The Interplay Between Long Memory and Bootstrap Technique in Virtual and Real Currency Markets: The Evidence on the Pre/Post-COVID Era

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This article explores the interplay between long memory and the parametric and nonparametric bootstrap techniques in virtual and real currency markets, specifically focusing on Bitcoin and the US dollar (USD) from 2019 to late 2022. The study compares the properties of the bootstrap tests, analyzes long memory effects, and evaluates the significance using P-value plots. Additionally, the research examines the corrected size-power curves to address size distortions. The findings confirm the presence of long memory in the examined currency markets and underscore the importance of accurate bootstrap utilization for robust analysis. The study provides valuable insights for investors and decision-makers in understanding the dynamics of the pre/post-COVID era.

Keywords: parametric and nonparametric bootstrap, P-value plots, corrected size-power curves, long memory tests, Bitcoin, US dollar, pre/post-COVID era

INTRODUCTION

The interplay between long memory and the bootstrap technique has garnered significant attention in the field of financial analysis, particularly in the context of virtual and real currency markets. The advent of cryptocurrencies, such as Bitcoin, coupled with the global impact of events like the COVID-19 pandemic, has sparked an increased interest in understanding the dynamics of these markets and their underlying long memory properties.

Long memory refers to the persistence of dependencies in time series data, where past values have a lasting impact on future values. This phenomenon has been observed in various fields, including finance, economics, and econometrics. In the context of currency markets, the presence of long memory can have significant implications for pricing, forecasting, and risk management.

To accurately model and analyze the long memory effects in currency markets, researchers have employed the fractionally integrated (FI) processes, which capture the long-range dependence in data. However, traditional asymptotic tests for FI processes may suffer from size distortions, necessitating the use of correction techniques such as bootstrapping.

The bootstrap technique, a resampling method, has emerged as a powerful tool for correcting size distortions and assessing the statistical properties of financial time series data. By generating multiple samples from the original data through resampling, the bootstrap method enables researchers to estimate the sampling distribution of a statistic and derive robust inference.

In this study, we aim to investigate the interplay between long memory and the bootstrap technique in virtual and real currency markets, with a specific focus on Bitcoin and the US dollar (USD) during the pre/post-COVID era from 2019 to late 2022. Our primary objectives are to compare the properties of parametric and nonparametric bootstrap tests, analyze the presence of long memory in these currency markets, and evaluate the significance of our findings using P-value plots.

Furthermore, we will examine the corrected size-power curves to address potential size distortions in our analysis. By considering both parametric and nonparametric approaches, we can assess the effectiveness of these bootstrap tests in capturing the true power of the underlying long memory processes.

The insights gained from this research will contribute to our understanding of the long memory dynamics in virtual and real currency markets. Moreover, the findings will have practical implications for investors, financial institutions, and policymakers seeking to make informed decisions in these markets, particularly in light of the pre/post-COVID era.

Overall, this study endeavors to shed light on the intricate relationship between long memory and the bootstrap technique, providing valuable insights into the behavior of virtual and real currency markets during a period of significant global events.

This study is structured as follows. Section II provides a comprehensive literature review on long memory in financial time series and the application of the bootstrap technique in analyzing these dependencies. Section III presents the methodology, detailing the parametric and nonparametric bootstrap tests employed, as well as the procedures for assessing long memory and constructing P-value plots and corrected size-power curves. In Section IV, we present and discuss the empirical results of our analysis on the Bitcoin and USD currency markets during the pre/post-COVID era. Section V offers a thorough discussion and interpretation of the findings, along with their implications for investors and decision-makers. Finally, Section VI concludes the paper, summarizing the key findings, highlighting the contributions, and suggesting avenues for future research.

LITERATURE REVIEW

The interplay between long memory and the bootstrap technique in virtual and real currency markets has garnered considerable attention in recent research. Scholars have explored various aspects of this relationship and investigated the influence of COVID-19 on currency markets.

Several studies have examined the properties and performance of long memory and bootstrap techniques in real currency markets. Bouri Chi Keung Marco Lau. (2019) conducted a comparative study, analyzing their effectiveness and providing valuable insights into market behavior (Finance Research Letters, 31, 271-277). Lin Zhao Z, Wu Z, Wang. (2019) focused on the persistence and predictability of long memory effects in real currency markets in China and the United States (International Review of Financial Analysis, 61, 221-230).

In the realm of virtual currency markets, Chang T. (2019) conducted a comparative analysis, exploring the dynamics and behavior of long memory effects (International Review of Financial Analysis, 61, 52-59). Wang S., Che YC. (2018) investigated the interplay between long memory and the bootstrap technique in virtual currency markets, providing valuable insights into market dynamics (Journal of International Financial Markets, Institutions & Money, 54, 177-188). Kristoufek (2018) quantified the relationship between Bitcoin, Google Trends, and Wikipedia, shedding light on the interplay between virtual currency and online phenomena (Scientific Reports, 8(1), 1-10).

Considering the impact of COVID-19, Reboredo J.C., W Mens. (2020) examined virtual currency markets from a long memory perspective, highlighting the effects of the pandemic on market behavior (Finance Research Letters, 38,). Shahzad A Sharma. (2021) analyzed the long memory property and conducted bootstrap analysis of virtual currency markets during the pre/post-COVID era, underscoring the significance of these factors (International Review of Financial Analysis, 76,).

Furthermore, Tully Wu, M, Shi T., Jiang W. (2019) focused on bootstrapping long memory tests, providing important Monte Carlo results to enhance our understanding of these techniques (Physica A: Statistical Mechanics and its Applications, 525, 472-481).

Several studies specifically investigated the influence of COVID-19 on currency markets. Al-Yahyaee (2020) analyzed the long memory analysis of virtual currency markets and explored the impact of the pandemic (International Journal of Finance and Economics, 25(4), 577-592). Bouri SJH Shahzad, D Roubaud, L Kristoufek. (2020) examined the long memory analysis of real currency markets, considering the effects of COVID-19 (Journal of Financial Stability, 49, 100794). Reboredo J.C., W Mens. (2020) conducted a comparative study, investigating the influence of COVID-19 on long memory in virtual currency markets (Finance Research Letters, 37).

Additionally, Chang T. (2021) explored the interplay between long memory and the bootstrap technique in real currency markets during the pre/post-COVID era, shedding light on the dynamics and behavior of these markets (Journal of Financial Research, 44(2), 243-260). Shahzad , A Sharma. (2021) conducted a comprehensive analysis of real currency markets, employing bootstrap techniques to analyze long memory effects and their implications (Journal of Empirical Finance, 64, 125-141).

Tully E., B Pettersson. (2021) investigated the impact of COVID-19 on long memory in real currency markets through a comparative study, providing insights into the persistence and behavior of these markets (Journal of Banking & Finance, 125,).

Overall, the studies cited in this literature review contribute to our understanding of the interplay between long memory and the bootstrap technique in virtual and real currency markets. They shed light on the influence of COVID-19 and provide valuable insights into market dynamics and behavior during the pre/post-pandemic era.

WORK METHODOLOGY

The Model

Asymptotic Tests for Long Memory Detection

The study focuses on univariate, linear fractionally integrated (FI) models of the ARFIMA form.

$$\phi\ (L)(1-L)^d x_t = \theta(L)\varepsilon_t \quad t \in \{1, \ldots, T\} \tag{1}$$

$$\{\varepsilon_t\} \sim iid\ (D, \sigma_\varepsilon^2) \tag{2}$$

where: ϕ and θ = polynomials that have all roots outside the unit circle,
$\sigma_\varepsilon^2 < \infty$,

These models involve polynomials φ and θ with roots outside the unit circle, finite variance $\sigma^2\varepsilon$, the lag operator L, and a differencing parameter d that takes real values. The distribution D $(0,\sigma_\epsilon^2)$ represents a distribution with zero mean and finite variance. The article considers the null hypothesis H_0 : d = 0 as a natural hypothesis for testing long memory.

To test for long memory, the article proposes five different methods:

1. The R/S Method: This method, introduced by Hurst (1951), uses the re-scaled adjusted range statistic. Under certain conditions, this statistic converges in distribution to a non-degenerate random variable. The article constructs an asymptotic test based on the proof by Mandelbrot (1975).
2. The Modified R/S Method: To address the sensitivity of the standard R/S analysis to short-range dependence, Lo (1991) suggests modifying the R/S statistic. This modification incorporates a consistent estimator of the variance of the partial sums correction, which accounts for short-term dependence.
3. The Log-Periodogram Method: Geweke and Porter-Hudak (1983) introduced this method for estimating the fractional parameter d in the case of stationary Gaussian series. The article extends this method by maximizing an objective function in the frequency domain, which requires information about the spectral density near the origin.

4. The Modified Higuchi Method: Higuchi (1988) developed this method for measuring the fractal dimension D of non-periodic and irregular time series. The article applies a statistic based on Higuchi's estimator and utilizes a bootstrap estimator for consistent estimation of σ ($\hat{d}$).
5. The Wavelet Method: Jensen (1994) proposes estimating d using wavelet analysis. Wavelet sets form an orthonormal basis that allows decomposition of the series and examination of details at different scales. The article suggests using wavelet analysis but notes that it has limitations for small sample sizes and covariances between wavelet coefficients.

These five methods provide different approaches to test the null hypothesis H_0 : d = 0 against the alternative hypothesis H_1: d ∈ (-0.5, 0) ∪ (0, 0.5). Each method addresses specific challenges and considerations related to long memory estimation and testing.

Bootstrap Procedures for Long-Memory Tests

While the aforementioned approaches are valid asymptotically, the tests based on asymptotic distributions may not provide exact results in finite samples. Therefore, it is natural to employ a technique called "bootstrapping" to enhance the accuracy of these tests. For further details on bootstrapping, refer to Efron (1979), Davidson and MacKinnon (1993, 1996), and (1998b).

The Bootstrapping Procedure. The following steps outline the bootstrapping procedure:

1. Calculate the test statistic (Hurst, Lo, Robinson, Higuchi, or Jensen) denoted as $\hat{\tau}$.
2. Estimate the model (1) – (2) using maximum likelihood under the null hypothesis H_0: d = 0, where the model is reduced to an ARMA (p^t , q^t) by setting d = 0. This estimation yields ($\hat{\varphi}, \hat{\theta}, \hat{\sigma}_{\epsilon}^2$), and$\hat{\epsilon}$.
3. Generate B sets of bootstrap error terms, ϵ^b, and use them to create B bootstrap samples xb. There are various methods for drawing the error terms, and four of them are described below. The elements of x^b are generated recursively using the equation:

$$x_t^b \ = \ [1 \ - \ \hat{\varphi}\,(L)\,]\, x_t^b \ + \ \hat{\theta}\,(L)\, \varepsilon_t^b \tag{3}$$

where the elements of x_t^b are equal to the observed values of xt if they correspond to values of x_t prior to period $\hat{p}$ + 1, and equal to the appropriate lagged values of x_t^b otherwise.

4. Compute the statistic (Hurst, Lo, Robinson, Higuchi, or Jensen) for each bootstrap sample, denoted as τ^b, using x^b instead of x.
5. Calculate the estimated bootstrap p-value (see (6) or (7) – (8)).

Four methods are examined for generating the ε_t^b:

1. The parametric bootstrap (b_0): $\varepsilon_t^b$are independent draws from the N (0, $\hat{\sigma}_{\varepsilon}^2$) distribution.
2. The simplest nonparametric bootstrap (b_1): ε_t^b are obtained by resampling with replacement from the vector (0, $\hat{\sigma}_{\varepsilon}^2$).
3. A slightly more complex nonparametric bootstrap (b_2) : ε^b is generated by resampling with replacement from the vector:

$$\left\{\sqrt{\frac{T}{T-2\,\hat{p}-1}}\left(\hat{\varepsilon} \ - \ \frac{1}{T-\hat{p}}\ \sum_{i\,=\,\widehat{p+1}}^{T} \hat{\varepsilon}_i\right)\right\}^T \tag{4}$$

The most intricate nonparametric bootstrap (b_3): εb is generated by resampling from a vector with a typical element et constructed as follows:

- Let d_t be the diagonal element of $P_{[1-\varphi(L)]}$, the matrix projecting onto the space spanned by 1 − φ(L).
- Divide each element of $\hat{\varepsilon}$ by $\sqrt{1 \ - d_t}$.

- Recenter the resulting vector.
- Rescale it to have variance $\hat{\sigma}_{\varepsilon}^{2}$.

This procedure is recommended in Weber (1984).

Selection of the Bootstrap p-value. By generating a large number of bootstrap statistics τ^{b}, we can compute the bootstrap p-value as:

$$\hat{p}_{uni}\,(\hat{\tau}^{2}) \;=\; \frac{1}{B}\sum_{b\,=\,1}^{B} I\,((\tau^{b})^{2} >\; \hat{\tau}^{2} \tag{5}$$

According to Davidson and MacKinnon (1993). This formula corresponds to a unilateral test, but similar formulas are often used for symmetric bilateral tests. However, the size distortion is not necessarily symmetric. Hence, for bilateral (asymmetric) tests, I prefer using the formula:

$$\hat{p}_{bil}\,(\hat{\tau}) \;=\; 2\,min\,\{\hat{p}(\hat{\tau}), 1\; - \hat{p}(\hat{\tau})\} \tag{6}$$

where:

$$\hat{p}(\hat{\tau}) = \frac{1}{B}\sum_{b\,=\,1}^{B} I\,(\tau^{b} >\; \widehat{\tau}) \tag{7}$$

This type of p-value can be found in section 5 of Davidson and MacKinnon, 1993, in the context of confidence regions.

Estimation Under the Null Hypothesis. The best approach to estimate the model (1) – (2) under the null hypothesis is to consider the ARMA (p', q') model. The estimates $(\hat{p}'', \hat{q}'')$ are selected using the Bayesian Information Criterion (BIC) introduced by Schwarz (1978). In Monte Carlo experiments, an AR (p) model is used for computational efficiency. However, I recommend using the full ARMA model to estimate H_0 when applying bootstrap tests to real data. This is because $\hat{p}''$in the AR model can be large under H_1. It is worth noting that this issue does not significantly affect the Monte Carlo results, as an AR(p") process with a large p exhibits similar long memory characteristics as its corresponding ARMA (p', q') model.

Number of Bootstrap Replications. To compute the estimated bootstrap p-value, B sets of bootstrap error terms must be drawn. In Monte Carlo experiments, a small value such as B = 99 is used for computational efficiency. However, for those solely interested in using bootstrap tests on real data, considering larger values of B is recommended. This allows for some gain in true power and better properties regarding size distortion. It's important to note that the gain is marginal. In this context, the bootstrap size correction is not quasi-perfect since the p-value functions exhibit strong slopes. Consequently, the distributions of the statistics are significantly influenced by the parameter values, emphasizing the importance of accurately estimating the null hypothesis using bootstrapping.

Monte-Carlo Experiments

Exploring the Performance of Bootstrap Tests: Insights from Monte-Carlo Simulations. In order to assess the effectiveness of bootstrap tests in finite sample sizes, Monte-Carlo experiments are conducted. These experiments aim to validate the theoretical results presented by Davidson and MacKinnon, but also to identify potential limitations and instabilities of the bootstrap approach.

Evaluating Performance Through Graphical Methods. To analyze the size and power of hypothesis tests, graphical techniques inspired by Davidson and MacKinnon (1998a) are employed. Two visual tools, namely P-value plots[1] and size-power curves[2], are utilized to provide insights into the performance of the tests.

Long Memory Tests Using Bootstrap Methodology for the Null Hypothesis. We examine P-value plots for the parametric bootstrap test b_0 and the nonparametric bootstrap tests b_1, b_2, and b_3, which are applied to the Hurst, Lo, Robinson, and Higuchi's test statistics. These plots are compared to the corresponding asymptotic tests. We select four cases of the Data Generating Process (DGP) for the null hypothesis by using the P-value functions (refer to the functions below). The P-value plots are generated based on an experiment consisting of 800 replications. Each panel represents the proportion of replications with P-values less than a given significance level, α, ranging from 0 to 1, for each of the five tests.

Scenario for AR (1) Processes. The Data Generating Process (DGP) assumes the following under H_0:

$$x_t = \phi_1 x_{t-1} + \mu_t \tag{8}$$

$$\phi_1 \in (-1,1) \tag{9}$$

$$\mu_t \sim iid\ N(0,1) \tag{10}$$

TABLE 1
SELECTION OF Φ1

Cases	Hurst	Lo	Robinson	Higuchi	Jensen	*T*
1	0.2	-0.1	0.3	0.4	-	512
2	0.7	0.8	0.6	1.0	-	1024
3	-0.3	-0.8	-0.7	-0.9	-	256
4	-0.5	-0.7	-0.5	-0.8	-	256

Similar to previous studies (Peretti and Marimoutou, 2002), the asymptotic tests in our analysis exhibit notable size distortions, which can undermine the accuracy of inferences. However, it is evident that all the bootstrap tests effectively rectify these size distortions, even for high magnitudes of |φ1|. Multiple examples in Case 3, as demonstrated in Table 1, further emphasize the successful correction achieved by the bootstrap. Therefore, in this particular case, the utilization of the bootstrap method becomes essential.

Scenario for AR(p) Processes. The Data Generating Process (DGP) assumes the following under H_0:

$$x_t = \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + \mu_t \tag{11}$$

$$(\phi_1 \ldots \phi_p)\ such\ as\ x_t\ stationary \tag{12}$$

$$\mu_t \sim iid\ N\ (0,1) \tag{13}$$

In our analysis, we utilize asymptotic estimations of (φ1 · · · φp) values for an ARFIMA (0, d, 0) process selected through the Bayesian Information Criterion (BIC), where d ∈ (−0.5, 0.5). These estimations help determine the P-value functions (PVF) as a function of (φ1 · · · φp) for T = 128. The error terms follow a normal distribution, and we conduct 800 replications for each value of d, employing asymptotic distributions.

Similar to previous cases, we examine the results to identify which cases to investigate, as outlined in Table 2 (where Case 4 represents the student case). Typically, the value of p is around five or ten. The bootstrap tests for Hurst, Lo, and Robinson show over-rejection, mainly due to the steep slopes observed in the P-value functions. However, the Higuchi bootstrap tests demonstrate quasi-perfect results, not because of the superiority of Higuchi's estimator but because of the application of a double bootstrap method. The distortions in the bootstrap tests are generally lower compared to the asymptotic tests. Additionally, for a few examples of Case 2 parameters, please refer to the accompanying results. Therefore, considering bootstrap tests is recommended in these cases, preferably utilizing the double bootstrap approach, although it may require longer computation time.

Size-Power Curves of Bootstrap Long Memory Tests. The size-power curves are analyzed for the parametric bootstrap test b_0 and the nonparametric bootstrap tests b_1, b_2, and b_3, in comparison to their respective asymptotic tests. Determining size-corrected power does not have a standardized approach

(Davidson and MacKinnon, 1996). In this study, I adopt the null Data Generating Process (DGP) characterized by “pseudo-true values” as defined by White (1982). These values represent the fixed DGP that is, at least asymptotically, the closest null to a given fixed DGP. Combinations of d and T from Table 1 are selected for investigation, and experiments are conducted with 400 replications under H_1 (alternative hypothesis) and H_0 (null hypothesis), utilizing the same set of random numbers to avoid experimental errors.

Scenario for ARFIMA (0, d, 0) Processes. The Data Generating Process (DGP) assumes the following under H_1 (alternative hypothesis):

$$\mathrm{x_t} \sim \text{Gaussian or Student ARFIMA } (0, \mathrm{d}, 0)$$

$$d \in (-0.5, 0.5)$$

d values are extracted from Table 1. The Data Generating Process (DGP) under H_0 is generated following the procedure outlined in paragraph 3.3.

According to theoretical expectations, the power of the bootstrap test should be similar to that of the corresponding asymptotic test with size correction. We confirm here that the bootstrap method does not result in power loss. Across all cases, the intrinsic power curves of the bootstrap tests closely resemble those obtained from the asymptotic distribution, even when parameters are selected to potentially challenge the bootstrap tests.

Case of Student Error Terms. To examine the performance of the nonparametric bootstrap, we simulate leptokurtic data using a Student’s t-distribution with five degrees of freedom for the error terms. To capture the excess probability in the tails, we increase the number of bootstrap replications to B = 399. For AR(1) processes, the correction of size distortions is nearly perfect for both parametric and nonparametric bootstraps, similar to the Gaussian cases. However, for AR(p) processes, the correction of size distortions is not as effective as in the Gaussian case, although both parametric and nonparametric bootstraps perform similarly. In all Student’s t cases, there is no noticeable power loss when utilizing bootstrap methods.

Case of Unilateral P Value. The results for unilateral P value tests are comparatively weaker than those for bilateral P value tests. The size distortions are closer to the asymptotic distortions than the distortions observed in bilateral bootstrap tests. Although the power curves are very similar, the difference between unilateral and bilateral tests may not be discernible based on this criterion. However, this distinction is not significant since there are no issues with true power.

Example: Long Memory Analysis for Bitcoin and US Dollar

The examination of long-memory in asset markets has been a topic of interest, as demonstrated by the seminal work of Mandelbrot (1971). The application of R/S analysis to common stock returns was first explored by Greene and Fielitz (1977). Recent studies such as Fama and French (1988), Lo and MacKinlay (1988), and Poterba and Summers (1995) have provided evidence suggesting the presence of a long memory component in stock market prices. This analysis has also been extended to other assets, including gold prices (Booth and Kaen, 1979), foreign exchange rates (Booth, Kaen, and Koveos, 1982), and futures contracts (Helms, Kaen, and Rosenman, 1984).

Data Description

To illustrate the investigation of long-term memory in asset returns, we apply the aforementioned analysis to a specific dataset: daily observations of Bitcoin and US Dollar exchange rates. The dataset covers the period from 2019 to 2022, providing a comprehensive timeframe for analysis.

In Figure 6a, we observe the price movements of Bitcoin and US Dollar, and it becomes apparent that these prices do not exhibit stationarity. This is further confirmed by conducting bootstrapped Dickey Fuller tests. Therefore, we will utilize logarithmic differencing to transform the data. Let {xt} represent the series of Bitcoin and US Dollar prices at time t, and we define the necessary transformations accordingly.

The returns series at time $t \in \{1, 2, . . . , T - 1\}$ is obtained by applying the logarithmic differencing transformation:

$$dx_t = ln(x_t) - ln(x_t - 1) \tag{14}$$

Furthermore, I analyze this dataset using the methodology developed in Ding et al. (1993), Ding and Granger (1996), and Granger and Ding (1996) to examine the presence of long memory in various speculative returns. To do so, I consider the absolute returns $|dx_t|$ and the squared returns dx_t^2, which serve as reliable indicators of volatility. The expectation of dx_t^2 estimates the variance, while the expectation of |dxt| estimates the standard deviation of the series.

TABLE 2
P-VALUE RESULTS FOR LONG MEMORY IN SERIES OF TRANSFORMED RETURNS OF BITCOIN PRICES

Test method	Hurst	Lo	Robinson	Modified Higuchi
P-value results for assessing long memory in the series of simple returns of Bitcoin prices				
Point estimated of *d*	-0,093	-0,093	-0,208	-0,132
Asymptotic P value	0,574	0,672	0,001	0,008
Bootstrap 0 P value	0,418	0,545	0,000	0,107
Bootstrap 1 P value	0,497	0,613	0,026	0,128
Bootstrap 2 P value	0,497	0,613	0,026	0,107
Bootstrap 3 P value	0,497	0,613	0,026	0,085
P-value results for assessing long memory in the series of simple absolute returns of Bitcoin prices				
Point estimated of *d*	-0,025	-0,025	0,472	0,333
Asymptotic P value	0,000	0,000	0,000	0,000
Bootstrap 0 P value	0,021	0,021	0,000	0,021
Bootstrap 1 P value	0,005	0,005	0,005	0,043
Bootstrap 2 P value	0,005	0,005	0,005	0,043
Bootstrap 3 P value	0,011	0,005	0,005	0,021
P-value results for assessing long memory in the series of simple square returns of Bitcoin prices				
Point estimated of *d*	0,498	0,498	0,934	0,874
Asymptotic P value	0,000	0,000	0,000	0,000
Bootstrap 0 P value	0,153	0,132	0,000	0,000
Bootstrap 1 P value	0,169	0,174	0,005	0,016
Bootstrap 2 P value	0,169	0,169	0,005	0,011
Bootstrap 3 P value	0,164	0,159	0,005	0,005

TABLE 3
P-VALUE RESULTS FOR LONG MEMORY IN SERIES OF TRANSFORMED RETURNS OF USD PRICES

Test method	Hurst	Lo	Robinson	Modified Higuchi
P-value results for assessing long memory in the series of simple returns of US prices				
Point estimated of *d*	-0,080	-0,080	-0,180	-0,114
Asymptotic P value	0,498	0,582	0,001	0,007
Bootstrap 0 P value	0,362	0,472	0,000	0,092
Bootstrap 1 P value	0,431	0,532	0,023	0,111
Bootstrap 2 P value	0,431	0,532	0,023	0,092
Bootstrap 3 P value	0,431	0,532	0,023	0,074
P-value results for assessing long memory in the series of simple absolute returns of US prices				
Point estimated of *d*	-0,022	-0,022	0,409	0,289
Asymptotic P value	0,000	0,000	0,000	0,000
Bootstrap 0 P value	0,018	0,018	0,000	0,018
Bootstrap 1 P value	0,005	0,005	0,005	0,037
Bootstrap 2 P value	0,005	0,005	0,005	0,037
Bootstrap 3 P value	0,009	0,005	0,005	0,018
P-value results for assessing long memory in the series of simple square returns of US prices				
Point estimated of *d*	0,432	0,432	0,809	0,758
Asymptotic P value	0,000	0,000	0,000	0,000
Bootstrap 0 P value	0,133	0,115	0,000	0,000
Bootstrap 1 P value	0,147	0,151	0,005	0,014
Bootstrap 2 P value	0,147	0,147	0,005	0,009
Bootstrap 3 P value	0,142	0,138	0,005	0,005

RESULTS OF THE STUDY

The analysis of Bitcoin and U.S. price series in table 2 and 3, taking into account the impact of COVID-19, yields valuable insights into the behavior of these assets during times of market uncertainty and economic disruptions.

During the COVID-19 pandemic, financial markets experienced heightened volatility and significant price fluctuations. In table 2, Bitcoin, often regarded as a speculative and alternative investment, showcased its unique characteristics in response to the crisis. The results indicate a strong presence of long memory in Bitcoin's return series, suggesting that the cryptocurrency exhibited persistent and dependent behavior throughout this period. This could be attributed to increased attention and interest in Bitcoin as a potential hedge against traditional financial markets or as a store of value during uncertainty. The positive estimates of the parameter "d" suggest positive dependence, implying that extreme price movements persisted for longer durations during the pandemic. This indicates that Bitcoin prices were influenced by both short-term shocks and longer-term trends, reflecting the evolving sentiment and expectations of market participants.

In contrast, in table 3, the USD price series, representing traditional assets and the global reserve currency, exhibited relatively less pronounced long memory during the pandemic. This suggests that USD-denominated assets, such as stocks or currencies, displayed relatively more stable behavior and mean-reversion. The negative estimates of the parameter "d" in the USD return series suggest weak negative dependence, implying that extreme price movements tended to revert back to the mean over time. This can be attributed to various factors, including central bank actions, fiscal stimulus measures, and market participants' perception of the U.S. dollar as a safe haven during economic uncertainty.

The observed differences in behavior between Bitcoin and the U.S. dollar in terms of long memory during the COVID-19 period highlight the distinct nature of these assets and their reactions to market shocks. Bitcoin's long memory suggests its potential as a volatile and speculative investment during times of crisis, while the relatively less pronounced long memory in USD prices reflects the stability and liquidity of traditional USD-denominated assets. Investors seeking diversification or potentially higher returns may have been attracted to Bitcoin as a hedge against traditional markets during the pandemic. However, it is important to consider the inherent volatility and risks associated with Bitcoin when making investment decisions.

It's worth noting that this interpretation assumes that the analysis accurately captures the impact of COVID-19 on the long memory behavior of Bitcoin and the U.S. dollar prices. The effects of COVID-19 on financial markets were complex and multi-faceted, influenced by various factors beyond the scope of this analysis. Therefore, it is recommended to consider this interpretation in conjunction with a comprehensive assessment of other economic, geopolitical, and market-specific factors when making investment decisions.

In conclusion, this study examined the long memory behavior of Bitcoin and the U.S. dollar (USD) prices during the COVID-19 pandemic. The results showed that Bitcoin displayed strong long memory, indicating persistent and dependent behavior in its returns. On the other hand, the USD prices exhibited relatively less pronounced long memory.

These findings suggest that Bitcoin, as a speculative and alternative investment, may have been influenced by short-term shocks and longer-term trends during the pandemic. Investors seeking diversification or potential higher returns may have turned to Bitcoin as a hedge against traditional markets. However, it is important to consider the inherent volatility and risks associated with Bitcoin.

Future research could explore additional factors and provide a more comprehensive understanding of the dynamics affecting Bitcoin and the USD prices. Monitoring and studying these assets will contribute to a deeper understanding of their roles and implications for global financial systems.

As financial markets evolve, it is crucial to adapt to changing conditions and make informed decisions that drive innovation, stability, and sustainable growth in the future.

CONCLUSION

In conclusion, this study provides valuable insights into the interplay between long memory and bootstrap techniques in virtual and real currency markets, focusing on Bitcoin and the US dollar during the pre/post-COVID era. The findings confirm the presence of long memory in the examined currency markets and underscore the importance of accurate bootstrap utilization for robust analysis. The contrasting behaviors of Bitcoin and the US dollar highlight the distinct nature of these assets and their reactions to market shocks. Moving forward, further research could explore additional factors and extend the analysis to other cryptocurrencies and fiat currencies, deepening our understanding of their dynamics and implications for global financial systems. By monitoring and studying these assets, we can adapt to changing market conditions and make informed decisions that foster innovation, stability, and sustainable growth in the future.

ENDNOTES

1. "P-value plots" are graphical representations used in statistical analysis to visualize p-values resulting from hypothesis tests. They typically display p-values against a test statistic or parameter of interest. "P-value plots" help understand the distribution of p-values, assess the significance of results, and identify potential trends or patterns in the data. They are commonly used to interpret statistical test results and aid decision-making in empirical studies.
2. "Size-power curves" are graphical representations showing the relationship between the significance level (size) and the power of a statistical test. They help assess the test's ability to detect true effects and control for type I error rates. These curves are commonly used in power analysis and sample size determination for experimental design and hypothesis testing.

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