# Regime-switching Volatility of Stock Returns and Exchange Rates in Latin America

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*A SWARCH model was applied to the stock returns and exchange rates of Brazil, Chile and Mexico. Two volatility states are present and the high volatility periods were identified and dated. The length of the high volatility period was greater for stock returns during the financial crises of the nineties whereas during the global financial crisis exchange rates experienced a longer period of high volatility. The European sovereign debt crisis and the end of the Fed's QE program caused high volatility on the exchange rates only. The degree of concordance between the high volatility regimes of stock returns and exchange rates is low in the three countries and only Brazil reported a significant positive correlation between the states.*

## INTRODUCTION

Time series models that incorporate a time-variant volatility such as the Autoregressive Conditional Heteroskedastic (ARCH) family of models have been extensively applied to financial variables such as stock market returns and exchange rates. A typical result of these models is that the shocks to the error term, the so-called ARCH effects, remain for several time periods in the future and that they are highly persistent. Lamoreaux and Lastrapes (1990) suggested that this implied high degree of persistence could be spurious if structural breaks are present in the time series of the financial variable. It can be argued that low probability events such as financial crises, recessions and stock market crashes could tentatively be viewed as structural breaks and that they may lead to changes in regime that could alter the underlying conditions of the market. Explicitly incorporating such structural breaks in either a deterministic or probabilistic way typically leads to a reduction in the degree of persistence of the ARCH effects and provides a better fit to the data than traditional ARCH models without such breaks. Hamilton and Susmel (1994) developed a Markov regime-switching model that allows for the ARCH effects of the model to be state dependent. In their investigation the switching ARCH (SWARCH) model was applied to U.S. stock market returns and it was found that the degree of persistence of the shocks was substantially reduced and it provided a better fit than the traditional ARCH model.

In the present study, the SWARCH model is applied to the time series of the stock market returns and the exchange rates of three emerging Latin American economies: Brazil, Chile and Mexico for the period 1997 to 2015, a period that includes several economic and financial crises such as the Asian financial crisis of 1997, the Russian and Brazilian crises of 1998 and 1999, the dot-com bust of 2000, the Argentinean economic crisis of 2001-2002 and the global financial crisis of 2008 which may tentatively imply changes in regime. One of the benefits of Markov regime-switching models such as the SWARCH model is that they allow for the objective dating and characterization of the regimes in financial time series. A comparison of the occurrence of the regimes across the countries will shed light on important topics such as the contagion or spill-over effects between the countries.

This research will also look at the relationship between exchange rates and stock market returns in these three Latin American economies. This relationship is of vital importance for both academicians and practitioners. The predictions of different theories about how these two financial variables interact are contradictory and can be summarized in two main branches. The first one states that when the local currency depreciates, exports become more attractive for foreign consumers and an increased demand leads to an improvement of the prospects of domestic companies which will be reflected on higher stock market returns. In short, the value of the local currency is negatively related to stock market returns. This effect is expected to be particularly important for export-oriented manufacturing economies such as Mexico. A second theory about exchange rates and stock returns predicts that a decreased level of economic activity in a country will deteriorate the prospects of the domestic companies and in consequence the returns paid in the stock market. Capital mobility will accelerate as investors seeking higher returns will rush to leave assets denominated in local currency which in turn will cause currency to depreciate. In short, this theory predicts a positive relationship between stock returns and exchange rates. In order to gain insights into the direction of the relationship between exchange rates and stock returns in Brazil, Chile and Mexico, the degree of concordance between the regimes of stock returns and exchange rates is computed and used to conduct a hypothesis test about the correlation between the two. This analysis will shed light on whether the high volatility regimes in stock market returns are in sync with similar episodes on the exchange rates.

## LITERATURE REVIEW

A few papers have applied regime-switching models to stock market returns and exchange rates in Latin American countries. Bazdrech and Werner (2005) found evidence of two regimes in the mean exchange rate of the Mexican peso. A long-lasting regime with an appreciating trend and low volatility is followed by a short-lived state with large depreciations and high volatility. Tovar-Silos (2012) also applied a Markov regime-switching model to the time series of exchange rates and stock returns. He found evidence of two distinct regimes in the two time series and assessed the degree of concordance between the regimes. He found that short duration depreciationary periods with high volatility in exchange rates are correlated with short periods of negative stock returns and high volatility. Chkili and Nguyen (2014) applied a Markov switching VAR model to the BRICS countries which includes Brazil among others and found evidence of two (high and low) volatility regimes and that stock markets have a bigger influence over exchange rates rather than the opposite in both calm and turbulent periods.

The majority of the applications of regime-switching models to exchange rates and stock markets in Latin-American economies do not explicitly incorporate switching ARCH effects but focus on switching means and the interactions between the regimes of the two time series. Canarella and Pollard (2007) applied the SWARCH model to the stock returns of six Latin American economies that include among others Brazil, Chile and Mexico. They documented the existence of two regimes in the ARCH effects of the time series and that the high volatility regimes are triggered by financial crises. They also found that the degree of persistence of the ARCH effects decreases substantially once regime-switches are incorporated in the model. This research extends the work of Canarella and Pollard by applying the SWARCH model to the exchange rates of the studied countries. The interaction between the two markets is also explored by assessing the degree of concordance between the regimes of the series and formally testing for correlation between exchange rates and stock returns.

Edwards and Susmel (2001), using stock market data for a group of Latin American countries found that high-volatility episodes are short-lived lasting from two to twelve weeks. They also found strong evidence of volatility co-movement among Latin American countries. In another study, Edwards and Susmel (2003) used interest rate data for a group of Asian and Latin American countries to analyse the behaviour of volatility through time. They found that high volatility episodes are short-lived, lasting from 2 to 7 weeks and that there is evidence of interest rate co-movements across countries. Diamandis (2008) used the SWARCH model to examine the dynamic behaviour of stock market volatility for four Latin American markets during the financial liberalization period that extended from January 1988 to July 2006. She found evidence of the existence of more than one volatility regime and that there are three episodes of high volatility for all markets around the Asian and Russian financial crises.

Only a few research articles have applied the SWARCH model to the analysis of exchange rates. Fong (1998) applied the SWARCH model to the DM/£ exchange rate in the period 1987 to 1994 which includes the entrance and exit of the UK to the Exchange Rate Mechanism (ERM). He found that the ARCH effects are substantially less severe once structural breaks are included in the model and that volatility switched to a low state shortly after Britain's ERM entry and it remained in that state until the September 1992 ERM crisis. Brunetti et al. (2007) analysed exchange rate turmoil in four Southeast Asian countries: Taiwan, Singapore, the Philippines and Malaysia with a Markov switching GARCH model. They found evidence of two regimes: an "ordinary" regime characterized by low exchange rates and low volatility and a "turbulent" regime characterized by high exchange rate movements and high volatility. They also found that real effective exchange rates, money supply relative to reserves, stock index returns and bank stock index returns contain information for identifying these regimes.

The paper is structured in the following way: in the second section the data is presented. The third section presents the model and results whereas the fourth section analyses the concordance and correlation between the two time series. The fifth and final section concludes.

## DATA

The central banks of Brazil, Chile and Mexico were the sources for the time series data of the weekly exchange rates of the Brazilian Real, Chilean Peso and Mexican Peso. Data was collected for the sample period extending from January 1997 to December 2015.

Also, data for the stock market of the three countries was obtained from Yahoo Finance and consists of the weekly closing values of the main stock indexes of Brazil, Chile and Mexico which are IBOVESPA, IGPA and IPC respectively. After eliminating some incompatible data, sample sizes and summary statistics are presented in Tables 1A and 1B.

## MODEL AND RESULTS

### Model Specification

In this research the SWARCH model developed by Hamilton and Susmel (1994) will be used to model the time series of the volatility of stock market returns and exchange rates in Brazil, Chile and Mexico. In regime-switching models it is assumed that the outcomes of a time series are drawn from one of several different probability distributions with a well-defined probability rule governing the process by which the time series switches from one regime to another. The transition probabilities are summarized in the following matrix:

$$P = \begin{bmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{bmatrix} \quad (1)$$

where $P_{ij}$ denotes the probability that the regime switches from regime i to regime j. The switching ARCH model (SWARCH) is an ARCH-type model that allows for a switching ARCH component. The first-order autoregressive equation that follows will be used to model both the time series of the stock market returns and the percentage change of the exchange rates:

$$\Delta x_t = \mu + \Phi \Delta x_{t-1} + \varepsilon_t \quad (2)$$

where:

$$\Delta x_t = ln(x_t) - ln(x_{t-1}) \tag{3}$$

and $x_t$ represents the financial variable being modelled.

In a typical ARCH model, it is assumed that the error term $e_t$ exhibits a non-constant variance that depends on the magnitude of past error realizations. ARCH models differ in the particular form that the variance process is modelled. The following equations summarize this time-dependent variability:

$$e_t = u_t \sqrt{g(s_t)} \tag{4}$$

where $u_t$ follows a conventional ARCH(q) process:

$$u_t = \sqrt{h_t} v_t,\ v_t \sim \text{i.i.d. } D(0, 1) \tag{5}$$

and $h_t$ obeys the ARCH(q) process:

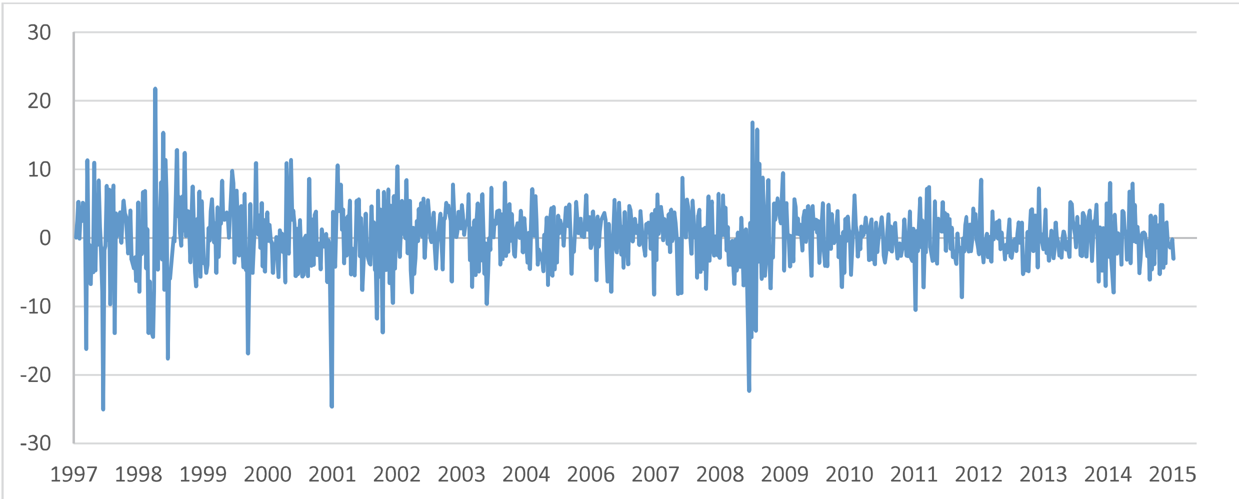
$$h_t = \propto_0 + \sum_{j=1}^{q} \propto_j u_{t-j}^2 \tag{6}$$

In the SWARCH model, g(st) is a state-dependent parameter whose function is to scale the magnitude of the ARCH effect. Note that if this parameter equals 1, the SWARCH model becomes a traditional AR(1)-ARCH(q) model so that it could be said that this model is "nested" in the SWARCH model. Also, D denotes the probability distribution of the error term. In this research, two underlying specifications of D will be assumed: the standard normal distribution and the student's t distribution. The decision to use these probability distributions relies on the fact that the former has been traditionally used whereas the latter has proved to better reflect the chances of occurrence of "unusual" realizations in financial data. Note also that the only parameter that is state-dependent is the volatility of the error term via the scale parameter $g(s_t)$. It is possible to allow other parameters such as the mean μ in equation 2 to be state-dependent but this will substantially increase the computational work and may cause attention to be deviated from the main focus of this research which is the switching volatility.

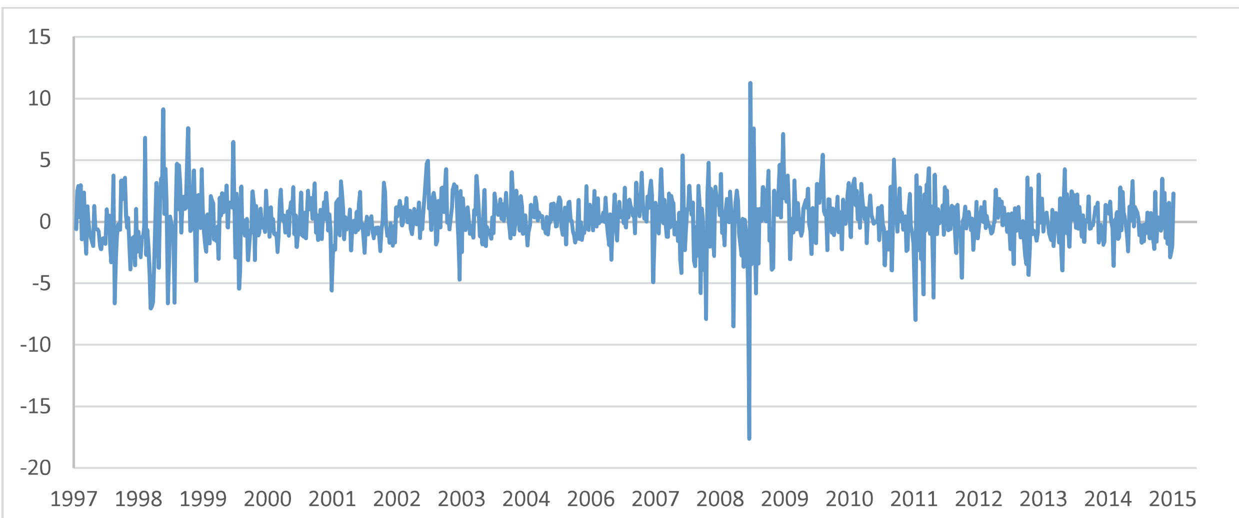
**Results of the Study**

Figure 1 exhibits the time series of the stock returns and the percentage change in exchange rates of Brazil, Chile and Mexico. In all six graphs volatility clustering, the tendency of small values to be followed by small values and of large values to be followed by large values, is identified which is a well-documented fact in financial time series. An informal inspection of these graphs permits the identification of several periods of heightened volatility, the most important being the one experienced during the global financial crisis of 2008-2009 where both exchange rates and stock markets suffered excessive volatility. Some other episodes of high volatility can be informally identified during the period 1998-1999 which includes unusual events such as the Russian and the Brazilian financial crises. It is worth noting that not all periods of high volatility affected the three countries or both exchange rates and stock markets in the same magnitude. For example, the summer of 2011 which settled the beginning of the European debt crises had a greater impact on the volatility of exchange rates of emerging economies than on the volatility of stock market returns.

**FIGURE 1**
**STOCK MARKET RETURNS IN BRAZIL**




**STOCK MARKET RETURNS IN CHILE**



**STOCK MARKET RETURNS IN MEXICO**

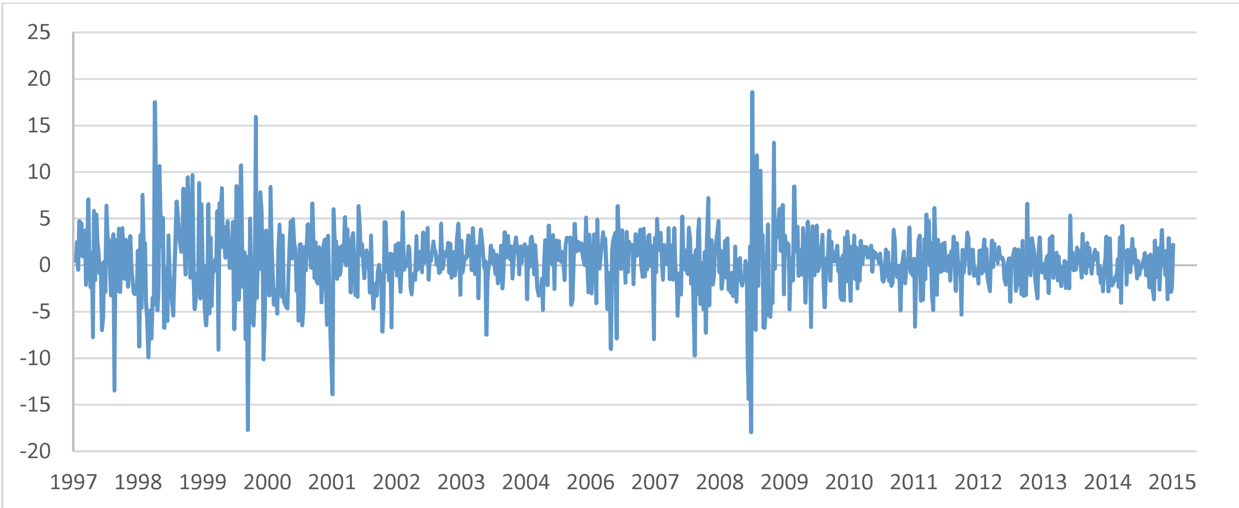
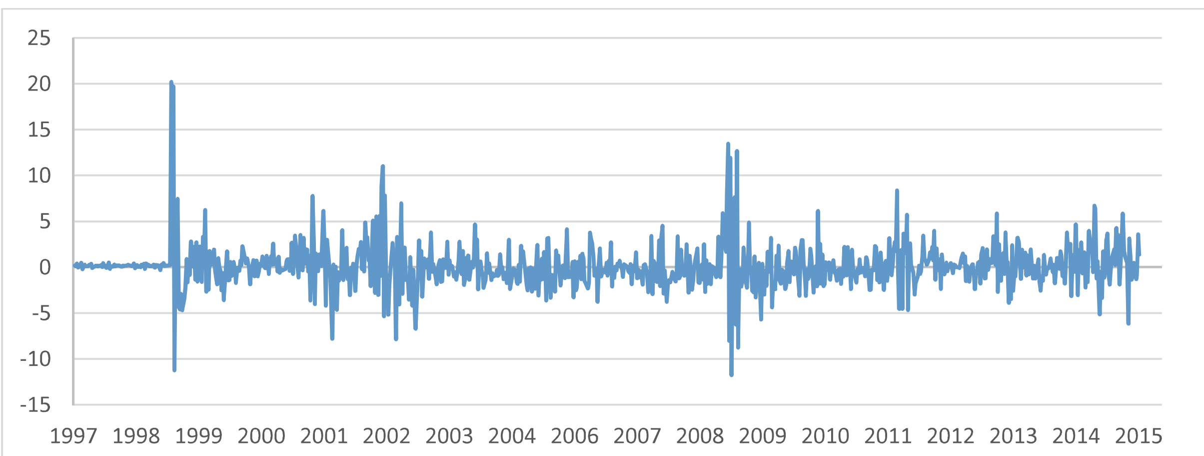
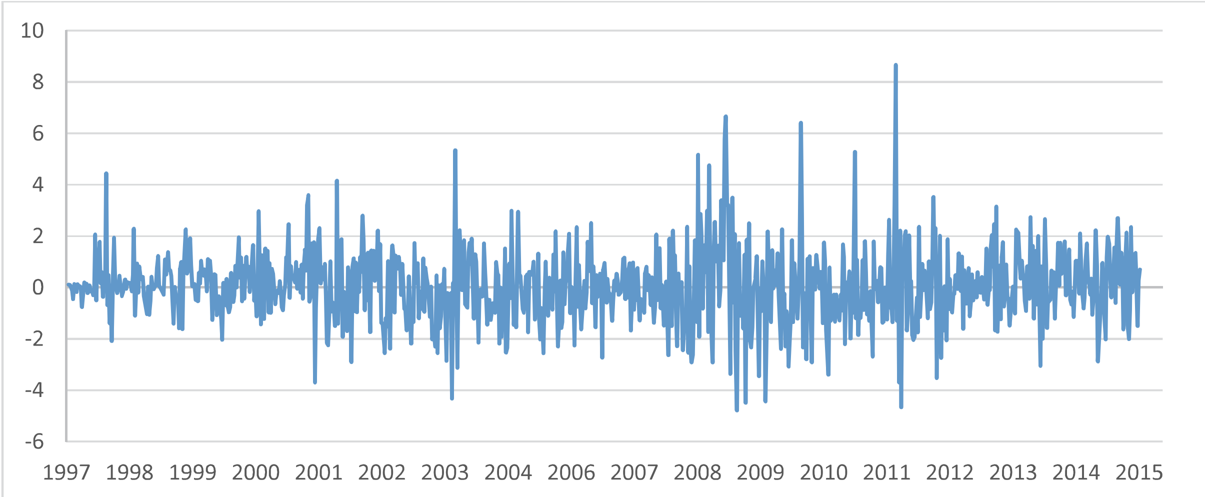


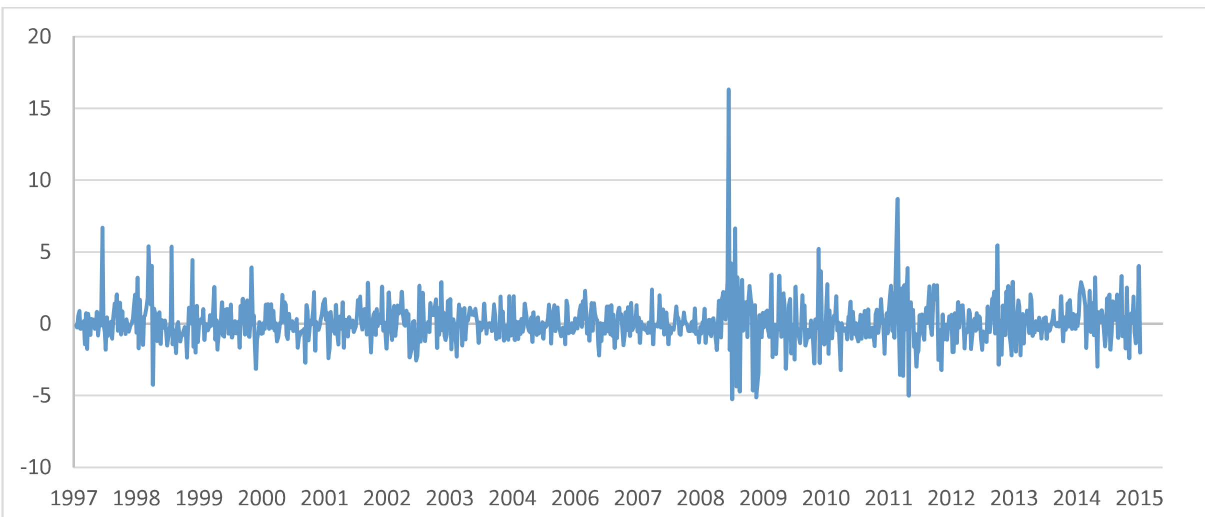

**FIGURE 1 (CONTINUED)**
**PERCENTAGE CHANGE OF THE BRAZILIAN REAL EXCHANGE RATE**



**PERCENTAGE CHANGE OF THE CHILEAN PESO EXCHANGE RATE**




**PERCENTAGE CHANGE OF THE MEXICAN PESO EXCHANGE RATE**




Other episodes of high volatility may be specific to one of the markets like the period that goes from 2003 to 2006 where the Chilean peso appreciated substantially as a result of an increase in the prices of copper that reached a maximum on May 2006. A formal identification and dating of these high volatility periods is one the benefits of the estimation of the SWARCH model.

Tables 1A and 1B present some descriptive statistics of the stock market returns and the percentage change in exchange rates. In the case of the stock market returns, the three countries exhibit positive mean values, with Mexico providing the highest mean return with a value of 0.26% per week. In the case of the exchange rates, all three currencies exhibit a mean weekly depreciation against the U.S. Dollar with Brazil exhibiting the highest value of 0.14% per week. Volatility as measured by the standard deviation and the coefficient of variation (not shown) is high in all six markets. The results of the Jarque-Bera test provide strong evidence against the null hypothesis of normality, which is a typical result of financial data. The high kurtosis (an indication of heavy tails) is also commonly observed in this type of data. In all of the time series, the results of the Ljung-Box test suggest that the null hypothesis of white noise residuals can be rejected and that significant autocorrelation is present. Also, the Ljung-Box test applied to the squared residuals indicates that there is strong evidence against the null of no autocorrelation and provides evidence for the existence of ARCH effects in the conditional variance.

**TABLE 1A**
**DESCRIPTIVE STATISTICS OF THE STOCK MARKET RETURNS**

| | **Brazil** | **Chile** | **Mexico** |
|---|---|---|---|
| Mean | 0.16 | 0.13 | 0.26 |
| Standard Deviation | 4.51 | 2.14 | 3.45 |
| Skewness | -0.57 | -0.67 | -0.18 |
| Kurtosis | 4.03 | 7.15 | 4.39 |
| Jarque-Bera | 657.16** | 2,012.70** | 742.14** |
| Ljung-Box (10 lags) | 17.28* | 21.68* | 29.58** |
| Ljung-Box$^2$ (10 lags) | 143.92** | 192.68** | 261.17** |
| Number of observations | 912 | 917 | 928 |

**TABLE 1B**
**DESCRIPTIVE STATISTICS OF THE PERCENTAGE CHANGE IN EXCHANGE RATES**

| | **Brazil** | **Chile** | **Mexico** |
|---|---|---|---|
| Mean | 0.14 | 0.06 | 0.08 |
| Standard Deviation | 2.49 | 1.41 | 1.47 |
| Skewness | 1.69 | 0.52 | 1.90 |
| Kurtosis | 13.16 | 3.20 | 18.37 |
| Jarque-Bera | 6,959.60** | 430.23** | 13,47** |
| Ljung-Box (10 lags) | 37.36** | 19.18* | 28.16** |
| Ljung-Box$^2$ (10 lags) | 520.65** | 129.01** | 46.43** |
| Number of observations | 912 | 917 | 928 |

Under the null hypothesis of normality, the Jarque-Bera test has a chi-squared distribution with 2 degrees of freedom. The Ljung-Box test for returns with 10 lags has a chi-squared distribution with 10 degrees of freedom. The Ljung-Box test for the squared returns has a chi-squared distribution with 10 degrees of freedom.
* Statistically significant at the 10% level
** Statistically significant at the 1% level

The SWARCH model was estimated using different specifications on the number of lags. The estimation results of the different models suggest that when more than one lag is utilized either there is no significant gain in the likelihood function or it is not possible to estimate the model due to convergence problems in the optimization algorithm. The results presented on Tables 2A - 2B and 3A - 3B correspond to the estimation of a switching ARCH with 2 states and one autoregressive lag denoted as a SWARCH(2,1). The normal and the student's t were both used as the underlying probability distributions of the error term in the estimation of the SWARCH(2,1) model. The estimates of the model that assumes the normal probability distribution specification (SWARCH(2,1)-N) are shown on Tables 2A and 2B whereas Tables 3A and 3B show the results when the t distribution (SWARCH(2,1)-t) was used. Tables 2A and 3A show the estimates of the SWARCH model applied to the stock returns. The estimate of the first-order autoregressive coefficient ($\Phi$) is negative and small in the cases of the stock market returns of Mexico and Brazil suggesting that returns are negatively correlated and not easy to predict. Interestingly, in the case of Chile the coefficient is positive and greater in magnitude which suggests the existence of a predictable component that could be explored in future research. The ARCH parameter ($\alpha_1$) is small in all three countries and in both the normal and the t distribution specification. The magnitude of this coefficient ranges from 0.00 in the case of Brazil to 0.13 in the case of Chile regardless of the underlying probability distribution. This result is in line with what Lamoreaux and Lastrapes (1990) suggested in relation to the spurious nature of the persistence of ARCH effects once structural breaks or changes in regime are incorporated in the model. They noticed that in a regime-switching type of model the ARCH parameter decreases substantially as compared with a traditional ARCH model. This comparison will formally be made later along with the analysis of the ARCH model.

**TABLE 2A**
**PARAMETER ESTIMATES OF THE SWARCH(2,1)-N MODEL FOR STOCK MARKET RETURNS**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | 0.2870 | 0.1619 | 0.3565 |
| $\Phi$ | -0.0482 | 0.1296 | -0.0229 |
| $\alpha_0$ | 11.9757 | 2.0507 | 0.00049 |
| $\alpha_1$ | 7.0246e-017 | 0.1109 | 0.0264 |
| $P_{11}$ | 0.9897 | 0.9852 | 0.9889 |
| $P_{12}$ | 0.0633 | 0.0499 | 0.0278 |
| $P_{21}$ | 0.0103 | 0.0148 | 0.0111 |
| $P_{22}$ | 0.9367 | 0.9501 | 0.9722 |
| Ergodic Probability (State 1 low) | 0.8600 | 0.7710 | 0.7137 |
| Ergodic Probability (State 2 high) | 0.1400 | 0.2290 | 0.2863 |
| g(2) | 5.8680 | 4.8862 | 5.6340 |
| LogL | -2,572.80 | -1,884.06 | 1,928.26 |

**TABLE 2B**
**PARAMETER ESTIMATES OF THE SWARCH(2,1)-N MODEL FOR EXCHANGE RATES**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | -0.0586 | 0.0487 | 0.0097 |
| $\Phi$ | 0.0298 | 0.0602 | -0.0162 |
| $\alpha_0$ | 1.6368 | 0.0001 | 0.00009 |
| $\alpha_1$ | 0.2160 | 0.0565 | 0.1279 |
| $P_{11}$ | 0.9780 | 0.9737 | 0.9695 |
| $P_{12}$ | 0.0342 | 0.0722 | 0.1434 |
| $P_{21}$ | 0.0220 | 0.0263 | 0.0305 |
| $P_{22}$ | 0.9658 | 0.9278 | 0.8566 |
| Ergodic Probability (State 1 low) | 0.6081 | 0.7332 | 0.8244 |
| Ergodic Probability (State 2 high) | 0.3919 | 0.2668 | 0.1756 |
| g(2) | 4.9606 | 4.1180 | 7.6350 |
| LogL | -1,763.87 | 2,671.30 | 2,733.62 |

LogL is the value of the likelihood function; $P_{11}$, $P_{12,}$ $P_{21,}$ $P_{21,}$ $P_{22}$ are the estimated elements of the transition matrix; g(1) is the estimate of the magnitude of the high volatility state (state 1) relative to the low volatility state (state 0).

**TABLE 3A**
**PARAMETER ESTIMATES OF THE SWARCH(2,1)-t MODEL FOR STOCK MARKET RETURNS**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | 0.2917 | 0.1420 | 0.3802 |
| $\Phi$ | -0.0489 | 0.1234 | -0.0152 |
| $K$ | 82.6727 | 10.0390 | 10.7296 |
| $\alpha_0$ | 12.0114 | 2.1273 | 0.0005 |
| $\alpha_1$ | 3.6238e-017 | 0.1270 | 0.0576 |
| $P_{11}$ | 0.9898 | 0.9936 | 0.9934 |
| $P_{12}$ | 0.0621 | 0.0179 | 0.0142 |
| $P_{21}$ | 0.0102 | 0.0064 | 0.0066 |
| $P_{22}$ | 0.9379 | 0.9821 | 0.9858 |
| Ergodic Probability (State 1 low) | 0.8592 | 0.7353 | 0.6811 |
| Ergodic Probability (State 2 high) | 0.1408 | 0.2647 | 0.3189 |
| g(2) | 5.7704 | 3.7550 | 4.8397 |
| logL | -2,572.74 | -1,877.52 | 1,933.16 |

**TABLE 3B**
**PARAMETER ESTIMATES OF THE SWARCH(2,1)-t MODEL FOR EXCHANGE RATES**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | -0.0685 | 0.0530 | -0.0197 |
| $\Phi$ | 0.0126 | 0.0309 | -0.0459 |
| $K$ | 6.1495 | 6.8780 | 5.5196 |
| $\alpha_0$ | 1.1872 | 0.00006 | .0001 |
| $\alpha_1$ | 0.2346 | 0.0900 | 8.3839e-023 |
| $P_{11}$ | 0.9710 | 0.9859 | 0.9875 |
| $P_{12}$ | 0.0173 | 0.0072 | 0.0289 |
| $P_{21}$ | 0.0290 | 0.0141 | 0.0125 |
| $P_{22}$ | 0.9827 | 0.9928 | 0.9710 |
| Ergodic Probability (State 1 low) | 0.3733 | 0.3387 | 0.6990 |
| Ergodic Probability (State 2 high) | 0.6267 | 0.6613 | 0.3010 |
| g(2) | 3.7019 | 3.7018 | 4.4962 |
| logL | -1,755.09 | 2,682.68 | 2,762.09 |

LogL is the value of the likelihood function; $P_{11}$, $P_{12,}$ $P_{21,}$ $P_{21,}$ $P_{22}$ are the estimated elements of the transition matrix; g(1) is the estimate of the magnitude of the high volatility state (state 1) relative to the low volatility state (state 0).

An important aspect to consider when comparing the normal and the t specifications is the magnitude of the likelihood function which is always greater in the latter for the three countries. For this reason, the analysis that follows focuses on the estimates of the SWARCH(2,1)-t model.

The estimate of the parameter g(2) which represents the ratio of the volatilities in the high and low states ranges from 3.76 in the case of Chile to 5.77 for Brazil which informally suggests the existence of regime switches in the conditional variance (a formal hypothesis test procedure will be conducted later). Note that the magnitude of this scale parameter is always greater under the normal distribution specification which suggests that some unusual realizations of the time series may not necessarily imply a change in regime but can be considered "normal" given the heavy tails of the t distribution.

The estimates of the transition probabilities $P_{11}$ and $P_{22}$ provide a measure of the degree of persistence of the states. The formula $1/(1-P_{ii})$ provides the average duration in weeks of regime i. Note that both $P_{11}$ and $P_{22}$ are always greater than 0.93 suggesting that once the series enter into one regime it tends to stay there for an extended period of time. The results indicate that the high volatility period is shorter in duration than the low volatility period and varies from 16 weeks in the case of Brazil to 70 weeks in the case of Mexico. In the other hand, the low volatility period ranges from 98 weeks in the case of Brazil to

156 weeks in the case of Chile. In general, the results of Mexico and Chile are very similar and the regimes in Brazil are shorter in duration.

The ergodic probabilities represent the percentage of time in the sample period that the series stays in a particular regime. The three countries stay in the low volatility state most of the time with the lowest percentage being 68% in the case of Mexico and the highest being 86% in the case of Brazil.

The formal statistical procedure employed to test whether regime switches exist in the ARCH effects of the time series of stock market returns relies on the following hypotheses:

$H_0$: There is no regime-switching in the ARCH effects of the stock market returns.

$H_1$**:** There is regime-switching in the ARCH effects of the stock market returns.

The model implied by the null hypothesis is a traditional AR(1)-ARCH(1)-t model whose estimated parameters are shown on Tables 4A and 4B. Note that if the scale parameter g(2) in the SWARCH model equals 1, it becomes the model implied in the null hypothesis. In this sense, it can be said that the AR(1)-ARCH(1)-t model is nested in the SWARCH(2,1)-t model and a traditional likelihood ratio test can be used to test the hypotheses shown above. The test statistic has a $\chi^2$ distribution with the number of degrees of freedom being equal to the difference in parameters between the two models which is 3. The critical value when the level of significance is .05 is 7.815. The results of this hypothesis test are shown on Tables 5A and 5B for each of the three countries and in all cases the null hypothesis is strongly rejected suggesting that the regime-switching specification provides a better fit to the data than the simple ARCH model without switching effects.

**TABLE 4A**
**PARAMETER ESTIMATES FOR AR(1)-ARCH(1)-t MODEL FOR STOCK RETURNS**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | 0.3600 (0.1323) | 0.1389 (0.0583) | 0.0042 (0.0009) |
| $\Phi$ | -0.0733 (0.03649) | 0.1362 (0.0362) | 0.0030 (0.0376) |
| $K$ | 5.6697 (0.9799) | 4.9067 (0.7716) | 4.2221 (0.0583) |
| $\alpha_0$ | 16.4656 (1.4316) | 3.3224 (0.3141) | 0.0009 (0.0001) |
| $\alpha_1$ | 0.1566 (0.0535) | 0.2429 (0.0652) | 0.3077 (0.0810) |
| logL | -2,604.12 | -1,906.23 | 1,900.53 |

**TABLE 4B**
**PARAMETER ESTIMATES FOR AR(1)-ARCH(1)-t MODEL FOR EXCHANGE RATES**

| | Brazil | Chile | Mexico |
|---|---|---|---|
| $\mu$ | -0.0323 (0.0469) | 0.0003 (0.0004) | -0.0002 (0.0003) |
| $\Phi$ | 0.0402 (0.0377) | 0.0431 (0.0382) | -0.0128 (0.0377) |
| $K$ | 2.8090 (0.3270) | 5.4802 (0.0980) | 3.8165 (0.0511) |
| $\alpha_0$ | 3.5356 (0.8093) | 0.0002 (0.00001) | 0.0001 (0.00002) |
| $\alpha_1$ | 0.9142 (0.2976) | 0.2363 (0.0744) | 0.3267 (0.0875) |
| logL | -1,882.99 | 2,663.98 | 2,745.15 |

The estimated standard errors are in parentheses. LogL is the value of the likelihood function.

**TABLE 5A**
**TEST FOR REGIME SWITCHING IN THE ARCH EFFECTS OF STOCK RETURNS**

| | **Brazil** | **Chile** | **Mexico** |
|---|---|---|---|
| **Log Likelihood SWARCH(2, 1)-t model** | -2,572.74 | -1,877.52 | 1,933.16 |
| **Log Likelihood AR(1)-ARCH(1)-t** | -2,604.12 | -1,906.23 | 1,900.53 |
| **Likelihood Ratio** | 62.76* | 57.42* | 65.26* |

**TABLE 5B**
**TEST FOR REGIME SWITCHING IN THE ARCH EFFECTS OF EXCHANGE RATES**

| | **Brazil** | **Chile** | **Mexico** |
|---|---|---|---|
| **Log Likelihood SWARCH(2, 1)-t model** | -1,755.09 | 2,682.68 | 2,762.09 |
| **Log Likelihood AR(1)-ARCH(1)-t** | -1,882.99 | 2,663.98 | 2,745.15 |
| **Likelihood Ratio** | 255.80* | 37.40* | 33.88* |

*Statistically significant at the 5% level of significance.

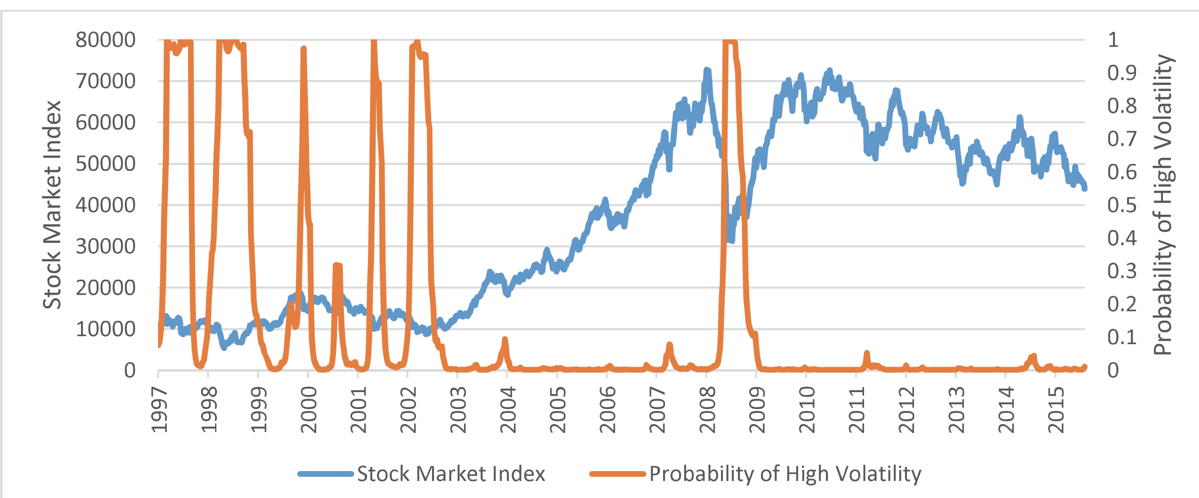
In Table 4A, the estimates of the parameter $\alpha_1$ show the degree of persistence implied by the traditional ARCH model. Note that the high values of this coefficient ranging from 0.16 in the case of

Brazil to .31 in the case of Mexico imply a very high degree of persistence which has been a subject of criticism in the literature. Some authors including Lamoreaux and Lastrapes (1990) have argued that this degree of persistence may be artificial if structural breaks exist in the time series. The results of the SWARCH model seem to support this conclusion as the estimates of the ARCH coefficient decrease substantially once structural breaks are incorporated in the model and range from 0.00 in the case of Brazil to .13 in the case of Chile.

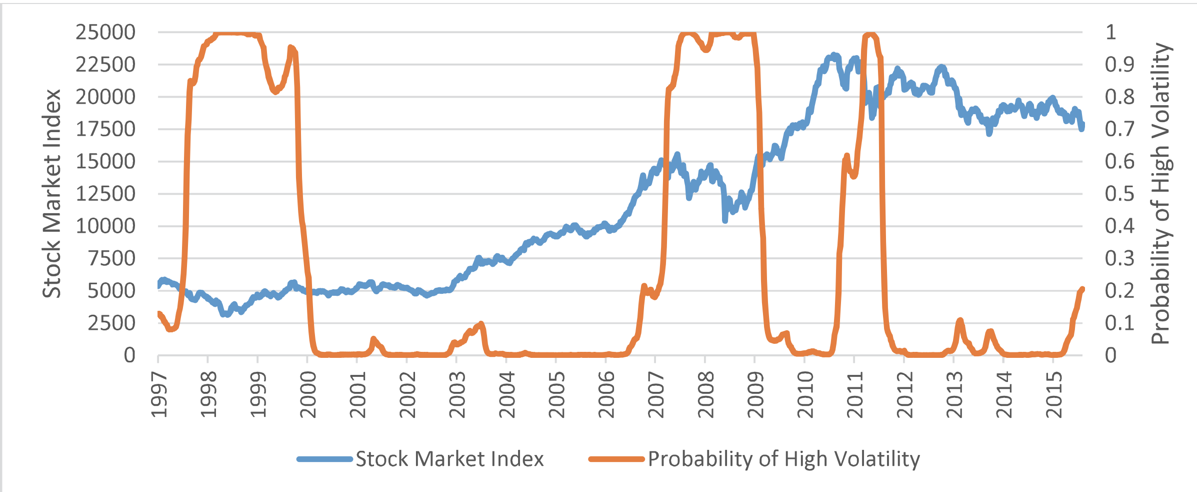
An important benefit of the SWARCH model is that it allows the objective identification of the occurrence of the regimes in the time series. This identification is based on the so-called smoothed probabilities which are ex-post inferences about the regime on a particular week and are calculated using all the information that is available at the end of the sample period. Figure 2 shows the weekly stock market index along with the probability of a high volatility regime.

Table 6A shows the dating and length of the high volatility periods identified in the stock markets of the three countries. In the case of the Mexican stock market four periods of high volatility were identified. The first period is also the longest in duration (216 weeks) and extended from May 1997 to October 2001 and coincides with several financial events such as the Asian, Russian and Brazilian financial crises of the end of the nineties and the dot-com bubble crises. Chile and Brazil also experienced high volatility in these years but in the case of Chile the length of the period was shorter (110 weeks) and in Brazil, the stock market entered and left the high volatility period four times during this period. The fact that all three economies experienced high volatility in these years is an indication that contagion was present. Another common volatility period among the three time series appears in the global financial crisis. Chile experienced the longest volatility period with a duration of 94 weeks and extended from August 2007 to June 2009. Brazil's stock market suffered high volatility for only 21 weeks from September 2008 to February 2009 whereas the Mexican stock market experienced heightened volatility from September 2008 to August 2009 for a total of 43 weeks. The remaining volatility periods do not seem to be linked to international financial crises so it is hypothesized that they could be caused by domestic events. These episodes are in general shorter in duration and the main event linked to them is indicated as "Domestic" on Table 6A.

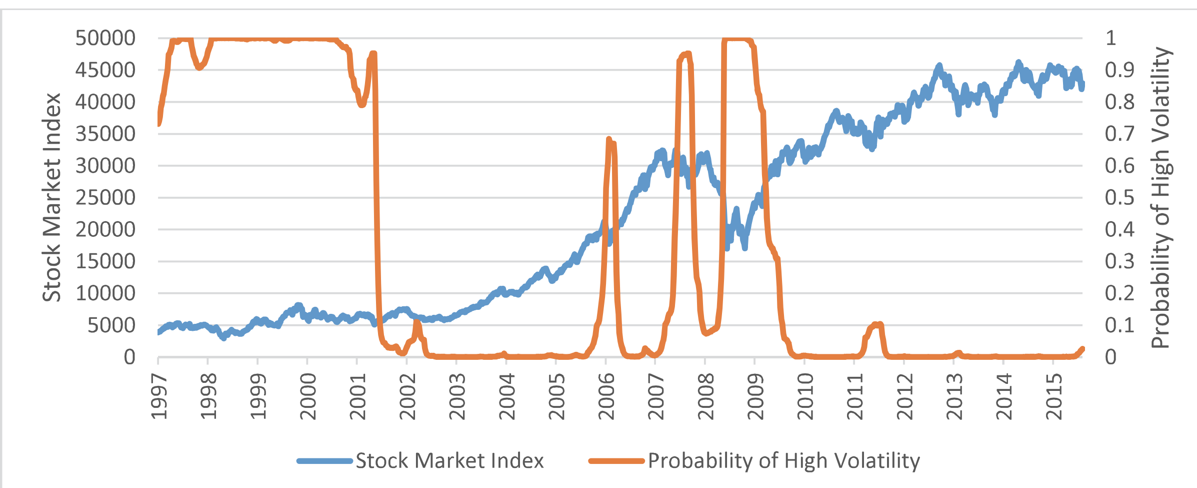
**FIGURE 2**
**STOCK MARKET AND PROBABILITY OF HIGH VOLATILITY FOR BRAZIL**



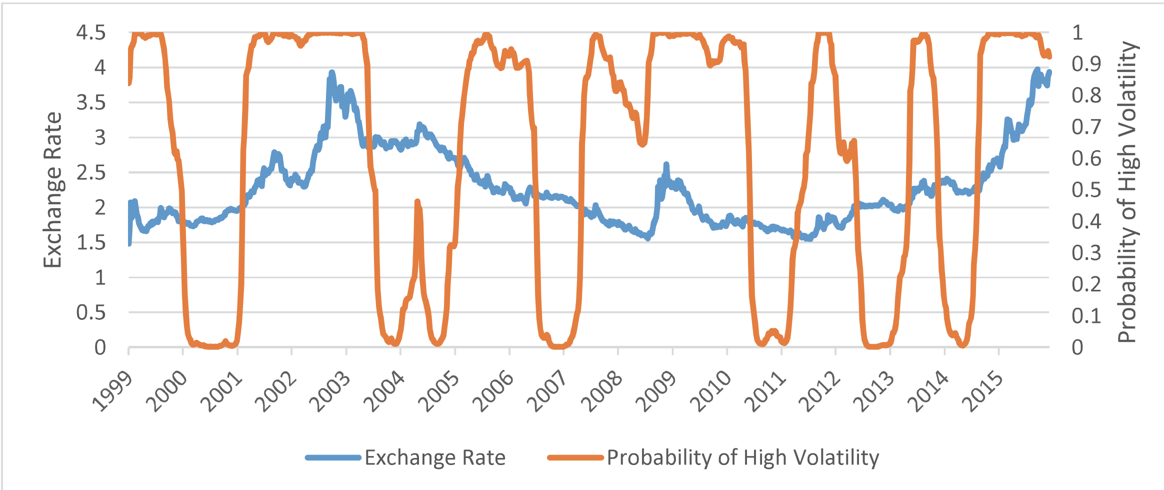
**STOCK MARKET AND PROBABILITY OF HIGH VOLATILITY FOR CHILE**



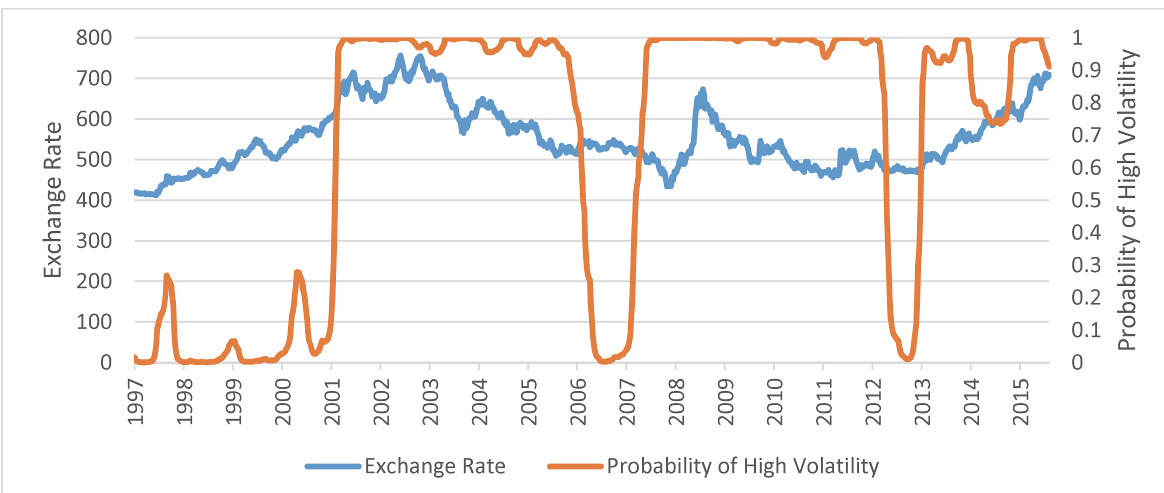

**STOCK MARKET AND PROBABILITY OF HIGH VOLATILITY FOR MEXICO**



**GRAPH 2 (CONTINUED)**
**EXCHANGE RATE AND PROBABILITY OF HIGH VOLATILITY FOR BRAZIL**




**EXCHANGE RATE AND PROBABILITY OF HIGH VOLATILITY FOR CHILE**



**EXCHANGE RATE AND PROBABILITY OF HIGH VOLATILITY FOR MEXICO**

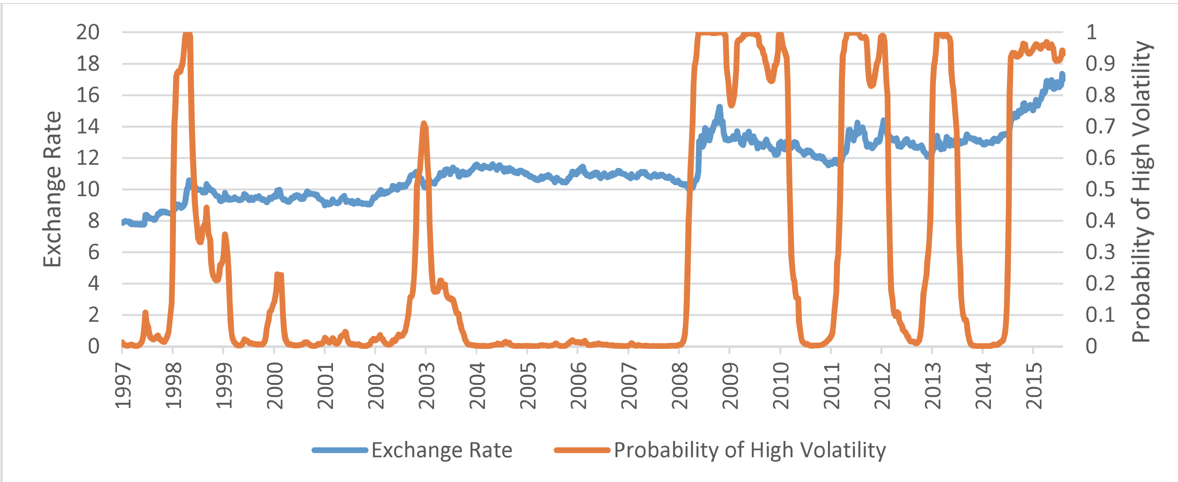



**TABLE 6A**
**STOCK MARKET HIGH VOLATILITY PERIODS**

| Country | Period | Duration (weeks) | Main event associated to period |
|---|---|---|---|
| Brazil | 7/11/97-1/09/98 | 27 | Asian financial crisis |
| | 7/10/98-3/19/99 | 35 | Russian and Brazilian financial crisis |
| | 3/31/00-5/12/00 | 6 | Dot-com bubble crisis |
| | 8/31/00-11/09/01 | 8 | Dot-com bubble crisis |
| | 6/14/02-10/25/02 | 20 | Argentinean economic crisis |
| | 09/26/08-02/06/09 | 20 | Global financial crisis |
| Chile | 12/12/97-3/17/00 | 110 | Asian, Russian and Brazilian financial crises and the dot-com bubble crises. |
| | 08/03/07-06/26/09 | 94 | Global financial crisis |
| | 02/25/11-12/09/11 | 41 | Domestic |
| Mexico | 5/16/97-10/05/01 | 216 | Asian, Russian and Brazilian financial crises and the dot-com bubble crises. |
| | 5/19/06-7/21/06 | 10 | Domestic |
| | 10/19/07-2/15/08 | 17 | Domestic |
| | 9/26/08-8/07/09 | 43 | Global financial crisis |

**Exchange Rates**

Tables 2B and 3B above show the estimates of the SWARCH model applied to the percentage changes of exchange rates. The estimate of the first-order autoregressive coefficient ($\Phi$) is positive and small in the cases of Chile and Brazil and negative and small in the case of Mexico. These results suggest that exchange rates are not easy to predict in these three countries. Note that a similar conclusion was obtained in the stock market returns case. The ARCH parameter ($\alpha_1$) is small in all three countries and in both the normal and the t distribution specification. The magnitude of this coefficient ranges from 0.00 in the case of Mexico to 0.23 in the case of Brazil regardless of the underlying probability distribution. These results indicate that volatility clustering is not as important as traditional ARCH models predict once regime switches are incorporated in the ARCH process. The estimates of the AR(1)-ARCH(1) model will be discussed later.

Note that the student's t distribution seems to provide a better fit (likelihood value is higher) to the data than the normal distribution specification just as in the case of the stock market returns. For this reason, the analysis will now focus on the parameter estimates of the SWARCH model resulting from the student's t specification.

The parameter g(2) which scales the ARCH process in the high volatility regime ranges from 3.70 in the case of Brazil and Chile to 4.50 in the case of Mexico indicating that volatility differs substantially between the two regimes.

The estimates of the transition probabilities $P_{11}$ and $P_{22}$ are greater than 0.97 in the three economies suggesting that, with the exception of Mexico, states in exchange rates are even more persistent than the states in stock market returns. In other words, the high volatility regimes in exchange rates have a greater average duration than the corresponding regimes in stock returns.

Another important difference between the results of the stock market returns and the exchange rates has to do with the dominant state in the series. In the case of stock returns it was the low volatility state that dominated the sample period whereas in the case of exchange rates that only happened in the case of Mexico where, according to the estimates of the ergodic probabilities, the low volatility regime prevailed 70% of the time. In the cases of Brazil and Chile, the dominant state in the sample period was the high volatility state which was present 63 and 66% of the time respectively. In the case of Chile, the fluctuations of the price of copper during the sample period may be the driving factor of the exchange rate of the Chilean peso. Brazil's currency could also be affected by the volatility of commodity prices whereas Mexico's currency was not as volatile during the sample period since its exports allocation has experienced a structural change and the proportion of exports corresponding to manufactured products has increased relative to oil exports.

The average length of the high volatility periods is 34.48 weeks in the case of Mexico and is shorter in duration than the low volatility regime which on average lasts 40 weeks. Note that this duration is almost half the duration of the high volatility period in the case of stock markets whose length is 70 weeks. However, a word of caution is needed about the generalization of these results. For example, the Asian, Russian and Brazilian crises of the late nineties had a more persistent effect on the volatility of the stock market than on the exchange rates. By contrast, the global financial crisis and some financial events that followed it had a more persistent effect on the exchange rates rather than the stock market. In the following section, a formal assessment of the level of synchronicity between these two financial markets, the so-called degree of concordance, will be performed.

In the case of Chile, the high volatility regime had an average duration of 139 weeks, almost twice the length of the low volatility period which had an average duration of 70.92 weeks. As Table 6B illustrates, two periods of high volatility are clearly identified in the time series of the Chilean Peso. The first went from June 15, 2001 to June 26, 2006 and it basically shows a depreciation of the Chilean peso which coincided with an initial decline in the price of copper from 2001 to 2003 followed by a steep recovery in the commodity's price and an appreciation of the currency that reached a peak on May 2006. This period lasted 252 weeks. When the high volatility periods of the stock market are contrasted with those of the Chilean peso it is noted that during this period the stock market was basically calm. On the other hand, a period of high volatility of the Chilean stock market, namely the international financial crises of the late nineties, had virtually no effect on the Chilean currency. A second extended period of high volatility had a similar duration (250 weeks) and extended from July 2007 to August 2012 and coincided with an initial depreciation of the currency followed by a strong appreciation that again was a consequence of a drop in copper prices during the global financial crises and the subsequent recovery after the crisis. The European sovereign debt crisis also had an effect on the volatility of the Chilean peso during this period. Note that the stock market experienced a period of high volatility during the global financial crisis that was substantially shorter as was shown on Table 6A.

**TABLE 6B**
**EXCHANGE RATES HIGH VOLATILITY PERIODS**

| **Country** | **Period** | **Duration (weeks)** | **Main event associated to period** |
|---|---|---|---|
| Brazil | 1/15/99-12/17/99 | 47 | Brazilian financial crisis |
| | 2/23/01-7/25/03 | 118 | Dot-com bubble and Argentinean economic crisis |
| | 2/04/05-7/07/06 | 70 | Domestic |
| | 5/18/07-6/11/10 | 149 | Global financial crisis and European sovereign debt crisis |
| | 06/10/11-06/11/12 | 49 | European sovereign debt crisis |
| | 05/24/13-11/29/13 | 27 | End of Fed's QE program |
| | 08/29/14-12/18/15 | 65 | Expectation of Fed's interest rate increase |
| Chile | 06/15/01-06/26/06 | 252 | Domestic |
| | 07/27/07-08/24/12 | 250 | Global financial crisis and European sovereign debt crisis |
| | 05/10/13-12/18/15 | 128 | End of Fed's QE program and expectation of Fed's interest rate increase |
| Mexico | 5/22/98-10/16/98 | 22 | Russian financial crisis |
| | 3/14/03-6/06/03 | 11 | Domestic |
| | 8/08/08-7/23/10 | 97 | Global financial crisis and European sovereign debt crisis |
| | 7/29/11-7/06/12 | 48 | European sovereign debt crisis |
| | 5/10/13-11/15/13 | 28 | End of Fed's QE program |

In the case of the Brazilian exchange rate, the sample period being used in the estimation of the SWARCH model was from January 1999 to December 2015. The reason was that the floating exchange rate system was adopted in 1999 and keeping the prior period in the sample would artificially create a completely different regime. Seven periods of high volatility where identified in the sample period, with an average duration of 58 weeks. The longest period of high volatility extended for 149 weeks from May 2007 to June 2010 and included the global financial crisis and the beginning of the European sovereign debt crisis. Note that the volatility in the exchange rates has been substantially larger than the volatility in the stock market. In particular, during the global financial crisis the stock market was in the high volatility state for only 20 weeks. This particular outcome was similar to the Chilean case which suggests that in addition to the global financial crisis other factors such as the volatility in the prices of commodities were the main sources of the high volatility in exchange rates during this period. The second longest period of high volatility in exchange rates occurred from February 2001 to July 2003 and was in sync with the dot-com bubble and Argentinean economic crisis which also had an effect on the stock market returns.

An extended period of high volatility was present on the final years of the sample period and affected the currencies of the three countries. As Table 6B illustrates, exchange rates have been subject to high volatility starting on May 2013 as a consequence of the Fed's announcement of the end of the quantitative easing (QE) program. Also the expectation that the Fed will increase interest rates has caused excessive volatility and a depreciation of the currencies of the studied countries and other emerging economies. It is worth noting that the stock market has been in the low volatility state during this period as opposed to other high volatility periods where both markets have experienced heightened volatility.

## CONCORDANCE

A test for the degree of synchronicity between the regimes of two time series was developed by Harding and Pagan (2006) and will be applied to the time series of the stock market returns and exchange rates of Brazil, Chile and Mexico. The concordance test is based on the percentage of time that two series were in the same regime. Specifically, the degree of concordance between two series r and x is determined by the following formula:

$$C_{x,r} = T^{-1}\sum_t^T [S_{x,t}S_{r,t} + (1 - S_{x,t})(1 - S_{r,t})] \qquad (7)$$

where $t$ denotes the period and T is the total number of periods. $S_{x,t}$ is the regime of the exchange rate time series at time $t$ and $S_{r,t}$ is the regime of the stock market return time series at time $t$. Table 7 exhibits the degree of concordance between the series for the three countries.

In the case of Mexico, the regimes of the stock market returns and exchange rates were in sync 55% of the time, which is the highest degree of concordance for the three countries. However, the correlation coefficient of the states was negative and small (-0.08) but statistically significant at the 5% level as shown on Table 7. This suggests that the main reason the states were in sync for most of the time in the case of Mexico was the high degree of persistence of the states rather than its correlation, which actually negatively affects the degree of concordance. For example, if the correlation coefficient was equal to zero the degree of concordance would be 59% instead. This degree of concordance is the expected value denoted as $\hat{E}(C_{x,r})$ and reflects the coefficient that is expected under the null hypothesis that the regimes of exchange rates and stock market returns are uncorrelated which need not be zero if one regime is more persistent than the other. Harding and Pagan prove that the degree of concordance $C_{x,r}$ can be rewritten as $C_{x,r} = 1 + 2\hat{\rho}_S(\hat{\mu}_{S_x}(1-\hat{\mu}_{S_x}))^{\frac{1}{2}}(\hat{\mu}_{S_i}(1-\hat{\mu}_{S_i}))^{\frac{1}{2}} + 2\hat{\mu}_{S_x}\hat{\mu}_{S_y} - \hat{\mu}_{S_x} - \hat{\mu}_{S_i}$, where $\rho_S$ is the correlation coefficient between $S_{x,t}$ and $S_{i,t}$, $\hat{\mu}_{S_x} = \hat{E}(S_{x,t})$ and $\hat{\mu}_{S_i} = \hat{E}(S_{i,t})$, then under the null of no correlation ($\rho_S$=0) the expected value of $C_{x,i}$, $\hat{E}(C_{x,i}) = 1 + 2\hat{\mu}_{S_x}\hat{\mu}_{S_i} - \hat{\mu}_{S_x} - \hat{\mu}_{S_i}$. To test whether the correlation coefficient between the series is significant the following hypotheses were tested:

$H_0$: $\rho_S=0$ vs. $H_a$: $\rho_S\neq0$ (8)

where $\rho_S$ is the correlation coefficient between $S_{x,t}$ and $S_{r,t}$. $\rho_S$ can be found from the regression:

$$\hat{\sigma}_{S_r}^{-1}S_{r,t} = a_1 + \rho_S\hat{\sigma}_{S_r}^{-1}S_{x,t} + u_t \quad (9)$$

The estimated value of $\rho_S$ is shown on Table 7 along with the p-value of the t test associated to its significance test. As mentioned before, in the case of Mexico, the correlation coefficient was found to be statistically significant but it is not of great importance. In the case of Chile, the degree of concordance is low, only 34% of the time the states of stock returns and exchange rates are in sync and this coefficient is greatly affected by the correlation between the states which as shown on Table 7 is negative and statistically significant. Only Brazil showed a positive and significant correlation between the states as shown on Table 7 but the degree of concordance between the states is low and equal to 43%. In summary, only in the case of Brazil, there is a positive and significant correlation between the regimes but the degree of concordance is low due to the fact that the high volatility states of the exchange rates are substantially longer than the corresponding states of the stock returns. In the cases of Chile and Mexico, there is evidence of a negative correlation between the regimes. In the case of Mexico, the concordance is high due to the persistence of the states whereas in the case of Chile, the concordance is low due to the fact that as in the case of Brazil exchange rates exhibit longer states than stock returns.

**TABLE 7**
**CONCORDANCE AND CORRELATION RESULTS**

| Country | Concordance | Null Expected Concordance | Correlation | Standard t-statistic | p-value |
|---|---|---|---|---|---|
| Brazil | 0.4294 | 0.3873 | 0.1638 | 4.7759 | 0.0000 |
| Chile | 0.3402 | 0.4129 | -0.1771 | -5.4429 | 0.0000 |
| Mexico | 0.5539 | 0.5852 | -0.0756 | -2.3070 | 0.0213 |

## CONCLUSION

A switching ARCH (SWARCH) model was used to model the stock market returns and exchange rate time series for three Latin American economies: Brazil, Chile and Mexico. The existence of two states in the volatility of the markets was significant in all cases. Also, the SWARCH model provided a better fit than the traditional ARCH model without switches and the persistence of the ARCH effects was substantially reduced once changes in regime were included. The SWARCH model allowed an objective identification and dating of the high volatility periods.

On average, the high volatility periods of the exchange rates are longer in duration than the corresponding periods of the stock returns. However, when specific episodes are compared, it was found that the length of the high volatility periods was greater for stock returns during the financial crises that occurred at the end of the nineties and the dot-com bubble, whereas during the global financial crisis exchange rates experienced a longer period of high volatility. A few high volatility periods in exchange rates were not present in all three countries at the same time and seem to be linked to the volatility of the prices of commodities, for example the Chilean peso fluctuated in the period 2001 to 2006 from a depreciation to an appreciation as a consequence of a decline in the prices of copper followed by an steep increase that reached a peak in May 2006. Finally, the end of the Fed's QE program and the expectation of higher interest rates caused a high volatility period in the exchange rates of the three countries. This period has extended from May 2013 until 2015 with short interruptions in the cases of Brazil and Mexico. The stock market was calm during this period.

In order to determine the level of synchronicity between the high volatility regimes of stock returns and exchange rates, the degree of concordance was calculated and a hypothesis test about the significance of the correlation was conducted. It was found that the concordance was low in the three countries and only Brazil reported a positive significant correlation between the states.

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