Using Data Envelopment Analysis to Analyze Academic Programs in a Business College

W. Brian Lambert
Metropolitan State University of Denver

Ann Murphy
Metropolitan State University of Denver

Abel A. Moreno
Metropolitan State University of Denver

Business colleges must efficiently manage academic programs which lead to degrees in various majors. In this paper we illustrate how data envelopment analysis (DEA) can be applied to analyze the relative efficiency of how different academic programs transform students into graduates. Assuming a set of common inputs and outputs, we use this optimization technique to compare the programs, and identify those which assume a maximum level of relative efficiency. Our DEA analysis also enables the analyses of why some programs are relatively inefficient and provide specific, quantitative guidance on how to make them more efficient.

Keywords: data envelopment analysis

BACKGROUND

In the business world there exists a demand for college graduates who have demonstrated knowledge and skills. Students historically respond to this demand by pursuing college degrees which expand their access to high-paying professional jobs. However, in a climate where this value of a college degree is increasingly questioned, university enrollments have declined since their peak in 2010. Reductions in enrollment create financial pressure on academic institutions, and so it is important that colleges offer degrees that remain both relevant and attractive to potential students. Also, once students are enrolled, it is essential that they remain engaged and progress through to graduation. Consequently, at the college level there should be interest in monitoring the efficiency with which departments manage their degree programs. This paper demonstrates the use of the data envelopment analysis (DEA) analytical technique to evaluate the relative efficiency of these programs.

Colleges offer degrees which students can earn to demonstrate their completion of a course of study at a certain level. Common academic degrees are an Associate, Bachelor's, Master's, or Doctoral degree. Colleges offer multiple fields of study, or majors, in which to earn a degree. To graduate with a degree in their major, students must complete a curriculum, which is a set sequence of courses. For the purposes of this paper, we will refer to a major and its associated curriculum as an academic program (AP).

A dean is the head of a college who manages different academic departments. For example, in a College of Business, the dean may oversee the Management, Accounting, and Computer Information Systems departments. A department is an administrative unit of the college, responsible in part for administering the academic programs that students must follow to obtain degrees in specific majors. For example, the Management Department might offer and be responsible for administering Bachelor of Science degrees in Business Administration, Human Resource Management, and International Business. To administer academic programs, the department must hire and manage professors who develop and teach the curriculum, and department staff who in part manage students' enrollments and degree progress.

Departments should strive to be both effective and efficient in regard to transforming students into graduates. The departments' effectiveness is their ability to produce graduates who are knowledgable in their field of study. This can be measured in an absolute sense by the quality of the graduate, and in a relative sense by the improvement in knowledge the student achieves throughout the academic program. The departments' efficiency is their ability to produce graduates with a minimal waste of time, energy, and expense. This can be measured in an absolute sense by factors such as total cost, number of graduates, and completion rate, and in a relative sense by comparison of these performance factors with other departments.

The dean manages and is ultimately responsible for the performance of all their subordinate departments, and as such is concerned with their relative effectiveness and efficiency. A dean's comparison of their departments' relative efficiency may be biased. Consider a department that is required to administer an academic program which is inherently inefficient, requiring the average amount of staffing, resources, and time but producing few graduates. This would unfairly cause such a department to look relatively inefficient when compared to other departments with no such burden.

Therefore, in this paper we consider the relative efficiency of each academic program, as opposed to the departments which administer the programs. Assessing the relative efficiency of the programs themselves focuses on the issue of concern, giving a more granular assessment of any inefficiencies. This comparison of efficiency highlights issues and supplements the operational knowledge of the dean and department heads who actively manage the departments and programs. The intent of this study is not to present relative efficiencies as a single metric for ranking of departments' performance, but instead to garner these insights.

DATA ENVELOPMENT ANALYSIS (DEA) EFFICIENCY MODEL

Data Envelopment Analysis (DEA) is an optimization technique developed by Charnes, et al. (1978) that produces a single measure of relative efficiency between multiple "decision-making units (DMU)." The beauty of DEA is that it allows measurement of production efficiency through the incorporation of non-homogeneous inputs and outputs, while still leveraging the properties of economic production functions. The resulting ubiquitous application of DEA in the literature is difficult to overstate, as it has been studied, applied, and documented across numerous domains for over 45 years (Cook, et al., (2009) and Liu, et al. (2013)). Comprehensive reviews such as Johnes, et al., (2017) and Witte, et al., (2017) demonstrate that the literature is quite rich even when limited to studies on higher education.

DEA studies focus on DMUs producing a "product" and what it means for the production process to be efficient. The freedom of input and output selection afforded by the DEA technique gives great latitude in the definition of the product(s). Resources are required for the DMU to produce the product, and in DEA studies those resources are referred to as inputs. While the product is obviously an output, often the attributes of the product and the production process are of interest, and so in DEA studies these are also referred to as outputs. Finally, efficiency is a ratio of the outputs produced to the inputs utilized. The lower the number of inputs required to produce a given number of outputs, the more efficient the DMU.

In DEA analyses specific to higher education institutions (HEIs), common products are graduates and research papers, each of varying quality. This paper focuses on the graduates produced by HEIs. Inputs can generally be classified as monetary, physical, or measures of effort. For example, HEIs require personnel salaries, classrooms, and instructional hours to produce graduates. Outputs are concerned with the quantity and/or quality of graduates. Along with the quantity of graduates themselves, overall graduate quality, the number of honor graduates, or the percentage of double majors are examples of additional HEI outputs.

DEA studies focused on HEIs vary in their purpose, level of focus, and factors included in the analyses. Studies generally assess the efficiencies of HEIs at different levels, such as across countries (Agasisti, 2009), within a country (Avkiran, 2001), within a state (Ahn, 1989), and subordinate units within an HEI such as departments (Kao, et al., 2008) or even within disciplines (Beasley, 1989). The motivation for many studies at these levels is to provide funders of the HEI with quantitative analyses for accountability purposes, such as Athanassopoulos, et al., (1997), Chalos (1997); Moreno, et al., (2002); Kuah, et al., (2011), and Kosor, et al., (2019).

The goal of this study is to gain insights about factors impacting the operational performance of different academic programs. As such, our focus is quite granular, providing quantitative, targeted guidance to inform managers' focus, decisions, and action. Taken in isolation, administrators may clearly see the impacts of a few inputs on outputs for a single DMU. However, an analytical approach is required to holistically understand the relative impact of multiple inputs and outputs across multiple DMUs. This is the precise purpose of the DEA approach, and as such is the proper tool for this analysis.

In this paper the DMUs are the academic programs, and graduates are the product. One input required to create graduates are students declaring their intention to complete the academic program (i.e., declaring a major). The average number of declarants enables calculating the conversion rate of declarants to graduates. Another input is the student credit hours (SCHs) of instruction specific to that academic program. While colleges require common courses of all students, only SCHs for courses specific to the major are included in our SCH input. The average number of SCHs are a measure of resources and effort required to transform the declarants into graduates.

While the average number of graduates is obviously an output, we also consider the duration of time from declaring to completing the academic program (i.e., the average time from declaring the major to graduating) as an output. This is the throughput rate of the AP, which is a measure of efficiency and an influencer of retention. The longer the average number of terms of an AP, the longer it takes the student to graduate, and the higher the probability that the student will drop out. This is an especially important factor for colleges with large numbers of non-traditional students, many of whom work full-time while trying to complete their degree.

DEA MODEL FORMULATION

Given a set of n DMUs, the linear program DEA_J calculates the relative efficiency of the j^{th} DMU. It maximizes the weighted sum of the j^{th} DMU's outputs by varying the weights applied to both the inputs and outputs of all n DMUs. Solving DEA_J once for each of the n DMUs in the set establishes the relative efficiency for all DMUs. The mathematical formulation of DEA_J follows.

Indices

- j = decision making unit $\in \{1..n\}$
- i = input $\in \{1..m\}$
- r = output $\in \{1..s\}$

Parameters

- y_{rj} = value of output r on unit j
- x_{ij} = value of input i on unit j

Decision Variables

- u_r = weight given to the r^{th} output
- v_i = weight given to the i^{th} input

Objective Function

$$\left(DEA_J\right) Max\ e_J = \frac{\sum_{r=1}^{s} u_r y_{rJ}}{\sum_{i=1}^{m} v_i x_{iJ}} \tag{1}$$

Constraints

$$\sum_{i=1}^{m} v_i x_{iJ} = 1 \tag{2}$$

$$\sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij}\ \forall j \tag{3}$$

$$u_r, v_i \ge 0 \forall\ r, i \tag{4}$$

The objective function (1) forces the j^{th} DMU, which we call DMU_J, to select those weights u_r and v_i which maximize its efficiency. The equation (2) constrains the denominator of (1) to the value of one, thereby preventing non-linearities and any unbounded solutions. All DMUs are prohibited from having an efficiency greater than 100% by constraints (3), where there exits one constraint (3) for each of the *n* DMUs in the set. Constraints (4) enforce the non-negativity of the decision variables.

After solving DEA_J the relative efficiency of DMU_J will equal the objective function value. It is important here to clarify that "relative efficiency" in DEA does not indicate that the DMU is operating at its own, maximum possible efficiency. Instead, it is a measure of how well the DMU transforms its inputs into outputs, *compared* to the other *n-1* DMUs in the set. Hence, a specific DMU could be relatively efficient in one set of DMUs and relatively inefficient in a different set of DMUs.

The concept of relative efficiency in DEA is somewhat technical. A relatively efficient DMU_j has the maximum objective function value of 100%. This indicates that the linear program could not find a linear combination among the other *n-1* DMUs with which to construct a hypothetical DMU, which we call DMU_h, capable of producing the same output using lower amounts of inputs. Conversely, a relatively inefficient DMU_j with an objective function value of less than 100% indicates that there does exist a linear combination among the other *n-1* DMUs with which to construct an efficient, hypothetical DMU_h.

In general, every constraint in a linear program has a corresponding "dual variable." After the linear program has been solved, the resulting value of that dual variable in the solution is called its "shadow price." The shadow price indicates how much better the objective function value could be by loosening that constraint. Therefore, a constraint with a non-zero shadow price indicates that that constraint is in some way preventing the linear program from finding a better objective function value.

For DEA linear programs specifically, shadow prices have an alternative interpretation (Nyhan, et al., 1999). The shadow prices of interest correspond to the *n* constraints (3) associated with each of the *n* DMUs. Recall that for an inefficient DMU_j there exists a combination of other DMUs with which to form a hypothetical, more efficient DMU_h. This combination of DMUs is collectively referred to as the reference set, and the constraints (3) associated with DMUs in the reference set have non-zero shadow prices. The hypothetical DMU_h is constructed by summing the inputs and outputs of each DMU in the reference set multiplied by their shadow prices. For an efficient DMU_j, the constraints (3) associated with the other *n-1* DMUs all have a shadow price equal to zero, and the constraint (3) associated with the efficient DMU_j will have a shadow price of one.

RESULTS

To demonstrate our approach, we solve the DEA linear programs using the inputs, outputs, and hypothetical data shown in Table 1. The DMUs are coded as AP1, AP2, etc. corresponding to the academic program (AP) for hypothetical majors Major #1, Major #2, etc. With *n* DMUs, *m* inputs, and *s* outputs, Banker (1989) suggests a rule of thumb that $n \geq 3(m+s)$, which is achieved with the n = 12 DMUs in our data set. While the inputs and outputs are described above, the average terms to graduate must be transformed for use in DEA_J. In DEA, the outputs must be of units such that larger values are preferred. However smaller values are preferred for the number of terms to graduate. Therefore, we use the inverse of the average terms to graduate to solve DEA_J, and then invert the solved values to interpret the results.

TABLE 1
INPUT AND OUTPUT ATTRIBUTES FOR THE ACADEMIC PROGRAMS

Major	Decision Making Unit	Inputs		Outputs		* Average Terms to Graduate
		Average Declarations	Student Credit Hours	Average Graduations	Inverse of Average Terms to Graduate	
Major 1	AP1	147.6	474.3	118.4	0.18	5.5
Major 2	AP2	2.0	68.2	1.3	0.20	5.0
Major 3	AP3	3.0	196.1	3.0	0.21	4.7
Major 4	AP4	90.2	509.0	76.8	0.17	6.0
Major 5	AP5	25.3	1560.8	15.6	0.21	4.8
Major 6	AP6	30.6	1687.6	29.6	0.15	6.5
Major 7	AP7	81.6	265.1	70.0	0.17	5.8
Major 8	AP8	9.8	1077.8	9.0	0.13	7.5
Major 9	AP9	12.4	158.7	12.4	0.13	7.5
Major 10	AP10	165.6	474.3	139.6	0.18	5.6
Major 11	AP11	106.0	207.7	89.6	0.17	5.7
Major 12	AP12	2.0	1622.5	2.0	0.25	4.0

Solving DEA_J for each of the DMUs produces their DEA relative efficiencies, and analyzing the non-zero shadow prices establishes the reference sets for inefficient APs, as shown in Table 2. Consider AP8 and its DEA efficiency of 0.92 in the second column. The single DMU in AP8's reference set is AP3, which itself must have a relative efficiency of 1.00. From the optimization perspective, this indicates that after the linear program DEA_8 is solved, the dual variable associated with constraint (3) for AP3 has the non-zero shadow price of 3.00.

TABLE 2
RELATIVE EFFICIENCIES, REFERENCE SETS, AND SHADOW PRICES FOR THE ACADEMIC PROGRAMS AND DMUS IN THEIR REFERENCE SET

Decision Making Unit	DEA Relative Efficiency	Reference Set	Shadow Prices		
			AP3	AP9	AP11
AP1	0.93	AP9, AP11	-	1.28	1.14
AP2	1.00		-	-	-
AP3	1.00		1.00	-	-
AP4	0.95	AP9, AP11	-	2.34	0.53
AP5	0.62	AP3, AP9	0.21	1.21	-
AP6	0.97	AP9	-	2.39	-

Decision Making Unit	DEA Relative Efficiency	Reference Set	Shadow Prices		
			AP3	AP9	AP11
AP7	0.99	AP9, AP11	-	0.78	0.67
AP8	0.92	AP3	3.00	-	-
AP9	1.00		-	1.00	-
AP10	0.98	AP9, AP11	-	1.09	1.41
AP11	1.00		-	-	1.00
AP12	1.00		-	-	-

To analyze what makes AP3 relatively more efficient than AP8, Table 3 includes an extract of the input and output data for these two APs from Table 1. AP8 has a higher average number of both declarations (input) and graduates (output) than does AP3. This shows that on average there are roughly 3 times the number of students pursuing Major #8 than there are pursuing Major #3. However, from an efficiency perspective, while AP3 produces only 33% of the graduates, it does so by using only 18% of the SCHs, in 63% of the time, and with an 8% higher conversion rate compared to those same measures for AP8. As such, the DEA model shows that while AP8 is more productive than AP3, it is relatively less efficient.

TABLE 3
INPUT AND OUTPUT ATTRIBUTES FOR THE INEFFICIENT ACADEMIC PROGRAM 8 AND THE EFFICIENT ACADEMIC PROGRAM 3 IN ITS REFERENCE SET

Decision Making Unit	DEA Relative Efficiency	Shadow Price AP3	Inputs		Outputs	
			Average Declarations	Student Credit Hours	Average Graduations	Average Terms to Graduate
AP8	0.92		9.8	1,077.8	9.0	7.5
AP3	1.00	3.00	3.0	196.1	3.0	4.7
AP8_3c			9.0	588.4	9.0	*1.6

To demonstrate the interpretation of shadow prices, consider that in the solution of DEA_8, AP3 has a shadow price of 3.00. Multiplying the inputs and outputs of AP3 by this shadow price produces the composite DMU AP8_3c, which would hypothetically produce the same or better outputs as AP8 but would do so using less inputs. The values of AP8_3c in the bottom row of Table 3 show that AP8_3c matches AP8's average graduates of 9.0 but does so in 21% of the time, while only requiring 92% of the declarants and 55% of the SCHs.

The calculation of the composite average terms to graduate is nuanced. Recall that the average terms to graduate must be inverted for use in solving the linear program, and then the solution values are inverted back for interpretation. The composite DMU must be created using the inverted values included in the linear program along with the associated shadow prices. In this calculation, the shadow price of 3.00 for AP3 is multiplied by the AP3 inverse terms to graduate of 0.21, giving a composite value of 0.63, which is then inverted and rounded to 1.6 terms to graduate, as shown in Table 3.

In the pair-wise comparison above, AP3 dominates AP8 across multiple measures, but this is not always the case. Table 2 shows that most of the relatively inefficient DMUs include multiple DMUs in their reference sets. To illustrate this more common case, compare the relatively inefficient AP1 with the efficient AP9 and AP11 in its reference set. Their input and output data are shown in Table 4.

TABLE 4
INPUT AND OUTPUT ATTRIBUTES FOR THE INEFFICIENT ACADEMIC PROGRAM 1 AND THE EFFICIENT ACADEMIC PROGRAMS 9 AND 11 IN ITS REFERENCE SET

			Inputs		Outputs	
Decision Making Unit	DEA Relative Efficiency	Shadow Prices AP9 & AP11	Average Declarations	Student Credit Hours	Average Graduations	Average Terms to Graduate
AP1	0.93		147.6	474.3	118.4	5.5
AP9	1.00	1.28	12.4	158.7	12.4	7.5
AP11	1.00	1.14	106.0	207.7	89.6	5.7
AP1_9_11c			137.2	440.8	118.4	2.7

Comparing AP1 to AP9 shows that on average AP1 produces 9.5 times the number of graduates using only 3 times the number of SCHs and taking only 74% of the time as does AP9. However, AP9 has a 20% higher conversion rate of declarants to graduates than does AP1. Next, comparing AP1 to AP11 shows that on average AP1 has 32% higher number of graduates, but requires 128% more SCHs at a 5% lower conversion rate than does AP11.

This example reflects the more common case where multiple and mixed trade-offs between DMUs account for the relative inefficiency of a DMU. While AP1 has a greater number of graduates than both AP9 and AP11 in its reference set, AP9 has a much higher conversion rate, while AP11 requires a much lower percentage of SCHs. The DEA optimization model and its resulting sensitivity analysis enables the identification of these trade-offs which would be hidden amongst the pair-wise comparisons of this many DMUs, with multiple inputs and outputs each.

The sensitivity analysis here proves to be directly applicable. Multiplying the inputs and outputs of APs 9 and 11 by their shadow prices produce the composite DMU AP1_9_11c, which would hypothetically produce the same or better outputs as AP1 using less inputs. The input values of this composite DMU provide quantitative goals, such that matching them both would make AP1 relatively efficient.

Consider the conversion rates and SCHs of the inefficient AP1 and the hypothetical AP1_9_11c. Both have 118.4 graduates, yet AP1 and AP1_9_11c have 147.6 and 137.2 declarants, and 80% and 86% conversion rates, respectively. This provides management with a quantitative retention target of carrying 86% of declarants through to graduation for AP1. Similarly, AP1_9_11c requires 33.5 less SCHs than does AP1. Management here could evaluate options to consolidate current courses or look for synergies with other courses to achieve a 33.5 reduction in the SCHs required for AP1.

CONCLUSIONS

In an environment where the value of a college degree is being questioned, managing college departments and administering academic programs requires a focus on efficiency. Quantifying the impact of multiple factors for multiple academic programs is important yet precluded without the use of analytical techniques. In this paper, we illustrate the application of Data Envelopment Analysis to identify which academic programs are relatively efficient in their production of graduates for a given set of inputs and outputs. For those academic programs identified as inefficient, linear programming sensitivity analysis identifies specific efficient academic programs to compare and better understand the factors which make them inefficient. Additionally, the input values of the constructed hypothetical DMU provide quantitative goals for making the inefficient DMU efficient.

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