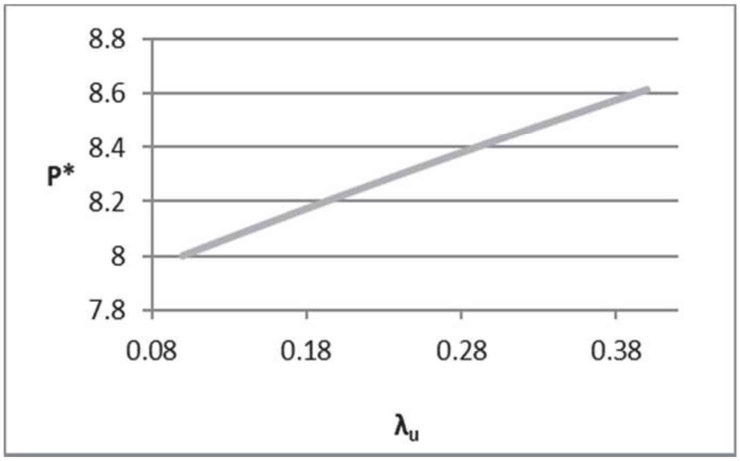
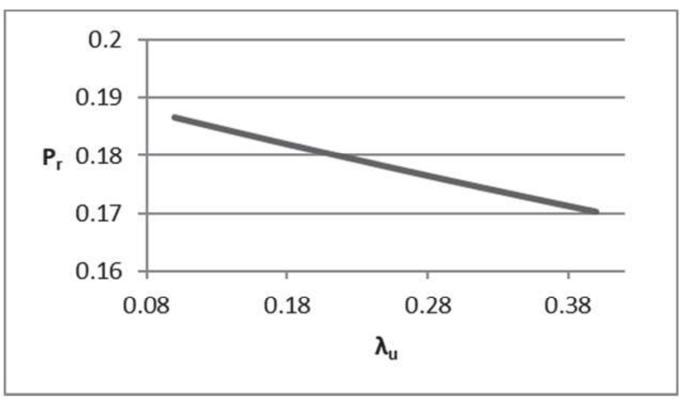


FIGURE 7
THE EFFECT OF λ_u ON P_r



The Effect of the Measure of the Extent of Distress, a, on the Distressed Public Company's Decision to Go Private Voluntarily

We assume that the good public company will achieve one unit of return per period and the distressed public company will achieve a units of return per period, where $a<1$. We can treat a as the measure of the extent of distress for the distressed public company, i.e., the lower the value of a, the more severe the extent of distress is.

In Figures 8 and 9, we study the influence of a on the distressed public company's reservation price P^* and on the probability of the distressed public company going private voluntarily. We find that when the distressed public company's return per period (a) increases from 0.4 to 0.8, the reservation price P^* goes up from 8.175 to 8.753, and the corresponding probability of the distressed public company going private voluntarily will go down from 18.20% to 16.66%. This finding is well consistent with our common sense that when the distressed public company's financial difficulty is less severe (compared to the normal good status), it is less likely that the distressed public company will accept the offer by a private equity fund to go private voluntarily.

FIGURE 8
THE EFFECT OF a ON P^*

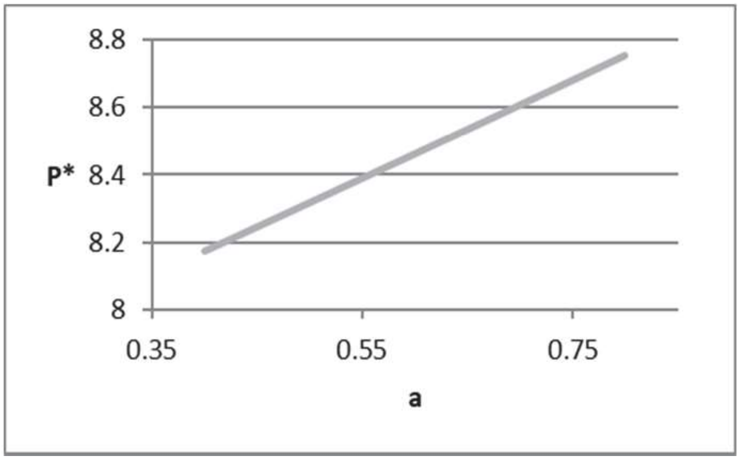
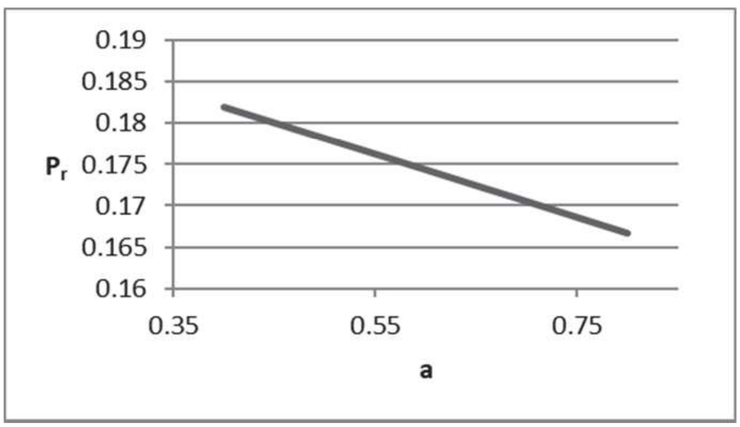


FIGURE 9
THE EFFECT OF a ON P_r



The Effect of the Return of the Involuntarily Delisted Company Per Period, b, on the Distressed Public Company's Decision to Go Private Voluntarily

Macey, O'Hara, and Pompilio (2008) find that there exists a huge direct and indirect cost borne by a distressed public company when delisted involuntarily by the stock exchange. Thus when we study the effect of the return achieved per period by the now-private, involuntarily delisted company, b, on a distressed public company's decision to go private voluntarily, we assume that b ranges from 0.1 to 0.5, which is always less than a, the return of distressed public company per period.

In Figures 10 and 11, we find that when b increases from 0.1 to 0.5, the reservation price P^* will go up from 5.826 to 9.204, and the corresponding probability of the distressed public company to go private voluntarily goes down from 24.46% to 15.46%.

Our results here illustrate again that there exists an intense interaction between going private involuntarily and voluntarily for the distressed public company. If the delisting imposed involuntarily by the stock exchange is not a formidable event for the distressed public company in terms of the loss of return per period, i.e., b is comparable with a in size, then the distressed public company doesn't need to choose to go private voluntarily once a purchasing offer from a private equity fund arrives. From this viewpoint, we can treat voluntary and involuntary delisting as alternative opportunities for the distressed public company.

FIGURE 10
THE EFFECT OF b ON P^*

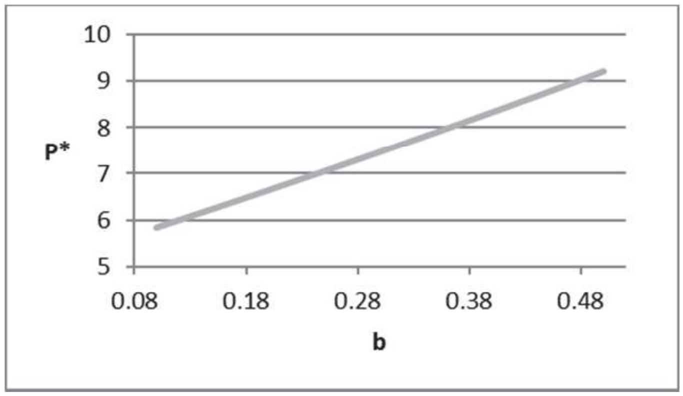


FIGURE 11
THE EFFECT OF b ON P_r

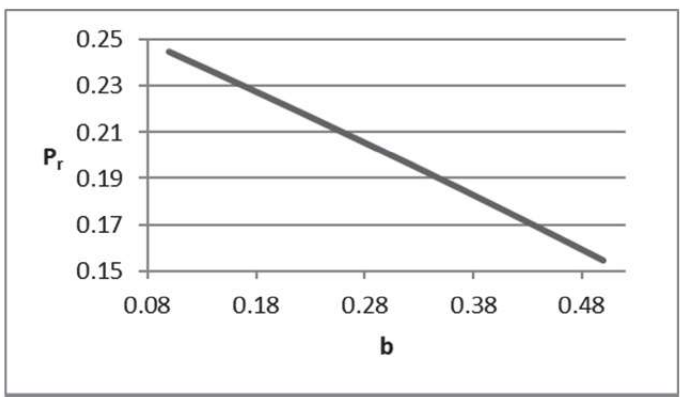


Table 3 summarizes our simulation results about the impacts of varieties of factors on the distressed public company's reservation price P^* and its probability of going private voluntarily P_r.

TABLE 3

	Factor	P^*	P_r
Interaction between voluntary delisting and involuntary delisting	**α**	Negative	Positive
	b	Positive	Negative
Other key factors' impact	**β**	Positive	Positive
	λ_u	Positive	Negative
	a	Positive	Negative

According to Table 3, we find that, except for the factor β, the effects of all other factors on the distressed public company's reservation price and on the distressed public company's probability of going private voluntarily are opposite. Typically, when the distressed public company's reservation price becomes smaller, it is more likely for the distressed public company to go private voluntarily, which is consistent with the empirical observations.

CONCLUSION

In this paper, we establish a random search based model to investigate the impact of the potential delisting risk by the stock exchange on the distressed public company's decision about going private voluntarily. One of the unique features of our model is that we distinguish between two types of going private: voluntary delisting and involuntary delisting, a difference which is largely ignored in the current finance literature. For the same reason, our model makes it possible for the first time to directly study the interaction between those two types of going private, specifically the effect of involuntary delisting on voluntary delisting.

With the help of the concept of the distressed public company's reservation price, we find that there exists a tight link between involuntary and voluntary delisting. When the distressed public company's probability of being delisted involuntarily by the stock exchange increases, it is more likely for the distressed public company to go private voluntarily. Furthermore, if the cost related to involuntary delisting in terms of the loss of return per period becomes larger, the same result obtains.

Our model also indicates that for any distressed public company, if it cannot return to normal status during a requisite time period, selling its entire equity voluntarily to a private equity fund at a price higher than its reservation price should not be considered as a curse, but rather as a blessing when compared to the worse scenario of being delisted involuntarily by the stock exchange.

In addition, our model's structure is also well suited to explore the influence of the other key factors on the distressed public company's decision to go private voluntarily. For instance, our simulation results illustrate that the likelihood of meeting a private equity fund buyer, the severity of financial hardship of the distressed public company, and the chance of getting out of this financial distress all have a profound impact on the distressed public company's probability of going private voluntarily.

No financial market is mature or healthy if it lacks an "exit mechanism". Going private is squarely the exit mechanism for the stock market. We hope that our results can fill in the gap in the literature on going private and that our paper can bring researchers' attention to this important but largely overlooked area.

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APPENDIX A: NOTATIONS

m	the mass of publicly-traded companies
n	the mass of privately-owned companies
State "H"	represents the state of a public company with high quality and good performance, i.e. a good public company
State "L"	represents the state of a public company with low quality and poor performance, i.e. a distressed public company
m_H	the mass of good pubic companies
m_L	the mass of distressed pubic companies
n_α	the mass of distressed companies who are delisted involuntarily by the stock exchange
n_β	the mass of distressed companies who go private voluntarily
a	a distressed public company will achieve a units of return per period if staying public while a good public company will achieve one unit of return per period, here a<1.
b	an involuntarily delisted company will achieve b units of return per period, here b<a.
λ_d	the falling-down rate from a good public company to distressed public company
λ_u	the jumping-up rate from a distressed public company to good public company
α	the probability of a distressed public company being delisted involuntarily by the stock exchange
β	the probability of a distressed public company meeting a potential private equity fund buyer
r	the (risk-free) discount rate
P^*	a distressed public company's reservation price
P_r	a distressed public company's probability of going private voluntarily
V_{mH}	the market value of a good public company traded over the stock exchange
V_{mL}	the market value of a distressed public company traded over the stock exchange
$V_{n\alpha}$	the market value of a private company involuntarily delisted by the stock exchange
$V_{n\beta}$	the market value of a private company voluntarily acquired by a private equity fund

APPENDIX B: PROOFS OF PROPOSITIONS

Proposition 1

From Equation (1), we can find out the relationship between V_{mH} and V_{mL}:

$$V_{mH}=\frac{1+\lambda_d V_{mL}}{r+\lambda_d} \tag{5}$$

From Equation (3), we obtain that:

$$V_{n\alpha}=\frac{b}{r} \tag{6}$$

Plug Equation (5) and (6) into Equation (2), we obtain that:

$$rV_{mL}= a + \lambda_u[\frac{1+\lambda_d V_{mL}}{r+\lambda_d} - V_{mL}]+\alpha(\frac{b}{r} - V_{mL})+\beta E[\max(P-V_{mL}, 0)] \tag{7}$$

According to the definition of the reservation price P^*, we know that:

$$P^*=V_{mL} \tag{8}$$

Substituting V_{mL} by P^*, we can thus derive the key equation to for the reservation price P^*,

$$rP^*= a + \lambda_u[\frac{1+\lambda_d P^*}{r+\lambda_d} - P^*]+\alpha(\frac{b}{r} - P^*)+\beta E[\max(P-P^*, 0)] \tag{9}$$

Replace the expectation notation E by the integral format and then simplify Equation (9), we obtain:

$$(r+ \alpha +\frac{\lambda_u r}{r+\lambda_d})P^*=a+\frac{\lambda_u}{r+\lambda_d} + \alpha\frac{b}{r} +\beta\int_{P^*}^{\overline{P}}(P - P^*)dF(P) \tag{10}$$

An alternative representation for the integral part could be:

$$\int_{P^*}^{\overline{P}} (P - P^*) dF(P)$$

$$= [(P - P^*)F(P)]_{P^*}^{\overline{P}} - \int_{P^*}^{\overline{P}} F(P) dP$$

$$= (\overline{P} - P^*) - \int_{P^*}^{\overline{P}} F(P) dP$$

$$= \int_{P^*}^{\overline{P}} [1 - F(P)] dP$$

Thus, Equation (10) changes to:

$$(r + \alpha + \frac{\lambda_u r}{r+\lambda_d})P^* = a + \frac{\lambda_u}{r+\lambda_d} + \alpha\frac{b}{r} + \beta\int_{P^*}^{\overline{P}} [1 - F(P)] dP \quad (10)$$

Proposition 2

Assume that F(P) follows a uniform distribution with a support of $[0, \overline{P}]$, thus:

$$F(P) = \frac{P}{\overline{P}} \quad (11)$$

Then the integral part of Equation (10) can be simplified further as below:

$$\int_{P^*}^{\overline{P}} [1 - F(P)] dP$$

$$= \int_{P^*}^{\overline{P}} [1 - \frac{P}{\overline{P}}] dP$$

$$= \frac{P^{*2}}{2\overline{P}} - P^* + \frac{\overline{P}}{2} \quad (12)$$

Replace the integral part of Equation (10) by Equation (12), we obtain that:

$$(r + \alpha + \frac{\lambda_u r}{r+\lambda_d})P^* = a + \frac{\lambda_u}{r+\lambda_d} + \alpha\frac{b}{r} + \beta\left(\frac{P^{*2}}{2\overline{P}} - P^* + \frac{\overline{P}}{2}\right) \quad (13)$$

Combine items and re-arrange them:

$$\frac{\beta}{2\overline{P}} P^{*2} - (r + \alpha + \beta + \frac{\lambda_u r}{r+\lambda_d})P^* + (a + \frac{\lambda_u}{r+\lambda_d} + \alpha\frac{b}{r} + \beta\frac{\overline{P}}{2}) = 0 \quad (14)$$

Define three parameters as below:

$$m = \frac{\beta}{2\overline{P}}$$

$$n = r + \alpha + \beta + \frac{\lambda_u r}{r+\lambda_d}$$

$$l = a + \frac{\lambda_u}{r+\lambda_d} + \alpha\frac{b}{r} + \beta\frac{\overline{P}}{2}$$

Consider P^* as the unknown variable X, thus Equation (14) is finally reduced into a quadratic format:

$$mX^2 - nX + l = 0 \quad (15)$$

Corollary 1 and 2

Solve Equation (15) which has two solutions if the condition of $n^2 \geq 4ml$ is satisfied, then

$$X_1 = \frac{n + \sqrt{n^2 - 4ml}}{2m} \quad (16)$$

$$X_2 = \frac{n - \sqrt{n^2 - 4ml}}{2m} \quad (17)$$

Consider the first solution X_1, here,

$X_1 = \frac{n + \sqrt{n^2 - 4ml}}{2m} \geq \frac{n}{2m}$, since $\sqrt{n^2 - 4ml}$ is always nonnegative.

Furthermore, replace n and m by their definition,

$$\frac{n}{2m} = \frac{r + \alpha + \beta + \frac{\lambda_u r}{r+\lambda_d}}{2\frac{\beta}{2\overline{P}}} = \overline{P}\left(\frac{r + \alpha + \beta + \frac{\lambda_u r}{r+\lambda_d}}{\beta}\right) > \overline{P}$$

Thus $X_1 > \overline{P}$. Only X_2 could be a reasonable solution for the system.

The reservation price P^* will be a function of a set of variables $(r, \alpha, \beta, \lambda_d, \lambda_u, a, b)$, i.e.

$$P^* = f(r, \alpha, \beta, \lambda_d, \lambda_u, a, b) = X_2 \quad (18)$$