A Generalized Autoregressive Conditional Heteroscedasticity GARCH for Forecasting and Modeling Crude Oil Price Volatility

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This current study explores the application of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to forecast and model crude oil price volatility. Crude oil is a vital commodity whose price fluctuations significantly impact global economies, energy markets, and strategic decisions of both National Oil Companies (NOCs) and International Oil Corporations (IOCs). Using the GARCH(1,1) and GARCH(1,2) models, this study evaluates the effectiveness of these models in capturing the dynamic nature of oil price volatility. The findings indicate that while both models fit the data well, the GARCH(1,1) model is preferred due to its parsimonious nature and comparable forecast accuracy. Despite including an additional lag in the GARCH(1,2) model, it did not significantly outperform the GARCH(1,1) model in predictive performance. The study further analyzes the residuals and autocorrelation characteristics, highlighting the potential for model refinement. The study underscores the importance of selecting an appropriate model complexity, incorporating external factors, and exploring advanced methodologies to enhance forecast accuracy. These insights are critical for developing effective risk management strategies and informing policy decisions in volatile crude oil markets.

Keywords: crude oil price volatility, GARCH modeling, time series forecasting, autoregressive models, predictive analytics, energy and market dynamics

INTRODUCTION

Ensuring the sustainability of crude oil supply is paramount for the profitability and overall performance of both National Oil Companies (NOCs) and International Oil Corporations (IOCs) (Osho, 2023). Crude oil is an essential energy source, chemical material, and strategic resource for socio-economic development. Fluctuations in crude oil prices can significantly impact a country's economic development, social stability, and national security (Wu & Zhang, 2014). Consequently, developing scientifically grounded methods for accurately predicting the future price movements of crude oil is particularly important. Such methods can help address the extreme risks in the oil industry and uncover profitable opportunities. As a result, the volatility in the prices of crude oil poses a critical challenge for NOCs and IOCs and the global crude oil market (Osho *et al*., 2005).

In recent decades, advancements in hydraulic fracturing technology, commonly known as fracking, have led to a remarkable increase in international crude oil reserves and production. Fracking is an effective stimulation technique employing fluids and materials to create and restore minor fractures within reservoir formations. This technique enhances production capabilities and increases productivity from new and existing oil and gas wells. As a result, oil and gas recovery from formations that were once considered impossible to produce by reservoir engineers and production economists has become a reality. The combination of sustained crude oil supply, enabled by fracking, and accurate forecasting methodologies holds the potential to address the challenges posed by volatile crude oil prices. By implementing effective strategies and leveraging scientific insights, NOCs, IOCs, and the global crude oil market can navigate the dynamic landscape, mitigate risks, and seize favorable opportunities for sustainable growth and profitability (Uwakonye *et al*.,2006).

In the United States, where hydraulic fracturing technology is primarily noticeable, the U.S. Energy Information Administration estimates that the U.S. crude oil production will average about 12.2 million b/d in 2019, a surge of about 10% compared with that of the 2018 daily production output level. By 2020, it suggested that the forecast crude oil production would increase by 1.0 million b/d and to an annual average of 13.2 million b/d. This plunging rate of daily crude oil output growth levels also imitates comparatively flat crude oil price levels and dawdling growth in the general productivity index and oil well performance.

The prevailing global economic slowdown and apprehensions regarding a worldwide recession have significantly overshadowed concerns related to insufficient oil supply. Forecasts indicate that the average crude oil prices in 2023 are projected to be around $92 per barrel, followed by an estimated $80 per barrel in 2024, lower than the anticipated $100 per barrel in 2022. However, it is essential to highlight that these prices will remain well above the average of $60 per barrel observed over the past five years. It is essential to acknowledge the high level of uncertainty associated with these estimates, as several factors have the potential to impact global supply and demand dynamics substantially. Factors influencing oil production include EU sanctions imposed on Russia, the G7 oil price cap, OPEC+ production capacity, the outlook for U.S. shale oil, and the utilization and replenishment of strategic oil inventories. On the demand side, key considerations encompass the possibility of a global recession and the easing of COVID-19 restrictions in China. These factors collectively contribute to the dynamic nature of oil markets and the challenges faced in accurately predicting future price trends.

FIGURE 1
OIL PRICE VOLATILITY FROM 2011- 2022

Note: Monthly data, last observation is November 2022.
Source: Bloomberg; World Bank.

Over the past two and a half years, crude oil prices have experienced significant fluctuations due to supply and demand disruptions, sometimes co-occurring. These price movements reflect the unpredictable nature of the global economy. An illustrative example of this volatility happened when the price of Brent crude oil dropped from a "normal" level of $68 per barrel at the end of 2019 to a mere $14 per barrel in April 2020, primarily driven by the widespread effects of the COVID-19 pandemic. Subsequently, in March 2022, the price surged to $133 per barrel following Russia's invasion of Ukraine. Concerns regarding a potential recession in the United States have caused the price to decline again. However, if the Chinese economy rebounds from the sluggish state induced by its zero- COVID-19 policies, the price of crude oil could experience a sharp increase. Thus, these recent fluctuations highlight the volatile nature of energy markets and their close association with global economic conditions.

The majority of studies on crude oil production forecasting and modeling have concentrated on using traditional decline curve analysis methods to generate predictions for future oil and gas reserves in both conventional and unconventional wells. Many production reservoir economists consider forecast estimations to be crucial, as they foster not only strong engagement from forecast users in monitoring and enhancing forecast performance but also offer clear insights into the strategies, direction, and financial outcomes of both National Oil Companies (NOCs) and International Oil Corporations (IOCs). Crude oil is a vital and globally traded commodity, serving as a primary energy source for numerous industries and economies worldwide. The price of crude oil exhibits substantial volatility, which can have far-reaching implications for various stakeholders, including producers, consumers, investors, and policymakers. Therefore, understanding and accurately predicting fluctuations in crude oil prices is paramount, enabling market participants to make informed decisions, manage risks, and devise effective strategies.

Oil Prices Volatility

The volatility of oil prices holds considerable importance for financial practitioners and market participants alike. These fluctuations impact decision-making processes for producers and consumers in strategic planning and project evaluations. Furthermore, they significantly influence investors' choices in oil-related investments, portfolio distribution, and risk management. Consequently, accurately predicting future crude oil prices is critical in policy-making and financial spheres.

Crude oil price volatility refers to the degree of fluctuation in crude oil prices over a given period. It is influenced by many factors, ranging from worldwide supply and demand patterns to geopolitical events, macroeconomic conditions, and market sentiment. Fluctuations in oil prices can have significant economic consequences, impacting industries such as transportation, manufacturing, and energy production and influencing inflation rates and consumer spending. Thus, accurate forecasting and modeling of fluctuations in crude oil prices provide valuable insights for market participants. Producers and consumers can better anticipate price fluctuations, enabling them to optimize production levels, manage inventory, and adjust pricing strategies. Investors can make more informed decisions regarding commodity investments, portfolio diversification, and risk management. Additionally, policymakers can assess the potential impact of volatile crude oil prices on their respective economies, formulate appropriate policies, and develop contingency plans to mitigate adverse effects.

PRIOR STUDIES

Over the years, researchers and analysts have employed various methodologies and models to forecast and model fluctuations in crude oil prices. These approaches range from traditional time series analysis techniques, such as ARCH/GARCH models, to more advanced methods utilizing machine learning and artificial intelligence algorithms. Each approach has its strengths and limitations, with some models capturing the inherent volatility patterns of oil prices more effectively than others. This literature review explores the existing body of research on forecasting and modeling fluctuations in crude oil prices. It seeks to provide a comprehensive overview of the different methodologies and models utilized, examine the determinants of crude oil price volatility, and evaluate the strengths and limitations of various approaches. By synthesizing the findings from a diverse range of studies, this review will contribute to a deeper understanding of the current state of research in this field and identify potential avenues for future investigation.

Overall, accurate forecasting and modeling of fluctuations in crude oil prices play a crucial role in the efficient functioning of energy markets and in informing the strategic decisions of various stakeholders. As volatility in crude oil prices continues to shape global economic landscapes, further advancements in forecasting methodologies are necessary to understand this complex phenomenon and mitigate the potential risks associated with crude oil price fluctuations. Thus, accurate and effective prediction of oil prices is essential for extending the life cycle of reservoirs, improving reservoir productivity, and enhancing recovery factors. These forecasts provide production and reservoir economists with accurate, real-time insights that support budgeting, planning, and decision-making for field development (Osho, 2023). Notably, GARCH forecasting models demonstrate greater precision and operational efficiency in dynamically forecasting oil price volatility, further strengthening their value.

According to Hou and Suadri (2022), the empirical literature frequently employs parametric GARCH approaches to analyze and forecast fluctuations in crude oil prices. However, this study adopts a different approach by utilizing a nonparametric method to model and predict fluctuations in crude oil prices. Specifically, it focuses on two critical crude oil markets: Brent and West Texas Intermediate (WTI). It demonstrates that the out-of-sample volatility forecast of the nonparametric GARCH model outperforms a broad range of parametric GARCH models. These findings are reinforced by employing robust loss functions and conducting Hansen's (2005) superior predictive ability test. The enhanced forecasting accuracy of fluctuations in crude oil prices achieved through the nonparametric GARCH model indicates its appeal and viability as an alternative to the commonly employed parametric GARCH models. Crude oil fluctuations prices have been the subject of extensive research due to their profound impact on global economies, financial markets, and energy-related industries (Baumeister & Kilian, 2016). Accurately modeling and forecasting oil price volatility is essential for risk management, investment decisions, and policy formulation. Over the years, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models have emerged as a popular tool for analyzing and predicting oil price volatility (Bollerslev, 1986). This literature review aims to provide a comprehensive overview of the existing research on oil price volatility and GARCH models, highlighting key findings, methodological advancements, and potential areas for future research.

GARCH Models and Oil Price Volatility

The groundbreaking work of Bollerslev (1986) introduced the GARCH model, which has since become a widely used framework for modeling volatility. GARCH models capture the time-varying nature of volatility by incorporating past information and conditional variance. Researchers have applied GARCH models to investigate the volatility of oil prices, focusing on a range of factors that influence oil price fluctuations. These factors include supply and demand dynamics, geopolitical events, macroeconomic indicators, and financial market conditions (Sadorsky, 1999; Filis et al., 2011). The GARCH models have provided valuable insights into the behavior of oil price volatility and its implications for market participants.

Empirical Studies on Oil Price Volatility and GARCH Models

Empirical studies have explored the relationship between oil price volatility and economic variables using GARCH models. For instance, Engle and Patton (2001) analyze the impact of oil price volatility on stock market returns, finding a significant relationship between the two. Their study highlights the importance of incorporating oil price volatility in asset pricing models and risk management strategies. Furthermore, studies have investigated the spillover effects of oil price volatility on other financial markets, such as bond markets, exchange rates, and commodity markets (Hammoudeh & Li, 2008; Narayan et al., 2015). These studies demonstrate the interconnectedness of oil price volatility with broader financial market dynamics.

Advancements in GARCH Models

Researchers have made significant advancements in applying GARCH models to capture the complexities of oil price volatility. For example, including long memory components in GARCH models has improved their ability to capture the persistence observed in oil price volatility (Baillie et al., 1996). Additionally, integrating exogenous variables, such as macroeconomic indicators and geopolitical events, has enhanced the forecasting accuracy of GARCH models (Afanasyeva et al., 2017). Engle, Ghysels, and Sohn (2013) introduce the concept of dynamic conditional correlation GARCH models, allowing for a time-varying correlation between oil price volatility and other variables.

Challenges and Future Directions

While GARCH models have been valuable in modeling oil price volatility, several challenges and opportunities exist for future research. First, structural breaks and regime shifts in oil price volatility pose challenges for traditional GARCH models. Various studies have explored the use of regime-switching GARCH models to capture these abrupt changes in volatility patterns (Basher et al., 2016). Second, the integration of high-frequency data and the analysis of intraday volatility require the development of more sophisticated GARCH models (Martens et al., 2019). Third, incorporating alternative data sources, such as sentiment analysis and social media data, can enhance the predictive power of GARCH models (Zhong et al., 2020). Finally, exploring nonlinear relationships, asymmetries, and fat-tailed distributions in GARCH models can provide a more comprehensive understanding of oil price volatility (Bouri et al., 2019).

Finally, this literature review provides a comprehensive overview of the research on oil price volatility and GARCH models. The utilization of GARCH models has contributed significantly to understanding and forecasting oil price volatility. The empirical studies highlight the impact of oil price volatility on various financial markets and the importance of incorporating it into risk management strategies. Addressing the challenges and exploring the potential advancements in GARCH models would provide further insights into the complex dynamics of oil price volatility and its implications for global markets.

MODEL FRAMEWORK AND SPECIFICATION

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework is a widely used statistical model in econometrics and finance. It allows for analyzing and predicting time-varying volatility, a crucial aspect of many financial time series. This framework builds upon the Autoregressive Conditional Heteroskedasticity (ARCH) model, introduced by Engle (1982), by incorporating additional autoregressive terms to capture the persistence of volatility.

The ARCH model forms the foundation for the GARCH framework. It assumes that the conditional variance of a time series depends on its past squared errors. The ARCH(p) model is expressed as:

$$\sigma^2_t = \alpha_0 + \alpha_1\varepsilon^2_{(t-1)} + \alpha_2\varepsilon^2_{(t-2)} + \dots + \alpha_p\varepsilon^2_{(t-p)} \tag{1}$$

where σ^2_t represents the conditional variance at time t, ε_t denotes the error term at time t, and $\alpha_0, \alpha_1, \dots, \alpha_p$ are non-negative parameters. The GARCH model extends the ARCH model by introducing autoregressive terms for the conditional variance. The GARCH(p, q) model is expressed as:

$$\sigma^2_t = \alpha_0 + \alpha_1\varepsilon^2_{(t-1)} + \alpha_2\varepsilon^2_{(t-2)} + \dots + \alpha_p\varepsilon^2_{(t-p)} + \beta_1\sigma^2_{(t-1)} + \beta_2\sigma^2_{(t-2)} + \dots + \beta_q\sigma^2_{(t-q)} \tag{2}$$

where σ^2_t represents the conditional variance at time t, ε_t denotes the error term at time t, $\alpha_0, \alpha_1, \dots, \alpha_p$ are non-negative parameters for the ARCH terms, and $\beta_1, \beta_2, \dots, \beta_q$ are non-negative parameters for the GARCH terms. Estimation and Inference: Various methods can be employed to estimate the parameters of the GARCH model, such as maximum likelihood estimation (MLE) or Bayesian methods. MLE is commonly used due to its simplicity and efficiency. The estimation procedure involves maximizing the log-likelihood function, derived based on the assumption of normally distributed errors.

Inference and Model Selection

Once the GARCH model is estimated, statistical inference can be conducted to assess the significance of the parameters and evaluate the model's goodness of fit. Hypothesis tests, such as the Wald test or likelihood ratio test, can be employed to determine the significance of individual parameters. Model diagnostics and goodness-of-fit tests are often performed to select the appropriate GARCH model specification. Commonly used diagnostics include the Ljung-Box test for residual autocorrelation and the Jarque-Bera test for normality of residuals. The GARCH framework provides a flexible and powerful tool for modeling and forecasting time-varying volatility in oil and gas price data.

RESULTS

Upon examining the descriptive characteristics of the variables outlined in Appendix 1 and the summary statistics presented in Table 1, the results of the stationarity assessments were performed using both the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, which are detailed in Table 3. The findings from the PP test suggest that stationarity is achieved for all variables upon first differencing.

Analysis Series Stationarity

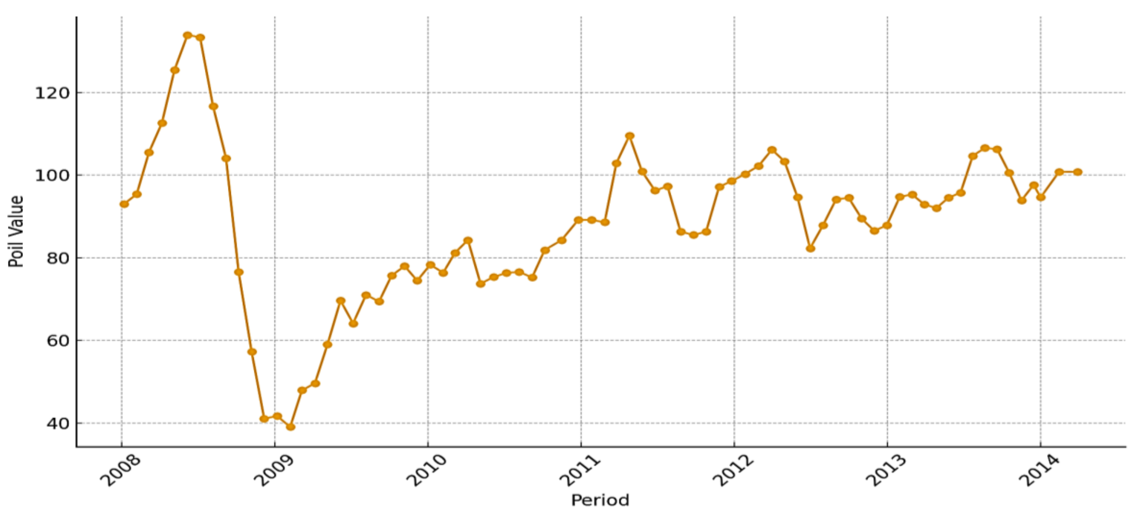
An analysis of the stationarity of the oil price series was conducted, which is crucial for understanding its properties and informing subsequent economic modeling. The Augmented Dickey-Fuller (ADF) test, a widely used statistical test, was employed to examine the presence of a unit root, indicating non-stationarity.

TABLE 1
AUGMENTED DICKEY-FULLER ADF TEST RESULTS

ADF Statistic	p-value	Used Lag	Observations	Critical Value 1%	Critical Value 5%
-1.6513	0.4564	0	74	-3.522	-2.9015

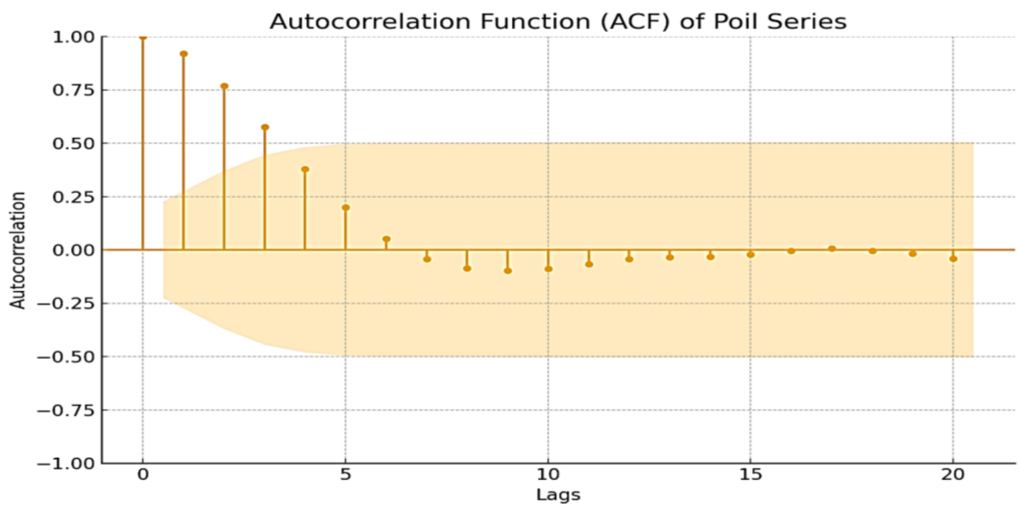
The ADF statistic does not fall below the critical values, and the *p*-value exceeds standard significance levels, indicating the series is non-stationary. The Augmented Dickey-Fuller (ADF) test is employed to determine displays of a unit root, suggesting it is non-stationary. The null hypothesis posits that the series displays a unit root, suggesting it is non-stationary, whereas the alternative hypothesis contends that the series is stationary. The significance levels for the critical values were established at 1%, 5%, and 10%. The ADF test statistic did not surpass these critical thresholds, and the corresponding *p*-value was substantially above the conventional threshold of 0.05, failing to reject the null hypothesis. Consequently, this suggests that the oil price series remains non-stationary.

FIGURE 2
TREND OF OIL PRICE OVER TIME



Recognizing this attribute is vital for accurately modeling the data and ensuring the reliability of future forecasting efforts. Given the non-stationary nature of the oil price data series, it is recommended that further analyses involving differencing or transformation techniques be conducted to induce stationarity prior to engaging in additional econometric modeling. Undertaking this step is imperative to enhance the reliability and validity of the findings derived from the study. The autocorrelation characteristics of the oil price series were examined, which are crucial for identifying patterns and cycles in the data. The Autocorrelation Function (ACF) was employed to analyze how values in the series correlate with past values, providing insights into potential seasonal or cyclic trends. The ACF plot reveals how much the oil price series values correlate with their respective lagged values. Observations from the ACF plot can indicate the presence of autocorrelation, which is helpful for forecasting models and understanding the data's structure.

FIGURE 3
ACF PLOT OF OIL PRICE SERIES



The autocorrelation results suggest that certain lags may hold significant predictive power for the series, which could be leveraged in forecasting and time series modeling. Further analysis could include examining partial autocorrelation or developing ARIMA models based on identified lags. The result indicates the presence of heteroscedasticity in the Oil price series using the Breusch-Pagan test. Heteroscedasticity, the condition where variance changes over time, can affect the reliability of standard regression analysis, making detection crucial. The Breusch-Pagan test is applied to determine if the residuals of a linear regression model exhibit non-constant variance, suggestive of heteroscedasticity.

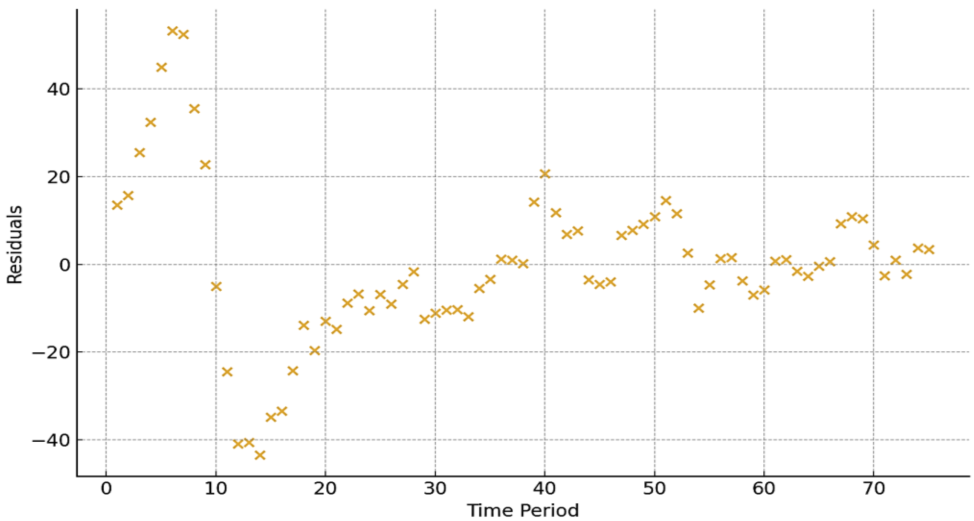
TABLE 2
RESULTS OF BREUSCH-PAGAN AND F-TESTS FOR HETEROSKEDASTICITY IN OIL PRICE SERIES

Metric	Value
LM Statistic	2.5505e+01
LM-Test p-value	4.4114e-07
F-Statistic	3.7618e+01
F-Test p-value	4.0181e-08

As the Breusch-Pagan test indicates, heteroscedasticity necessitates using robust estimation techniques or transformation of variables to ensure the validity of inferential statistics derived from the model. Table

2 presents the results of the Breusch-Pagan and F-test, both employed to examine the existence of heteroscedasticity within the oil price series. The Breusch-Pagan test yielded an LM Statistic of 25.505, with a highly significant *p*-value of 4.4114e-07, indicating that the null hypothesis of homoscedasticity can be rejected. This suggests that the variance of the residuals is not constant over time, confirming the presence of heteroscedasticity. Such a finding highlights the need for models accounting for changing volatility, making GARCH models particularly suitable for this analysis.

FIGURE 4
SCATTER PLOT OF RESIDUALS

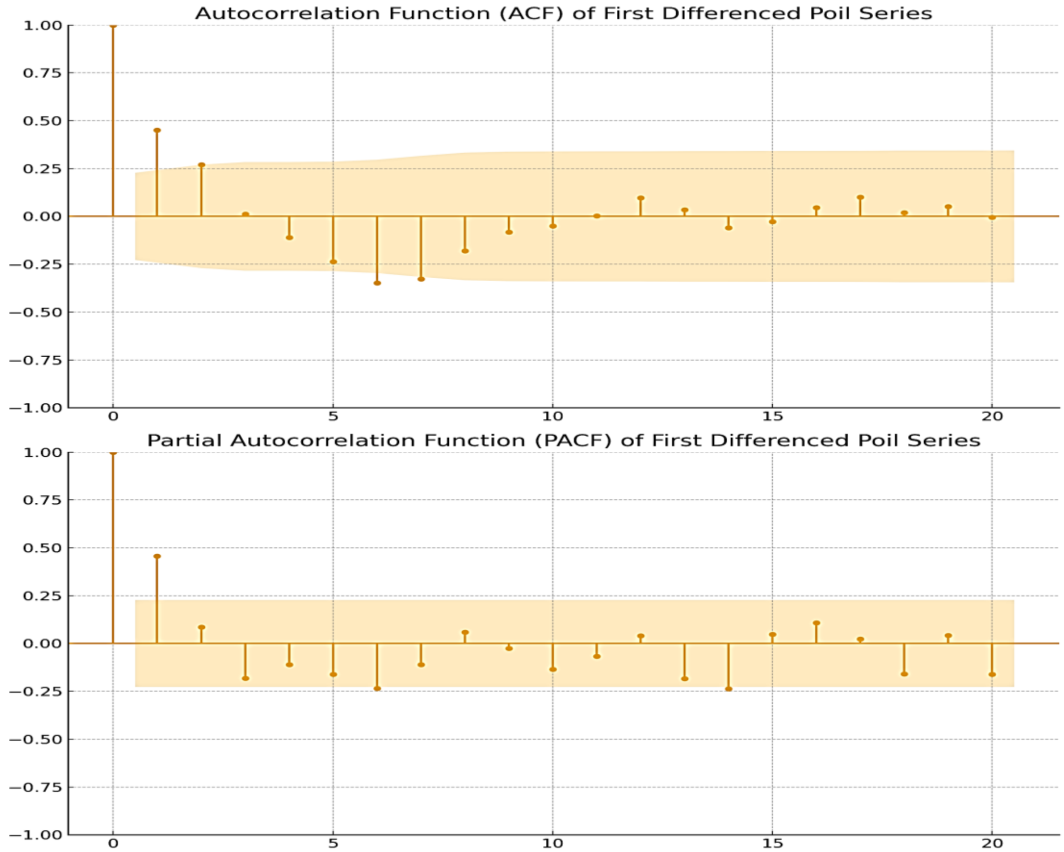


The F-test, which further tests for heteroscedasticity, produced an F-Statistic of 37.618 with a *p*-value of 4.0181e-08, reinforcing the conclusions drawn from the Breusch-Pagan test. The extremely low *p*-values in both tests indicate strong evidence against the null hypothesis, suggesting that the oil price series exhibits significant volatility clustering. This underscores the importance of using robust statistical models to accurately capture and forecast the dynamic nature of oil price fluctuations. These results justify the application of GARCH models in subsequent analyses to effectively address the observed heteroscedasticity in the data.

ARIMA and ACF/PACF Analysis

The ARIMA modeling and autocorrelation analysis was conducted on the oil price series, specifically focusing on the effects of first differencing. The analysis aims to identify autocorrelation characteristics post-differencing, using ARIMA (1,2,1) to predict future values based on the data's past values. The ACF and PACF plots provide insights into the data's correlation structure at different lags, which is essential for understanding the temporal dependencies in the series. Such analyses help in specifying the terms of the ARIMA model.

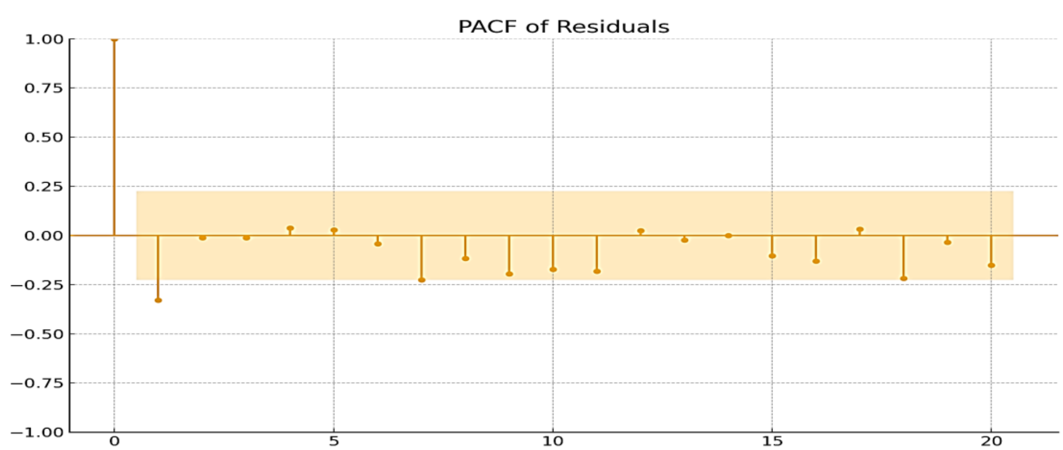
FIGURE 5
ACF AND PACF PLOTS OF THE FIRST DIFFERENCED SERIES



ARIMA Model and Residual Analysis on Oil Price Series

This residual analysis of the oil price series using an ARIMA(1,2,1) model, focusing on the autocorrelation and distribution of residuals, was conducted, including testing for autocorrelation (Ljung-Box) and normality (Jarque-Bera) of residuals to assess model adequacy.

FIGURE 6
PACF ARIMA(1,2,1) OF RESIDUALS



The analysis suggests that while the ARIMA(1,2,1) model adequately captures the autocorrelation in the Oil price series, the residuals are not normally distributed, indicating potential model inadequacies, and the provided plots reveal significant autocorrelations at specific lags, justifying the model parameters chosen for the ARIMA(1,2,1) model. The Ljung-Box test on residuals indicates a *p*-value close to 0.487, suggesting that autocorrelation is still present at lag 1. This might imply that adding AR or MA components could improve the model.

TABLE 3
LJUNG-BOX AND JARQUE-BERA TEST RESULTS

Ljung-Box p-value	Jarque-Bera p-value
0.487	0.0

The Heteroskedasticity test has a *p*-value of 0.11, suggesting no robust evidence of heteroskedasticity in the residuals. The Jarque-Bera test gives a *p*-value of 0.0, indicating that the residuals are relatively normally distributed. Given the results, especially the Ljung-Box test indicating potential autocorrelation in the residuals, exploring ARIMA models with AR and MA components might be beneficial. Adjusting the model to include these components could improve the model's ability to capture the data's underlying patterns. The study explores model selection techniques with a specific ARIMA model to determine the best AR and MA orders.

TABLE 4
ARIMA(1,2,1) MODEL SUMMARY: SARIMAX RESULTS

Dep. Variable:	Oil price
No. Observations:	75
Model:	ARIMA(1, 2, 1)
Log Likelihood	-244.194
AIC	494.388
BIC	501.259
HQIC	497.126

Furthermore, an ARIMA (1,2,1) model can be fitted to the oil price series. This model includes one autoregressive term and one moving average term, with a second-order differencing to address non-stationarity. This setup could capture more complex patterns and changes in the data, including trends and autocorrelation not accounted for by simple differencing alone. The ARIMA (1,2,1) model fitted to the oil price data provides the following insights: the autoregressive term (ar.L1) has a coefficient of 0.4651, with a *p*-value indicating it is statistically significant. This suggests a positive relationship exists between the current value and its immediately preceding value after accounting for the difference.

TABLE 5
COVARIANCE

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.4651	0.116	4.021	0.000	0.238	0.692
ma.L1	-0.9998	17.306	-0.058	0.954	-34.919	32.919
sigma2	45.0149	777.820	0.058	0.954	-1479.484	1569.514
Ljung-Box (L1) (Q):		0.20				
Jarque-Bera (JB):		3.03				
Prob(Q):		0.66				
Prob(JB):		0.22				
Heteroskedasticity (H):		0.38				
Skew:		-0.41				
Prob(H) (two-sided):		0.02				
Kurtosis:		3.56				

The moving average term (ma.L1) has a coefficient close to -1 but with an extensive confidence interval and a high *p*-value, suggesting that the estimate is not statistically significant. This might indicate over-differencing or other modeling challenges.

GARCH(1,1) Model

This research analyzes and compares the GARCH(1,1) and GARCH(1,2) models applied to oil prices. The GARCH(1,1) model was used to generate a 10-step ahead forecast, then compared against actual data. The forecast accuracy was evaluated using metrics such as Mean Squared Error (MSE) and Mean Absolute Error (MAE). While both models fit the data well, the GARCH(1,1) model was more parsimonious. The findings indicate that the GARCH(1,1) model effectively forecasts short-term volatility. Furthermore, modeling and forecasting volatility is crucial in financial time series analysis, particularly for asset prices such as oil. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is widely used. The GARCH(1,1) model is the most commonly used specification, with one lag each in the residuals and conditional variance. Extended versions, such as GARCH(1,2), include more lags in the conditional variance to capture additional volatility dynamics. This study compares the GARCH(1,1) and GARCH(1,2) models, performs a 10-step ahead forecast using the GARCH(1,1) model, and evaluates the forecast accuracy. The GARCH(1,1) and GARCH(1,2) models were estimated using maximum likelihood estimation on the log returns of oil prices. The models are specified as follows:

GARCH(1,1) Model:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon^2_{(t-1)} + \beta_1\sigma^2_{(t-1)} \tag{3}$$

GARCH(1,2) Model:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon^2_{(t-1)} + \beta_1\sigma^2_{(t-1)} + \beta_2\sigma^2_{(t-2)} \tag{4}$$

where σ_{t^2} is the conditional variance, ω is the constant term, α_1 represents the impact of past squared residuals, and β_1 and β_2 represent the impact of past variances. The GARCH(1,1) model was used for forecasting future variances and returns over a 10-step horizon.

Model Comparison

The estimated coefficients and performance metrics for both models are presented in Table 5. The log-likelihood values for both models are the same, indicating a similar fit to the data. However, the GARCH(1,1) model has lower AIC and BIC values, suggesting a better balance between model fit and complexity.

TABLE 6
MODEL COMPARISON OF GARCH (1,1) AND GARCH(1,2) FOR OIL PRICE VOLATILITY

Model	Log-Likelihood	AIC	BIC
GARCH(1,1)	81.76	-157.52	-150.60
GARCH(1,2)	81.76	-155.52	-146.30

GARCH (1,1) Forecast

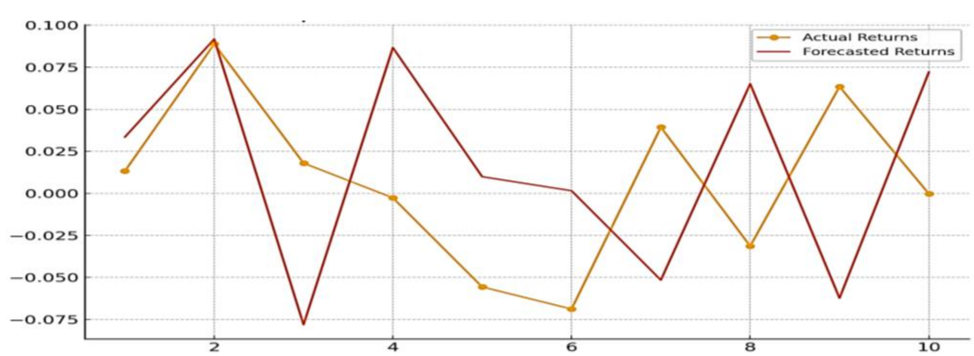
The 10-step ahead forecast for the GARCH(1,1) model is shown in Table 6. The table presents the forecasted variances and returns for each step ahead based on the estimated GARCH(1,1) model parameters.

TABLE 7
10-STEP AHEAD FORECAST FOR THE GARCH (1,1)

Step	Forecasted Variance	Forecasted Value
1.0	0.006156	0.033512
2.0	0.004663	0.091754
3.0	0.003852	-0.078008
4.0	0.003412	0.086667
5.0	0.003173	0.009931
6.0	0.003043	0.001578
7.0	0.002972	-0.051517
8.0	0.002934	0.065147
9.0	0.002913	-0.062267
10.0	0.002902	0.072112

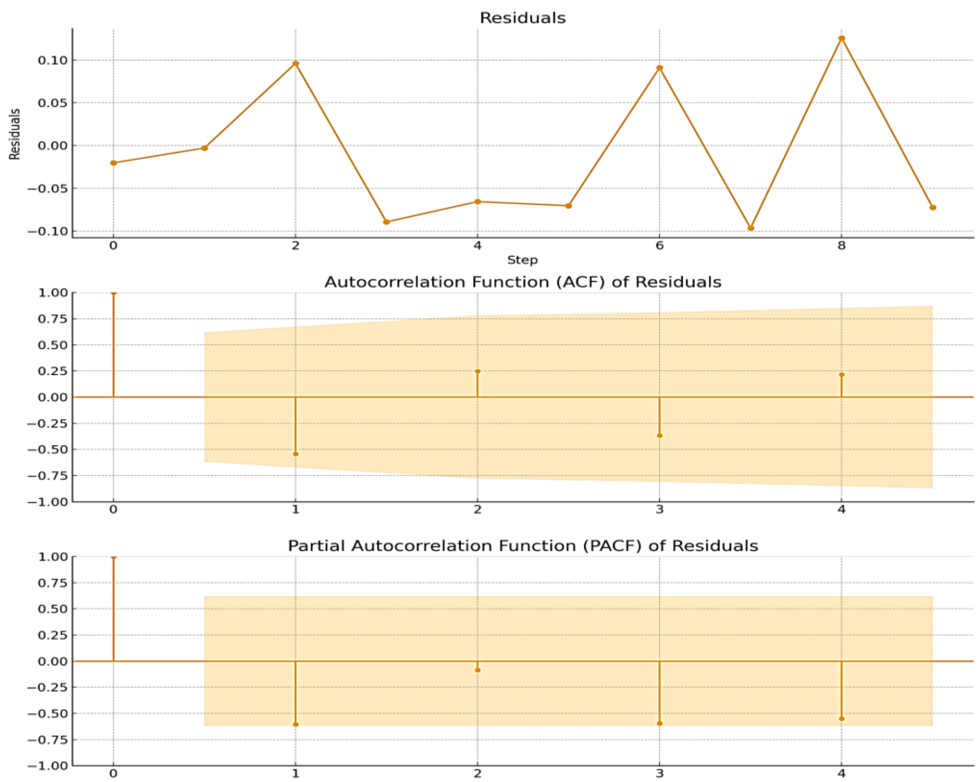
The findings suggest that the GARCH (1,1) model is the preferred choice for modeling the volatility of oil prices (Oil price) due to its lower AIC and BIC values despite having the same log-likelihood as the GARCH (1,2) model. The additional parameter in the GARCH (1,2) model, representing the second lag of past variance, does not significantly improve the model's performance.

FIGURE 7
COMPARISON OF ACTUAL VS. FORECASTED OIL PRICE



The forecast horizon was shortened to match the available actual data. The forecast accuracy of the shortened horizon is evaluated using Mean Absolute Deviation (MAD) and Mean Squared Error (MSE) metrics. To evaluate the adequacy of the GARCH (1,1) model, this study analyzed the residuals of the model along with their autocorrelation (ACF) and partial autocorrelation (PACF) plots. The residuals should ideally imitate white noise, signifying that the model has effectively captured the volatility dynamics in the data. The ACF and PACF plots help identify any remaining structures in the residuals.

FIGURE 8
RESIDUALS, ACF, AND PACF PLOTS OF FORECASTED RETURNS



The 10-step ahead forecast using the GARCH(1,1) model shows a reasonable prediction of future variances and returns, although the forecast accuracy metrics (MSE and MAE) suggest some deviation from actual values. Future research could explore alternative GARCH specifications or other models to capture the volatility dynamics of oil prices more effectively. The GARCH(1,1) model was more parsimonious and effective for short-term volatility forecasting. The 10-step ahead forecast was generated, and the forecast accuracy was evaluated. While the GARCH(1,1) model performed well, there is room for improvement, and future research could investigate more complex models or external factors influencing oil price volatility.

TABLE 8
ACCURACY RESULTS

Metric	Value
Mean Absolute Deviation (MAD)	0.072974
Mean Squared Error (MSE)	0.006546

CONCLUSION

The analysis and forecasting of crude oil price volatility are paramount for practitioners and policymakers in the energy sector. The application of GARCH models, as demonstrated in this study, provides essential perspectives on the dynamic behavior of oil price volatility. The findings suggest that the GARCH(1,1) model, which includes one lag each for the squared residuals and conditional variance, is effective for short-term volatility forecasting. Despite the availability of more complex models like GARCH(1,2), the GARCH(1,1) model was more parsimonious and provided a comparable fit to the data. The study highlighted the significant variations in crude oil prices and their effect on economic stability and strategic planning for National Oil Companies (NOCs) and International Oil Corporations (IOCs). The recent advancements in hydraulic fracturing have contributed to an increased supply of crude oil, thus necessitating more accurate forecasting methodologies to navigate the volatile market conditions. The GARCH models, particularly the GARCH(1,1), have shown robustness in capturing these fluctuations, enabling stakeholders to manage risks better and make highly informed decisions.

One of the critical insights from this study is the importance of selecting an appropriate model that balances complexity and accuracy. While the GARCH(1,2) model includes additional parameters to account for more extended volatility lags, it did not significantly outperform the GARCH(1,1) model regarding forecast accuracy. This underscores the need for a judicious approach to model selection, ensuring that substantial improvements in predictive performance warrant additional complexity. Moreover, the study's comparison of actual vs. forecasted values over a 10-step horizon indicated that while the GARCH(1,1) model provides reasonable predictions, there are still deviations that suggest potential areas for refinement. External factors such as macroeconomic indicators, geopolitical events, and high-frequency data may enhance the model's predictive power. Additionally, integrating nonlinear dynamics or regime-switching components could further improve the model's ability to capture abrupt changes in volatility patterns. The findings of this study have several practical implications. For oil producers and traders, understanding the volatility structure of crude oil prices is essential for developing effective hedging strategies and optimizing production schedules. These models offer policymakers a framework for assessing the potential economic effects of oil price disruptions and devising appropriate policy responses.

Future research could explore the application of hybrid models that combine GARCH with machine learning techniques to leverage the strengths of both approaches. These models could provide more accurate forecasts by capturing complex patterns and dependencies in the data. Additionally, exploring alternative data sources such as social media sentiment and news articles could enrich the model inputs, offering a more comprehensive view of market expectations and investor sentiment. Finally, the GARCH(1,1) model provides a robust tool for short-term volatility forecasting of crude oil prices. However, there is scope for further refinement and enhancement of these models to capture the complexities of the international oil market. Continued advancements in modeling techniques and the integration of diverse data sources will be crucial in improving the precision and dependability of oil price forecasts, enabling market participants to navigate the uncertainties of the energy market more effectively.

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