US APR vs EU APR, and the Substitute Tax on Loan Effects on These Formulas

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This paper explores the theory, goals, outcomes and difficulties of consumer finance disclosure process, focusing mainly in the Substitute Tax (especially the Italian one) effects on EU APR vs US APR formula and the compound mechanism intrinsic subsistent in a constant mortgage for additional research and study, and a general technical analysis about IRR and EU/US APR relations.

In particular, this study demonstrates that the supposed equivalence of IRR formula with APR formula is definitively incorrect under many aspects and empiric chases, while the most used constant mortgage has a typical anatocism deriving from compound interest-quota in each installment.

Keywords: (Italian) substitute tax, influence on loanable funds, IRR vs EU APR

IS THE EU APR AN OFFSHOOT OF THE US APR?

The conflict between lenders and debtors is significant: on one hand, debtors borrow from banks and issue debt to raise funds to maintain and grow their business. While some business loans are simple interest loans, other take the form of fully amortized loans, whereby annuity payments are composed of a declining interest portion and a rising principal repayment portion over the life of the loan.

Debtors must invest these funds lend in capital projects that will generate excess cash flows sufficient in amount to provide lenders with their expected rates of return.

On the other hand, lenders expect to earn compound rates of return on their debt and investments held more than a year.

Therefore, the balance of this "conflict" between lenders and debtors depends, particularly, on these factors:

a) Value of the money lend;
b) Compounding;
c) Discounting;
d) Cash flows.

For what is concerning the value of the money lend, we need to keep into account four variables:

i. Present value;
ii. Future value;
iii. Interest rate;
iv. Number of periods.

In fact, these four variables are included in the future value and present value equations. As long as the values for any three of these variables are known, we can solve for the fourth, or unknown, variable.

For what is concerning compounding, this is an arithmetic process whereby an initial value increases or grows at a compound interest rate over time to reach a value in the future. Compound interest involves earning interest on interest in addition to interest on the principal or initial investment.

This mechanism is known also as "anatocism" (from ancient Greek ἀνατοκισμός *anatokismós*, coming from ανα- "*over*, *again*" and τοκισμός "*usury*").

For what is concerning discounting, this is to determine present values through an arithmetic process whereby a future value decreases at a compound interest rate over time to reach a present value.

For what is concerning the interest rate for an ordinary annuity, it can be found by entering the future value, the periodic payments amount, and the number of periods, considering that an annuity is a series of equal payments (or receipts) that occur over a number of time periods. Therefore, an ordinary annuity would correspond to when equal payments (or receipts) occur at the end of each period.

When more frequent than annual compounding (or discounting) occurs, the stated annual interest rate must be divided by the number of compounding periods within one year to get the rate per period, and the total number of periods calculated multiplying the number of years by the compounding periods within a year.

To complete the basilar definitions, we can write that the present value (PV) is the value today of a saving amount or investment, while the future value (FV) is the value of a saving amount or an investment at a specified time or date in the future, as per the most common definitions taught in schools.

After this important clarification, we can write the relation among the present value, the future value, and compounding interest.

In fact, we must clear that compounding is an arithmetic process whereby an initial value increases or grows at a compound interest rate over time to reach a value in the future. Thus, the compounding concept also can be expressed in equation form as,

$$FV_n = PV\ (1+r)^n$$

Where FV is the future value, PV is the present value, r is the interest rate, and n is the number of periods in years. Of course, this is just an equation-based process for finding a future value when compounding is involved, but there are other methods that could be used to find a future value, and the PV itself. For example, the two equations below give the present value as well:

$$PV = \frac{C_n}{(1+r)^n} \tag{1}$$

$$PV = \frac{C}{r}\ [1-\frac{1}{(1+r)^n}] \tag{2}$$

The equation 1 calculates the present value of a single cash flow, C_n, received at the end of the period. Instead, the equation 2 calculates the present value of an annuity with equal cash flows, C, paid at the end of each period for n periods. In both the equations, r is the interest rate used for discounting and it must be the effective rate.

For the goal of this paper, it is sufficient the above evidenced formula, to show the tight relation among future value, present value and compounding interest mechanism. In fact, after evidenced this relation, it clearly appears that APR, in general, recalls all the four variables above mentioned (PV, FV, interest rate and number of periods) and expressed through discount factors mechanism, or, better, through discount factors functions. For these reasons, the principle that sees APR well matching with savings concepts finds confirm, and this is why the US APR can be better defined like an APY (Annual Percentage Yield).

Correctly, the effective interest rate corresponds to be the compound interest rate equal to the number of period in interim, or fraction of a year, for each discounted cash flow [*rectus*: receipt], built under the present value and the future value factors. Therefore, US APR appears to be equivalent to an APY, and this evidence is perfectly suitable to the US laws regarding usury and consumer credit regulations, for whom anatocism (*rectus*: compound interest) is rightful and lawful.

While in the U.S.A., and, in some parts, even in the United Kingdom as well, anatocism is legitimated, in many EU Countries compound interest rate mechanism is forbidden, *id est* in Italy (*see* article 1283 of the Civil Code). In other words, what in the U.S. is lawful, in many European countries, it would be illegal.

Beside these evidences, we can pointing out all the effective discrepancies between the US APR and the EU APR, are very important, but if we must consider, primarily, the importance of the main principle on top of all this paper. In fact, the whole truth about the cost of credit really is not meaningfully available unless it states in terms that consumers in our society can understand. If the Legislator does not achieve this goal, everything we discuss here is useless; without easy knowledge of the APR as unit price for credit, it is virtually impossible for the ordinary person to shop for the best credit buy.

By the middle 80's, the European Union started to prepare all the EU country members to merge in a unique and common politic and economic policy, prospecting a harmonization of the laws, of the regulations and of the administration systems. This aim represents another aspect about the disclosure of the credit cost, because EU looked for a clear definition of finance charge, upon which an Annual Percentage Rate is calculated, and in a comprehensive and uniform way. It needs to be uniform to permit a meaningful comparison between alternative sources of credit, and in order to convey the true cost of lending.

One of these aspects was about the consumer credit legislation, in order to have a common rightful rule valid for the EU countries, legally accepted by everyone, considering their internal and personal legislation.

It is here important to underline that this last mentioned aspect focused on consumer credit, and not on savings or deposits. This evidence remembers what previously evidenced in this positioning paper, about the two different sides existing between lender/creditor and borrower/debtors positions. Therefore, EU required a valuation of a financial contract, or offer, only for loans and credit, without considering any kind of aspects about savings or deposits. The original idea was to emulate the US APR formula, but "purified" of those compound interest mechanisms that many EU countries would not allow as per their internal legislation and culture.

That is why the EU valuation kept the US acronym of APR, which is – beside all – the British-English name for the annual interest rate.

While the U.S.A. demanded a valuation for both (loans and savings), given by the American-British name APR, Annual Percentage Rate, the same word and abbreviation had been used in the United Kingdom and in the European Union, but coming to be differently intended.

In fact, in the US, the APR has not just a different meaning then the UK APR and the EU APR, but even different meaning from French APR and, probably, from other states in EU, before they had to change to the APR calculation considered the correct one by the European Commission by 1995. Therefore, the first difference between US APR and EU APR comes from the valuation of the Annual Percentage Rate. In other words, the US and the EU APR have completely different valuations from each other, and so **the EU APR is definitely not an offshoot of the US APR, but EU APR cannot exclude the US APR from its awareness**.

Specifically, in the US, the APR corresponds to be better like an APY (Annual Percentage Yield), rather than an APR, while, in the EU, the Annual Percentage Rate corresponds to be better like a generic EIR (Effective Interest Rate), rather than an APR, as commonly described in financial literacy. The EU Commission chose the British name for the effective interest rate, that it was not a correct choice as per some experts, because the UK APR does not correspond to the EU APR, and not even to the US APR, since none in Britain even made any mention about it in their British law.

Another main difference between US APR and EU APR comes from the Legislator. In fact, if U.S. is just one state, the European Union is a commission made by different and independent states, one from each other's, and this create a substantial main problem, because any EU Commission action or position have to be commonly agreed and accepted by every single European country member of the European Union.

The third difference between US APR and EU APR is the definition of the valuation, especially regarding the definitions of the financial and economic terms, in a lawsuit too.

This third difference derives by the aim of the measure chosen by the EU Commission to establish and to ensure harmonisation of the consumer credit internal market in the European Union, on both legal basis and mathematics. This third aspect is fundamental: in fact, several EU countries have different definitions for the same element, and same definition for different terms, such as for the use of the term “interest” and “interest rate”, or, for example, for the ways of interest-bearing money calculations.

This evidence has, consequently, direct effect even on the various different credit agreements, and the ways of calculating rates and costs, that might differ from one style of credit to another and from one UE Member State to another. For example, “interest” means the income from the use of money, worldwide, while “loan” may also be seen as a savings deposit made by the lender, and vice versa a savings deposit may be seen as a loan given by the saver. The basic difference between “loan” and “savings”, as commonly used, lies in the purpose of the transactions: the debt representing the loan shall decrease with time; the deposit representing savings shall grow with time.

In any case, “interest” is a reward for the lender. Since this reward is supplied by the borrower, interest has the implied subsidiary function and meaning “cost” and the effect 'burden' for the borrower, too. The “interest rate” is the same for both of them.

All the three highlighted before aspects find their influence in EU when the EU Commission, with its panel, had to merge the different ways of calculation ad consideration of the legal terms related to the characteristics of interest-bearing money, and the connection between interest rate and time with money, and, again, among “interest rate”, “reference period” and “period rate” mechanism.

In interest calculus, for its correct calculation, it becomes extremely important to clarify all the factors description of the interest formula, because a hypothetical wrong representation and/or misleading of these financial concepts, could determine different results coming from further calculations, due to the wrong translation of the math formula, in its lawsuit.

As per common law, the reference period is, generally, a year, and starting by this financial axiom, all the interim payments and/or over a year payments must maintain the effective interest rate to fulfil the known tenet “*pacta sunt servanta*”.

To satisfy the above-evidenced condition, that is the interest rate with different reference periods, the problem to solve is the conversion of interest rate using the exponential interest factors, which needs of a mathematical approach to interest-bearing money uses. This exponential growth of capital at any moment relative to itself leads to an interest-bearing capital increase with a growth-constant interest rate for a time span.

The period for which interest is calculated is the “interest period”, that is independent of the reference period in the interest rate, and it represents the interval between two claims [*rectus*: debits] and associated balances.

This scenario creates the preamble to the disbelief in the theory of existence of anatocism in compound interest factors belonging to this exponential growth of capital due by the interest-bearing money mechanism that has the following general characteristics:

a) An interest rate includes on it all the elements of income, without exceptions, regardless the different natures of the incomes. This global characteristic, worldwide, represents the fundamental principle by which even the default interest and the penalties, in general, must be included and considered as element of income, belonging to “interest rate” class;
b) An interest rate, as property, constitutes in an average growth rate, deriving from the exponential growth of money, in the time span, that differs from a reference period to another, in an interest rate;
c) The previous mentioned point *a)* and *b)* of this list, together, determine the assignment of an interest rate to a capital flow as a valuation that converts all elements of income into an average growth rate;
d) This natural exponential capital growth derives from the capital flows function, due to deposits or withdrawals that, at any time, can alter the interest-bearing capital;

e) Since interest, generally, is the income from the use of money, and it is connected with cash flows [*rectus*: capital flows], interest becomes, commonly, the result of an incessant and exponential growth of money between deposits and withdrawals.

After cleared these further properties connected to interest-bearing capital, interest rate and reference periods, it is relevant to evidence, once again, the "conflict" insurgent between the lender and the borrower, in respect of their own interests, opposing the investment and cost principles, in order to merge into a common set of rules for the consumer credit.

To null this ideological "conflict", it is necessary to proceed, mathematically, to certain mandatory steps, considering some specific financial and actuarial aspects. The first mathematical formula needed is an equation able to consider every transaction involving claims, counterclaims and capital interest.

This equation, correctly, must represents the balance at the end of the life span of the loan of a sequence of capital flows with constant interest rate consisting in in claims and counterclaims until the end of the loan life span.

To let the reader fully comprehension of this matter, we can expressly refer to the study of Prof. Robert Seckelmann ("*Final report on tender n° XXIV/96/U6/21 SECKELMANN, R., Methods of calculation, in the European Economic Area, of the annual percentage rate of charge, Final Report 31 October 1995, Contract n° AO 2600/94/00101*").

In this study, the above-evidenced equation comes to be the equation of the future value that is the following:

$$B(t_f) = \sum_{v=1}^{n} C_v \cdot q_A^{\frac{(t_f - t_v)}{A}} - \sum_{\mu=1}^{m} z_\mu \cdot q_A^{\frac{(t_f - t_\mu)}{A}}$$

where C_v is a claim at the time t_v and Z_μ is a counterclaim at the time t_μ and $B(t_f)$ is the balance at the end of the life span. t_f is the time at the ending transaction of the life span, while q_A is the interest factor for a period A that stands for "year" (from Latin "*annus*", "for year"), and C is the capital lent.

If the balance B derives from a loan in which there is only one claim, and the capital lent C at the time $t=t_0$, we will have the formula of the future value becoming like the following simplified equation:

$$B(t_f) = C(t_0) \cdot q_A^{\frac{(t_f - t_0)}{A}} - \sum_{\mu=1}^{m} Z_\mu \cdot q_A^{\frac{(t_f - t\mu)}{A}}$$

This last equation shows the balance as isolated quantity at any periods (t_m) of the life span of the loan, when all the other information are available, so to determine the balance through, basically, an iteration process of calculus.

This iteration process considers both claims and counterclaims that becomes the representation of the claims (deposits C) and counterclaims (withdrawals Z) that influence the interest rate, beside the balance itself, making it positive or negative, up to the number and the quantity of deposits and quantity of withdrawals. In such a case, we might obtain several mathematical solutions that could not state an unequivocal real interest rate, resulting in many series of complex numbers.

This fact brings to the second step that goes including other factors, such as the "time value" V(t) of the capital flow, that is the value at the point in time t of all claims (or payments) constituting the capital flow, compounded with interest from the time each claim is made to the time of valuation.

The third step is the "cash value", *a.k.a.* "present value", that is the time value of a planned sequence of claims (or payments) at a time of the life span of a loan. We need to observe that "time value" and "cash value" stress the importance of the cash flows dynamics, and this is the reason why this leads to the cash value formula as follows:

$$V(t_0) = \sum_{v=1}^{n} \frac{C_v}{q_A^{\frac{(t_v - t_0)}{A}}} - \sum_{\mu=1}^{m} \frac{Z_\mu}{q_A^{\frac{(t_\mu - t_0)}{A}}}$$

Assuming that a balance $B(t_f) <> 0$ is paid at the time t_f leads to a new balance $B(t_f) = 0$ and a cash value $V(t_0) = 0$, then one has $t_m=t_f$ or $t_n=t_f$. With this assumption, the cash value equation turns to become a "balanced cash value form" equation, as follows:

$$\sum_{v=1}^{n} \frac{C_v}{q_A^{\frac{(t_v - t_0)}{A}}} = \sum_{\mu=1}^{m} \frac{Z_\mu}{q_A^{\frac{(t_\mu - t_0)}{A}}}$$

This last formula comes to be equivalent to the other previous two and can be converted into each other, because in all of them the reference period cam be changed from A (year) to any period P, with q_p instead of $q_{A.}$

Furthermore, $(t_{m-}\ t_0)/P$ can be changed with $(t_{m-}\ t_0)/A$ and $q_A=q_p^{(A/P)}$, considering that all the equations can be written with $(1+y_A)$ instead of q_A.

Thus, the balanced cash value equation better redeem the equality of all claims and counterclaims, including a final payment downgraded with the interest rate, and this operation let the lender and/or the borrower to isolate the common value of all payments at a specific time.

By this principle, we can assure that the interest rate becomes the force that achieves this equality, but if only a capital amount C is paid out at a time t_0, the balanced cash value equation takes on a simpler form, as follows:

$$C(t_0) = \sum_{\mu=1}^{m} \frac{Z_\mu}{\frac{(t_\mu - t_0)}{A}}$$

Many experts consider this last equation plausible, although not everybody think that it reflects exactly debt or saving development.

This increased the problems to find a mutual and common "translation" of an APR formula valid for all the European Union countries is the European Commission itself. In fact, The EU Directive defines, in the Article 1.2.e, that the "*"Annual percentage rate of charge" means the total cost of the credit to the consumer, expressed as an annual percentage of the amount of the credit granted* (…)".

This definition is wrong, as correctly, Prof. Seckelmann evidenced too in his study here mostly in recall ("*Final report on tender n° XXIV/96/U6/21 SECKELMANN, R., Methods of calculation, in the European Economic Area, of the annual percentage rate of charge, Final Report 31 October 1995, Contract n° AO 2600/94/00101*").

Let us look to the EU APR formula as actually in use, especially, by 1995:

$$\sum_{K=1}^{K=m} \frac{A_K}{(1+i)^{2}K} = \sum_{K'=1}^{K'=m'} \frac{A'_{K'}}{(1+i)^{t}K'}$$

where

K is the number identifying a particular advance of credit

K' is the number identifying a particular instalment

A_k is the amount of advance K

A'_k is the amount of instalment K'

∑ represents the sum of all terms indicated
m is the number of advances of credit
m' is the total number of instalments
t_k is the interval, expressed in years, between the relevant date and the date of the second advance and those of any subsequent advances numbers three to m
$t_{k'}$ is the interval, expressed in years, between the relevant date and the dates of instalments numbered one to m'

In theory, this equation can be equivalent to the equation used for a monthly instalment loan as

$$s = \sum_{k=1}^{n} \frac{d}{(1+r)^k} = \frac{d - d(r+1)^{-n}}{r}$$

where

s is the principal
d is the periodic repayment
r is the periodic rate
n is the number of periods

$s = \frac{d-d(r+1)^{-n}}{r}$ cannot be tidied up for r that is resolved through or an iterative calculus or a graph process. This formula can be expressed in years as follows:

$$\sum_{k=1}^{k=m} \frac{A}{(1+i)^{\frac{k}{12}}}$$

where

A is the regular monthly payment
K is the intervals, normally expressed in months

Therefore, this general case of summation is possible to convert to a closed form by induction as follows:

$$\frac{A}{(1+APR)^{\frac{n}{p}}} + \frac{A}{(1+APR)^{\frac{n+1}{p}}} + \cdots + \frac{A}{(1+APR)^{\frac{m}{p}}} =$$

$$\sum_{k=n}^{m} \frac{A}{(1+APR)^{\frac{k}{p}}} = -\frac{A\left((APR+1)^{\frac{1-n}{p}} - (APR+1)^{-\frac{m}{p}}\right)}{1-(APR+1)^{\frac{1}{p}}}$$

Let make an empiric example:
Loan amount = € 30,000.00
Initial admin fee and other upfront costs: € 250.00
Duration of loan repayments: 3 years (36 monthly instalments)
Interest charge = 12.00%
The monthly payment results to be € 1,333.33 determining an APR of 24.13%
The monthly payment results from the following formula:

$$p = \frac{P_0 \cdot r \cdot (1+r)^n}{(1+r)^n - 1}$$

where

p is the payment made each period
P_0 is the initial principal
r is the percentage rate used each payment
n is the number of payments

Therefore, we process the following calculus:

$$p = \frac{30{,}000.00 * 0.12 * \left(1 + (0.12)\right)^{36}}{(1 + 0.12)^{36} - 1}$$
$$= \frac{212{,}888.06614296}{58.1355739289} = 3{,}661.92$$

We need to process the following calculus:
r = (1 + 24.13/100) ^ (1/12) -1 = 0.0181764855
d = 1,333.33
n = 36

s = (d – d (1 + r) ^-n) / r = 1,333.33 – 1,333.33 (1 + 0.0181764855) ^-36) /0.0181764855
s = 35,001.83

So to say
i = APR = 0.2413
A = 1,333.33
m = 36

$$\sum_{k=1}^{k=m} \frac{A}{(1+i)^{\frac{k}{12}}}$$ That must be equally to 35,001.83 in our example.

Therefore, the full equations is the following:

$$30{,}000.00 = 250.00 - \frac{\mathrm{A}\,(1 - (\mathrm{APR} + 1)^{-\frac{m}{12}}}{1 - (\mathrm{APR} + 1)^{\frac{1}{12}}}$$

where
A = 1,333.33
m = 36

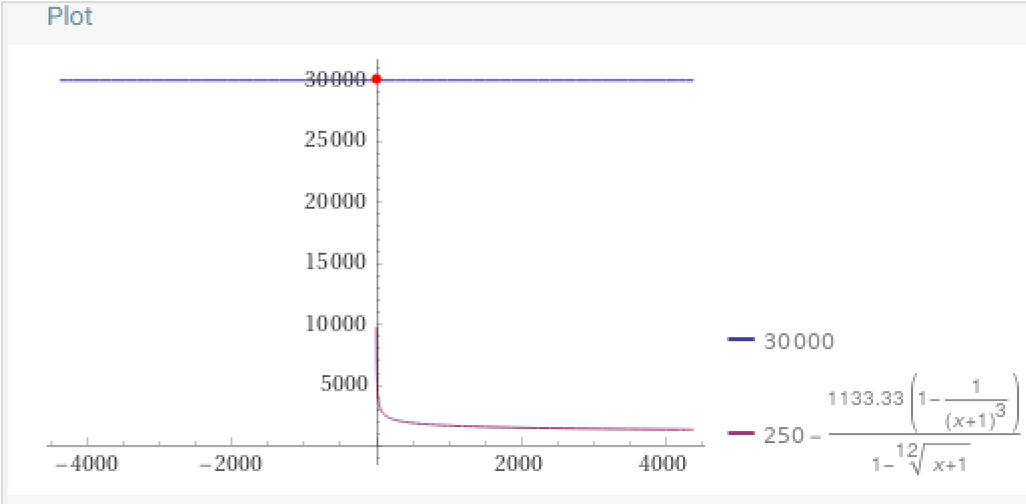
Solving with Wolfram Alpha

Input interpretation
$30000 = 250 - \frac{1133.33\left(1 - (1+x)^{-36/12}\right)}{1 - \sqrt[12]{1+x}}$
Result
$30000 = 250 - \frac{1133.33\left(1 - \frac{1}{(x+1)^3}\right)}{1 - \sqrt[12]{x+1}}$

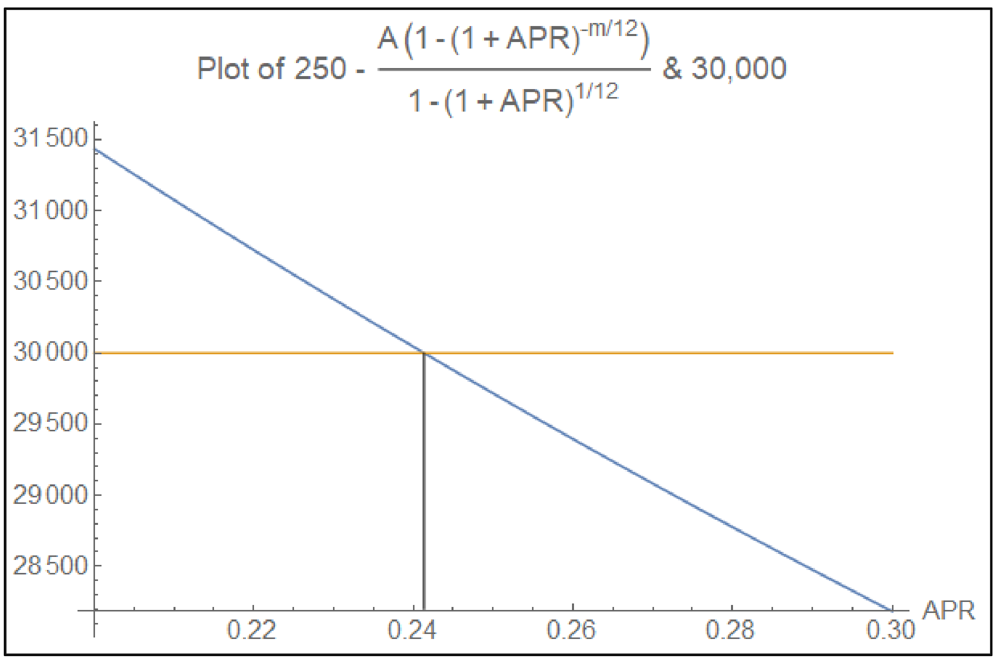
Giving this numerical solution, that confirms the APR at 24.13% as follows:

Numerical solution
$X = 0.241346599336412\ldots$

In addition, we obtain the following plot:



or, differently, finding APR by plotting



The two lines (blue and yellow) intersect at APR = 0.241347

Returning to the very first and simplest formula with the updated figures:

r = (1 + 24.13466/100) ^ (1/12) -1 = 0.0181796701

d = 1,333.33

n = 36

s = (d – d (1 + r) ^-n) / r = 1,333.33 – 1,333.33 (1 + 0.0181796701) ^-36) /0.0181796701
s = 29,750.00

which is the expected result.

What we can assume from this empiric example is that the bank rules to calculate the interest is not corresponding to the EU APR rules to calculate the same interests. This divergence comes from the EU APR rules that do not expect fees and the interest is added to the capital only after each completed year. This, basically, means that there should not be anatocism [compound interest] for the first 11 months.

The second assumption is that the upfront cost of € 250.00 is not a pay down payment, but, instead, just interest. In fact, let consider another general example as follows:

Mortgage loan $ 100,000.00; Duration: 30 years.

Let consider, as example, the possibility to choose either to pay down payment, or not, for this hypothetical mortgage.

For example, if not choosing for the pay down payment, the annual interest rate would be 12.00%; if, instead, choosing the offer of paying $ 2,000.00 (2.00% discount point off initial $ 100,000.00), the offer would be the 11.50% annual interest rate.

Case 1

No down payment, annual interest rate is 12.00%, therefore, monthly is 12.00%/12 = 1.00%. Compounding monthly:

Effective annual rate = $(1.01)^{12}$ - 1 = 0.1268, which is = 12.68% because of monthly compounding

Case 2

Down payment = $ 2,000.00 (so, the debt is $ 100,000.00 - $ 2,000.00 = $ 98,000.00)

Interest rate at 11.50%, therefore, monthly should be 11.50%/12 = 0.9583%.

In this case, using finance calculator, monthly payment would be $ 990.29 as follows:

Monthly Payments

$ 990.29

Total Principal Paid	$100,000
Total Interest Paid	$256,504.92

Now, compute the effective annual rate if you pay 2.00 discount points and let us assume that the amount of the loan is still $ 100,000.00. If you pay 2.00 points, instead of receiving $ 100,000.00 it comes to receive only $ 98,000.00 ($ 100,000.00 - $ 2,000.00). The payment is computed on the $ 100,000.00 but at a lower interest rate. Using a financial calculator, the monthly payment is $ 990.29 and the monthly rate is 0.9804% that does not correspond to the 0.9583% before evidenced.

The effective annual rate after compounding is $(1.00984)^{12}$ - 1 = 0.1242, which is = 12.42% as shown here below in the following table:

Year of Prepayment	Effective Rate of Interest (%)	Year of Prepayment	Effective Rate of Interest (%)
1	14.54	6	12.65
2	13.40	7	12.60
3	13.02	10	12.52
4	12.84	15	12.45
5	12.73	30	12.42

It is important to evidence that the 2.00 points for which it pays off $ 2,000.00 it is the buy of the lower interest rate and they are not a down payment, because the debtor still owes the full amount.

For this reason, the $ 2,000.00 is straight interest and if the debtor payoffs the loan before the breakeven point, the interest rate results very high and it will be, anyway, larger than 11.50%.

On another hand, it is possible this second scenario, taking the standard loan equation:

$s = (d - d\ (1 + r)^{-n})/r$

where

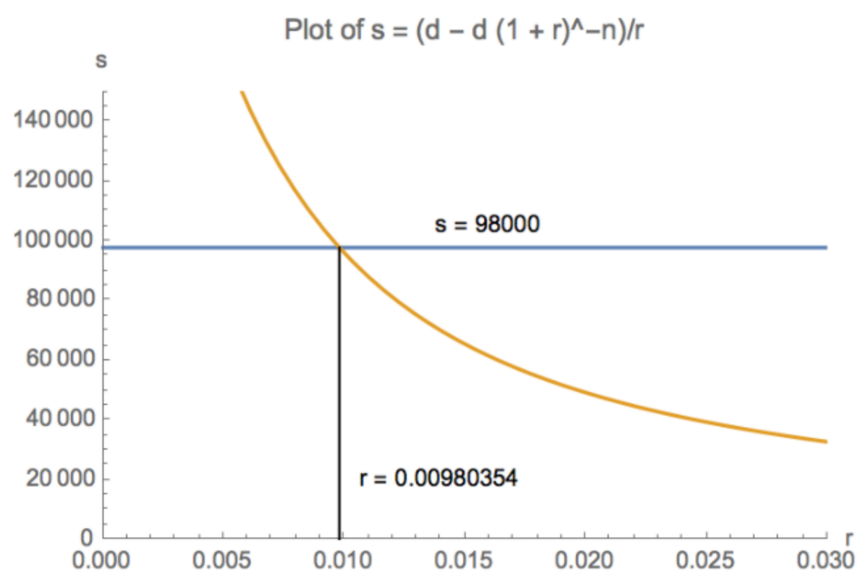
- s is the loan principal
- d is the periodic payment
- r is the periodic interest rate
- n is the number of periods

Let

- s = 100000
- r = 0.115/12 = 0.00958333
- n = 30*12 = 360

$d = (r\ (1 + r)^n\ s)/((1 + r)\ n - 1) = 990.291$

Now setting s = 98000, with d = 990.291 solve for r
r = 0.980354 %



THE EU APR AND THE SUBSTITUTE TAX IN ITALY: A PROBLEM TO BE RESOLVED

Before any following evaluation, it is important to clarify that what we will discuss hereafter focuses on the Substitute Tax applied in Italy, although this, in theory, could find expression in the entire EU.

In particular, the articles 15 and 17 of the D.P.R. ("Decreto del Presidente della Repubblica"- "*D.P.R.*") n. 601/1973 rule the Italian Substitute Tax ("*Imposta Sostitutiva*") that is the tax that banks operating in Italy own to the Italian State, coming from the credit lending.

The above-mentioned articles are the following:

TITOLO IV
Agevolazioni per il settore del credito

Art. 15 - Operazioni di credito a medio e lungo termine
[1] Le operazioni relative ai finanziamenti a medio e lungo termine e tutti i provvedimenti, atti, contratti e formalità inerenti alle operazioni medesime, alla loro esecuzione, modificazione ed estinzione, alle garanzie di qualunque tipo da chiunque e in qualsiasi momento prestate e alle loro eventuali surroghe, sostituzioni, postergazioni, frazionamenti e cancellazioni anche parziali, ivi comprese le cessioni di credito stipulate in relazione a tali finanziamenti, effettuate da aziende e istituti di credito e da loro sezioni o gestioni che esercitano, in conformità a disposizioni legislative, statutarie o amministrative, il credito a medio e lungo termine, sono esenti dall'imposta di registro, dall'imposta di bollo, dalle imposte ipotecarie e catastali e dalle tasse sulle concessioni governative.
[2] In deroga al precedente comma, gli atti giudiziari relativi alle operazioni ivi indicate sono soggetti alle suddette imposte secondo il regime ordinario e le cambiali emesse in relazione alle operazioni stesse sono soggette all'imposta di bollo di lire 100 per ogni milione o frazione di milione.
[3] Agli effetti di quest'articolo si considerano a medio e lungo termine le operazioni di finanziamento la cui durata contrattuale sia stabilita in più di diciotto mesi.

Art. 17 - Imposta sostitutiva
[1] Gli enti che effettuano le operazioni indicate negli artt. 15 e 16 sono tenuti a corrispondere, in luogo delle imposte di registro, di bollo, ipotecarie e catastali e delle tasse sulle concessioni governative, una imposta sostitutiva.
[2] Per gli istituti di credito costituiti ai sensi dei decreti-legge 2 settembre 1919, n. 1627, 15 dicembre 1923, n. 3148 [1] [2] , e 20 maggio 1924, n. 731, degli artt. 14 e 18 del decreto-legge 29 luglio 1927, n. 1509 [1] [3] , dei decreto-legge 13 novembre 1931, n. 1398 [1] [2] e 2 giugno 1946, n. 491 [1] [2] , del D.Lgs. 15 dicembre 1947, n. 1418 [1] [2] , della legge 22 giugno 1950, n. 445 [1] [2] , dell'art. 17 della legge 25 luglio 1952, n. 949 [1] [2] , e delle leggi 13 marzo 1953, n. 208 [1] [2] ,11 aprile 1953, n. 298 [1] [2] , e 31 luglio 1957,

n. 742 [1] [2] , nonché per gli istituti autorizzati all'esercizio del credito fondiario in base al testo unico 16 luglio 1905, n. 646 [1] [4] , per gli istituti soggetti alla disciplina di cui al D.Lgs. 23 agosto 1946, n. 370 [1] [5] , per le sezioni autonome opere pubbliche di cui alle leggi 6 marzo 1950, n. 108, e11 marzo 1958, n. 238, e per la sezione interventi speciali di cui alle leggi 18 dicembre 1961, n. 1470, e 18 maggio 1973, n. 274, l'imposta sostitutiva comprende anche le imposte di bollo e di registro, le imposte ipotecarie e catastali e le tasse sulle concessioni governative sugli altri atti ed operazioni che detti istituti pongono in essere per il loro funzionamento e per lo svolgimento della loro attività, in conformità alle norme legislative o agli statuti che li reggono, salvo quanto stabilito nel secondo comma dell'art. 15 per gli atti giudiziari e le cambiali.

(1) Per l'abrogazione di questa disposizione, con effetto 1° gennaio 1994, vedil'art. 161, comma1, D.Lgs. 1° settembre 1993, n. 385.

(2) Vediora l'art. 10, D.Lgs. 1° settembre 1993, n. 385.

(3) Vediora l'art. 43, D.Lgs. 1° settembre 1993, n. 385.

(4) Vediora l'art. 38, D.Lgs. 1° settembre 1993, n. 385.

(5) Vediora gli artt. 51 e ss., D.Lgs. 1° settembre 1993, n. 385.

In Italy, the Substitute Tax is ruled even by the Italian Law Decree n. 145/2013 (*a.k.a.* "*Decreto Destinazione Italia*"), and the Italian Law Decree n. 91/2014 (*a.k.a.* "*Decreto Competitività*").

On top of these decrees, we must mention also the article n. 8, paragraph n. 2 of the Law n. 212 of July the 27th 2000 that allows the banks operating in Italy to shift the Substitute Tax to the borrowers.

In fact, the switch to the borrowers of this tax is very common - not only in Italy - and it is valid and fully legit: the banks ask the borrowers to pay for their Substitute Tax on loans, often deducting this tax form the money lent at the time of the transfer of the credit form the bank to the debtors.

This tax is due by the banks in payment to the State, every 30th of April and 30th of October of every year, splitting this payment in 45% of the 95% of the total of the Substitute Tax due, every 30th of April, and the standing balance of this tax, every 30th of October.

Another evidence is that the Substitute Tax becomes part of the loan - in particular - it becomes part of the financed capital of the credit lent.

We must even say that the banks pay directly the Substitute Tax, and not the borrowers. This means that very likely banks finance their Substitute Tax to the borrowers, since banks can manage directly its payments, without involving, at any cases, the borrowers about the handling of this tax.

In other words, this financial operation provides three main extra advantages to the banks:

1) The first advantage is to cash interest accrued on this tax, because it becomes part of the amortization plan;
2) The second advantage is to cash compounded interest accrued on this tax, because of the very nature of the constant mortgage amortization plan method;
3) The third advantage is related to the time "t", both as "t_0", both as "t_m", because the banks cash this tax from the borrower at the time of the contract [*rectus*: capital transfer] and not when this tax is due to the State from the banks.

The most common aliquot of this tax is 0.25% of the credit lent, so, for this reason, we will consider this standard rate for the analysis of this study.

First, we will use the same example before used in order to better show the influence of the Substitute Tax on APR if capitalized in the loan that we will recap here as follows:

Loan amount = € 30,000.00
Initial admin fee and other upfront costs: € 250.00
Duration of loan repayments: 3 years (36 monthly instalments)
Interest charge = 12.00%
APR 24.13%

At this point, we should "clean" from the € 250.00 the Substitute Tax to understand the influence of it, in percentage points, in the APR.

We should consider that the Substitute Tax is normally included among the upfront costs, as – in theory – the EU APR formula considers standardly. Therefore, in our example, we should assume that the Substitute Tax is already included in the € 250.00 debited by the lender to the borrower and deducted from the effective capital lent and transferred to the borrower at the time of the contract, as follows:

€ 30,000.00 - € 250.00 = € 29,250.00 is the effective capital transferred to the borrower.

€ 30,000.00 * 0.25% = € 75.00 is the Substitute Tax amount, included in the € 250.00 upfront costs applied to the loan at the time of the contract.

€ 250.00 - € 75.00 = € 175.00

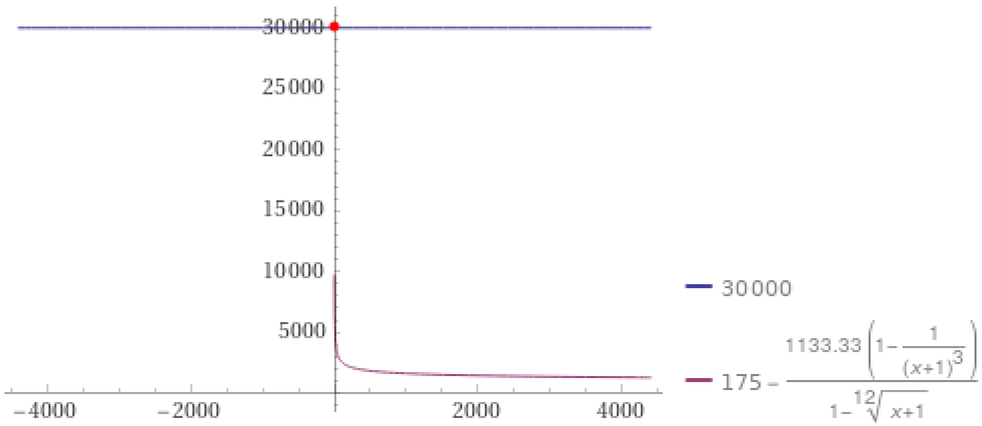
Input interpretation

$$30000 = 175 - \frac{1133.33\left(1-(1+x)^{-36/12}\right)}{1-\sqrt[12]{1+x}}$$

Result

$$30000 = 175 - \frac{1133.33\left(1-\frac{1}{(x+1)^3}\right)}{1-\sqrt[12]{x+1}}$$

Plot



Let make another example, this time on a general mortgage.

Kind of mortgage: constant

Loan amount: $ 950,000.00

Initial admin fee and other upfront costs: $ 10,485.00 deducted from the money transfer

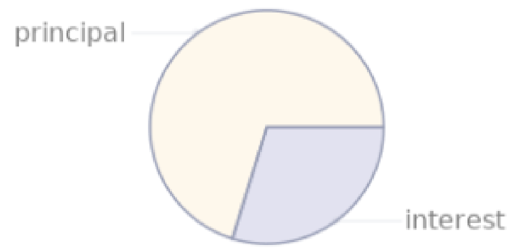
Duration of loan repayments: 15 years (180 monthly instalments)

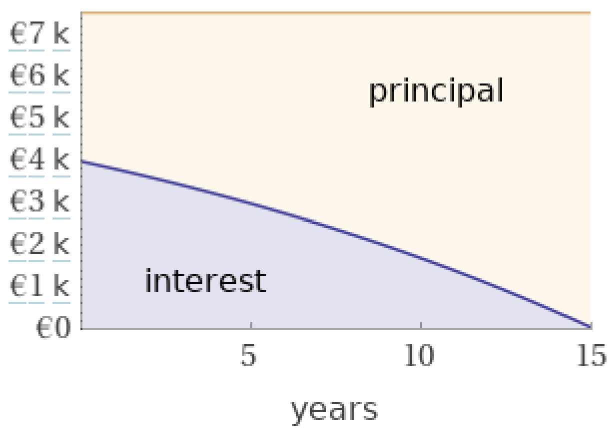
Nominal annual interest rate: 5.007% fixed-rate mortgage, 5.124% effective annual interest rate

In the $ 10,485.00 there is included even the Substitute Tax of 0.25% of the credit that corresponds to be $ 2,375.00 ($ 950,000.00 * 0.25%). Therefore, we should compare the APR results coming from the applying of the Substitute Tax with and without it.

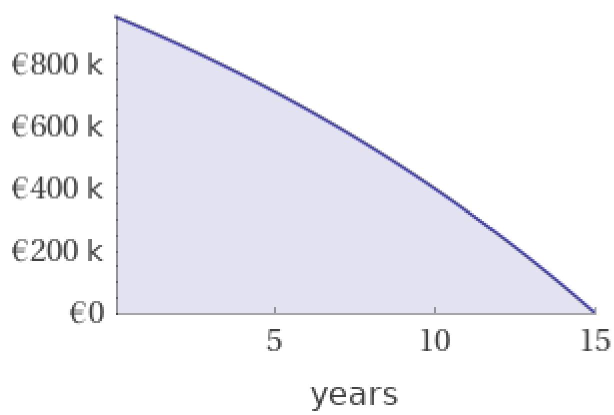
monthly payment	€7516
effective interest rate	5.124%

principal paid	€950 000
total interest paid	€402 881
total payments	€1.353 million





(monthly payment breakdown)



(remaining balance)

Payments table

year	monthly payment	ending balance	yearly principal paid	yearly interest paid
1	€7516	€906383	€43617	€46575
2	€7516	€860530	€45852	€44340
3	€7516	€812329	€48201	€41991
4	€7516	€761658	€50671	€39521
5	€7516	€708390	€53267	€36925
6	€7516	€652394	€55996	€34196
7	€7516	€593529	€58865	€31327
8	€7516	€531647	€61881	€28311
9	€7516	€466595	€65052	€25140
10	€7516	€398211	€68385	€21807
11	€7516	€326322	€71889	€18304
12	€7516	€250750	€75572	€14620
13	€7516	€171307	€79444	€10748
14	€7516	€87793	€83514	€6678
15	€7516	€0	€87793	€2399

$$PV = \frac{PMT\left(1-(1+i)^{-n}\right)}{i}$$

PV	present value
i	interest rate
n	number of periods
PMT	periodic payment

Convert known variables into appropriate units using the following:

The relevant equation that relates present value (PV), interest rate (*i*), number of periods (n) and periodic payment (PMT) is:

$$PV = \frac{PMT(1-(1+i)^{-n})}{i}$$

Substitute known variables into the equation:

Separate the numerical part, $\frac{8689.99959(1-(1+0.05007)^{-180})}{0.05007}$, from the unit part, US dollars:

Evaluate $\frac{8689.99959(1-(1+0.05007)^{-180})}{0.05007}$:

PV = 173530.7 US dollars

Convert 173530.7 US dollars into euros using the following:

1.00 US dollars = 0.86 euros:

Answer:

PV = 150 087.09 euros

That is our requested answer.

Case 1: With the Substitute Tax Deducted From the Loan (YES CAPITALIZATION OF TAX)

In this case, using a finance calculator, monthly payment would be $ 7,516.00 as follows:

Monthly Payments

$ 7,516.00

Total Principal Paid	$950,000
Total Interest Paid	$402,880.73

The amortization plan would be the following, if in monthly compound interest rate:

Payment Date	Payment	Principal	Interest	Total Interest	Balance
Nov-21	$7,516.00	$3.552.13	$3.963.88	$3.963.88	$946.447.87
Dec-21	$7,516.00	$3.566.95	$3,949.05	$7.912.93	$942,880.92
Jau 2022	$7,516.00	$3,581.83	$3,934.17	$11.847.10	$939,299.09
Feb-22	$7,516.00	$3.596.78	$3,919.23	$15.766.32	$935,702.31
Mar-22	$7.516.00	$3.611.79	$3,904.22	$19.670.54	$932,090.52
Apr-22	$7,516.00	$3,626.86	$3,889.15	$23.559.69	$928,463.67
May-22	$7.516.00	$3,641.99	$3,874.01	$27.433.71	$924,821.68

Jun-22	$7,516.00	$3.657.19	$3.858.82	$31,292.52	$921.164.49
Jul-22	$7,516.00	$3,672.45	$3,843.56	$35.136.08	$917,492.05
Aug-22	$7,516.00	$3,687.77	$3.828.24	$38.964.32	$913,804.28
Sep-22	$7,516.00	$3.703.16	$3,812.85	$42.777.17	$910.101.12
Oct-22	$7,516.00	$3,718.61	$3,797.40	$46.574.56	$906.382.51
Nov-22	$7.516.00	$3,734.12	$3,781.88	$50.356.44	$902.648.39
Dec-22	$7,516.00	$3.749.70	$3,766.30	$54,122.74	$898,898.69
Jan-23	$7,516.00	$3,765.35	$3,750.65	$57,873.40	$895,133.34
Feb-23	$7,516.00	$3,781.06	$3,734.94	$61,608.34	$891,352.28
Mar-23	$7.516.00	$3,796.84	$3,719.17	$65,327.51	$887,555.44
Apr-23	$7,516.00	$3,812.68	$3,703.33	$69,030.84	$883,742.76
May-23	$7,516.00	$3,828.59	$3,687.42	$72,718.25	$879.914.18
Jun-23	$7,516.00	$3,844.56	$3,671.44	$76,389.69	$876,069.61
Jul-23	$7.516.00	$3,860.60	$3,655.40	$80,045.09	$872,209.01
Aug-23	$7.516.00	$3,276.71	$3,639.29	$83,684.39	$868,332.30
Sep-23	$7,516.00	$3.892.89	$3,623.12	$87,307.50	$864,439.41
Oct-23	$7.516.00	$3.909.13	$3.606.87	$90.914.38	$860.530.28
Nov-23	$7,516.00	$3,925.44	$3,590.56	$94,504.94	$856.604.84
Dec-23	$7,516.00	$3,941.82	$3,574.18	$98,079.12	$852,663.02
Jan-24	$7.516.00	$3,958.27	$3,557.74	$101,636.86	$848.704.75
Feb-24	$7,516.00	$3,974.78	$3,541.22	$105,178.08	$844,729.97
Mar-24	$7,516.00	$3,991.37	$3,524.64	$108,702.72	$840,738.60
Apr-24	$7.516.00	$4,008.02	$3.507.98	$112.210.70	$836.730.58
May-24	$7,516.00	$4,024.75	$3,491.26	$115,701.96	$832,705.83
Jun-24	$7,516.00	$4,041.54	$3,474.47	$119,176.42	$828.664.29
Jul-24	$7,516.00	$4,058.40	$3,457.60	$122,634.02	$824.605.89
Aug-24	$7.516.00	$4.075.34	$3.440.67	$126.074.69	$820.530.55
Sep-24	$7.516.00	$4.092.34	$3,423.66	$129.498.35	$816.438.21
Oct-24	$7,516.00	$4.109.42	$3,406.59	$132,904.94	$812,328.80
Nov-24	$7,516.00	$4,126.56	$3,389.44	$136,294.39	$808.202.24
Dec-24	$7,516.00	$4.143.78	$3,372.22	$139,666.61	$804,058.46
Jan-25	$7,516.00	$4,161.07	$3,354.93	$143,021.54	$799,897.39
Feb-25	$7,516.00	$4,178.43	$3,337.57	$146,359.11	$795,718.95
Mar-25	$7,516.00	$4.195.87	$3,320.14	$149,679.25	$791.523.09
Apr-25	$7.516.00	$4,213.37	$3.302.63	$152,981.88	$787.309.71
May-25	$7,516.00	$4,230.95	$3,285.05	$156,266.93	$783,078.76
Jun-25	$7.516.00	$4.248.61	$3,267.40	$159,534.33	$778,830.15
Jul-25	$7,516.00	$4,266.34	$3.249.67	$162,784.00	$774,563.82
Aug-25	$7,516.00	$4.284.14	$3,231.87	$166,015.86	$770,279.68
Sep-25	$7.516.00	$4,302.01	$3,213.99	$169.229.86	$765,977.67
Oct-25	$7,516.00	$4,319.96	$3,196.04	$172,425.90	$761,657.70
Nov-25	$7.516.00	$4,337.99	$3,178.02	$175,603.91	$757,319.72
Dec-25	$7,516.00	$4.356.09	$3,159.92	$178,763.83	$752,963.63

Jan-26	$7,516.00	$4.374.26	$3,141.74	$181,905.57	$748,589.37
Feb-26	$7.516.00	$4,392.51	$3,123.49	$185,029.06	$744,196.85
Mar-26	$7,516.00	$4,410.84	$3.105.16	$188,134.22	$739,786.01
Apr-26	$7.516.00	$4.429.25	$3.086.76	$191,220.98	$735,356.76
May-26	$7,516.00	$4,447.73	$3,068.28	$194,289.26	$730,909.03
Jun-26	$7.516.00	$4,466.29	$3,049.72	$197,338.97	$726,442.75
Jul-26	$7,516.00	$4,484.92	$3,031.08	$200,370.06	$721,957.83
Aug-26	$7,516.00	$4.503.64	$3,012.37	$203.382.43	$717,454.19
Sep-26	$7,516.00	$4,522.43	$1,993.58	$206,376.00	$712,931.76
Oct-26	$7,516.00	$4,541.30	$2,974.71	$209,350.71	$708,390.47
Nov-26	$7,516.00	$4,560.24	$2,955.76	$212,306.47	$703,830.22
Dec-26	$7,516.00	$4.579.27	$2,936.73	$215,243.20	$699,250.95
Jan-27	$7,516.00	$4,598.38	$2,917.62	$218,160.83	$694,652.57
Feb-27	$7,516.00	$4.617.57	$2,898.44	$221,059.26	$690.035.01
Mar-27	$7.516.00	$4,636.83	$2,879.17	$223,938.43	$685,398.17
Apr-27	$7,516.00	$4,656.18	$2,859.82	$226,798.26	$680,741.99
May-27	$7,516.00	$4,675.61	$2,840.40	5229.638.65	$676,066.38
Jun-27	$7,516.00	$4,695.12	$2,820.89	$232,459.54	$671,371.27
Jul-27	$7,516.00	$4,714.71	$2,801.30	$235,260.84	$666,656.56
Aug-27	$7,516.00	$4,734.38	$2,781.62	$238,042.46	$661,922.18
Sep-27	$7,516.00	$4.754.13	$2,761.87	$240.804.33	$657,168.05
Oct-27	$7,516.00	$4.773.97	$2,742.03	$243,546.37	$652,394.08
Nov-27	$7,516.00	$4,793.89	$2,722.11	$246,268.48	$647,600.19
Dec-27	$7,516.00	$4.813.89	$2,702.11	$248,970.59	$642,786.29
Jan-28	$7,516.00	$4.833.98	$2,682.03	$251,652.62	$637,952.32
Feb-28	$7,516.00	$4,854.15	$2,661.86	$254,314.47	$633,098.17
Mar-28	$7,516.00	$4,874.40	$2,641.60	$256,956.08	$628,223.77
Apr-28	$7,516.00	$4.894.74	$2,621.26	$259,577.34	$623,329.03
May-28	$7,516.00	$4,915.16	$2,600.84	$262,178.18	$618,413.86
Jun-28	$7,516.00	$4,935.67	$2.580.33	$264,758.51	$613,478.19
Jul-28	$7,516.00	$4,956.27	$2.559.74	$267,318.25	$608,521.92
Aug-28	$7,516.00	$4,976.95	$2,539.06	$269,857.31	$603,544.98
Sep-28	$7,516.00	$4,997.71	$2,518.29	$272.375.60	$598,547.26
Oct-28	$7,516.00	$5,018.57	$2,497.44	$274,873.04	$593.528.70
Nov-28	$7,516.00	$5.039.51	$2,476.50	$277,349.54	$588,489.19
Dec-28	$7.516.00	$5,060.53	$2,455.47	$279,805.01	$583.428.66
Jan-29	$7.516.00	$5.081.65	$2,434.36	$282.239.36	$578,347.01
Feb-29	$7.516.00	$5,102.85	$2.413.15	$284,652.52	$573.244.16
Mar-29	$7,516.00	$5.124.14	$2.391.86	$287,044.38	5568.120.02
Apr-29	$7,516.00	$5,145.52	$2,370.48	$289,414.86	$562.974.50
May-29	$7,516.00	$5.166.99	$2,349.01	$291.763.87	$557,807.50
Jun-29	$7.516.00	$5.188.55	$2,327.45	5294.091.32	$552.618.95
Jul-29	$7,516.00	$5.210.20	$2,305.80	$296.397.12	$547.408.75

Aug-29	57.516.00	$5.231.94	$2,284.06	$298.681.19	$542,176.81
Sep-29	$7,516.00	$5,253.77	S2.262.23	$300.943.42	$536,923.04
Oct-29	$7,516.00	$5.275.69	$2.240.31	$303.183.73	$531.647.34
Nov-29	$7,516.00	$5.297.71	$2,218.30	$305.402.03	$526,349.64
Dec-29	$7.516.00	$5.319.81	$2.196.19	$307,598.22	$521.029.83
Jau 2030	$7.516.00	$5,342.01	$2.174.00	$309.772.22	$515,687.82
Feb-30	$7.516.00	$5,364.30	$2.151.71	$311.923.93	$510,323.52
Mar-30	$7,516.00	$5,386.68	$2,129.32	$314.053.25	$504,936.85
Apr-30	$7,516.00	$5,409.16	$2,106.85	$316.160.10	$499.527.69
May-30	$7.516.00	$5,431.72	$2.084.28	$318,244.38	$494,095.97
Juu 2030	$7.516.00	$5,454.39	$2.061.62	$320.306.00	$488,641.58
Jul-30	$7,516.00	$5,477.15	$2.038.86	$322,344.85	$483,164.43
Aug-30	$7.516.00	$5.500.00	$2.016.00	$324,360.86	$477.664.43
Sep-30	$7,516.00	$5,522.95	$1.993.05	$326,353.91	$472,141.48
Oct-30	$7,516.00	$5,545.99	$1,970.01	$328,323.92	$466,595.49
Nov-30	$7,316.00	$5,569.13	$1,946.87	$330,270.79	$461,026.35
Dec-30	$7.516.00	$5.592.37	$1,923.63	$332,194.42	$455.433.98
Jau 2031	$7,516.00	$5,615.71	$1,900.30	$334,094.72	$449,818.28
Feb-31	$7.516.00	$5.639.14	$1,876.87	$335.971.59	$444,179.14
Mar-31	$7,516.00	$5.662.67	$1,853.34	$337,824.93	$438.516.47
Apr-31	$7.516.00	$5.686.29	$1.829.71	$339.654.64	$432.830.18
May-31	$7,516.00	$5.710.02	$1.805.98	$341,460.62	$427,120.16
Juu 2031	$7,516.00	$5,733.85	$1,782.16	$343.242.78	$421,386.31
Jul-31	$7,516.00	$5,757.77	$1,758.23	$345,001.01	$415,628.54
Aug-31	$7,516.00	$5.781.79	$1,734.21	$346.735.22	$409.846.75
Sep-31	$7,516.00	$5,805.92	$1,710.09	$348,445.31	$404,040.83
Oct-31	$7.516.00	$5,830.14	$1,685.86	$350,131.17	$398,210.69
Nov-31	$7,516.00	$5,854.47	$1.661.53	$351,792.70	$392,356.22
Dec-31	$7.516.00	$5.878.90	$1,637.11	$353.429.81	$386.477.32
Jau 2032	$7,516.00	$5,903.43	$1,612.58	$355,042.39	$380,573.89
Feb-32	$7,516.00	$5,928.06	$1,587.94	$356,630.33	$374,645.83
Mar-32	$7,516.00	$5,952.79	$1,563.21	$358,193.54	$368,693.04
Apr-32	$7,516.00	$5.977.63	$1.538.37	$359.731.91	$362.715.41
May-32	$7,516.00	$6,002.57	$1,513.43	$361,245.34	$356,712.83
Juu 2032	$7,516.00	$6,027.62	$1,488.38	$362,733.73	$350,685.21
Jul-32	$7,516.00	$6.052.77	$1,463.23	$364.196.96	$344,632.44
Aug-32	$7,516.00	$6.078.03	$1.437.98	$365.634.94	$338.554.42
Sep-32	$7,516.00	$6.103.39	$1.412.62	$367,047.56	$332,451.03
Oct-32	$7,516.00	$6,128.85	$1,387.15	$368,434.71	$326,322.18
Nov-32	$7,516.00	$6,154.42	$1,361.58	$369,796.29	$320.167.75
Dec-32	$7,516.00	$6,180.10	$1,335.90	$371,132.19	$313,987.65
Jan-33	$7,516.00	$6.205.89	$1.310.11	$372,442.30	$307.781.76
Feb-33	$7,516.00	$6,231.78	$1,284.22	$373,726.52	$301,549.97

Mar-33	$7.516.00	$6,257.79	$1,258.22	$374,984.74	$295,292.19
Apr-33	$7,516.00	$6,283.90	$1,232.11	$376,216.85	$289,008.29
May-33	$7,516.00	$6.310.12	$1.205.89	$377,422.73	$282.698.17
Jun-33	$7.516.00	$6.336.45	$1,179.56	$378,602.29	$276,361.73
Jul-33	$7,516.00	$6,362.88	$1,153.12	$379,755.41	5269.998.84
Aug-33	$7,516.00	$6,389.43	$1,126.57	$380,881.98	$263,609.41
Sep-33	$7,516.00	$6.416.09	$1,099.91	$381,981.89	$257,193.31
Oct-33	$7,516.00	$6.442.86	$1,073.14	$383,055.03	$250,750.45
Nov-33	$7,516.00	$6,469.75	$1,046.26	$384,101.29	$244,280.70
Dec-33	$7,516.00	$6.496.74	$1,019.26	$385,120.55	$237,783.96
Jan-34	$7,516.00	$6.523.85	$992.15	$386,112.70	$231,260.11
Feb-34	$7,516.00	$6,551.07	$964.93	$387,077.63	$224,709.04
Mar-34	$7.516.00	$6.578.41	$937.60	$388,015.23	$218,130.63
Apr-34	$7,516.00	$6,605.85	$910.15	$388,925.38	$211,524.78
May-34	$7,516.00	$6.633.42	$882.59	$389,807.97	$204,891.36
Jun-34	$7,516.00	$6.661.09	$854.91	$390,662.88	$198,230.27
Jul-34	$7,516.00	$6,688.89	$827.12	$391,490.00	$191,541.38
Aug-34	$7,516.00	$6,716.80	$799.21	$392,289.20	$184,824.58
Sep-34	$7,516.00	$6,744.82	$771.18	$393,060.38	$178,079.76
Oct-34	$7,516.00	$6,772.97	$743.04	$393,803.42	$171,306.79
Nov-34	$7,516.00	$6,801.23	$714.78	$394,518.20	$164,505.56
Dec-34	$7.516.00	$6.829.60	$686.40	$395,204.60	$157,675.96
Jan-35	$7,516.00	$6,858.10	$657.90	$395,862.50	$150,817.86
Feb-35	$7,516.00	$6.886.72	$629.29	$396,491.79	$143,931.14
Mar-35	$7,516.00	$6,915.45	$600.55	$397,092.34	$137,015.69
Apr-35	$7,516.00	$6,944.31	$571.70	$397,664.04	$130,071.38
May-35	$7,516.00	$6.973.28	$542.72	$398,206.76	$123,098.10
Jun-35	$7,516.00	$7,002.38	$513.63	$398,720.39	$116,095.73
Jul-35	$7,516.00	$7,031.59	$484.41	$399.204.80	$109.064.13
Aug-35	$7,516.00	$7,060.93	$455.07	$399,659.87	$102,003.20
Sep-35	$7,516.00	$7,090.40	$425.61	$400,085.48	$94,912.80
Oct-35	$7,516.00	$7,119.98	$396.02	$400,481.50	$87,792.82
Nov-35	$7.516.00	$7,149.69	$366.32	$400,847.81	$80,443.13
Dec-35	$7,516.00	$7,179.52	$336.48	$401,184.30	$73,463.61
Jan-36	$7,516.00	$7,209.48	$306.53	$401,490.83	$66,254.14
Feb-36	$7,516.00	$7,239.56	$276.45	$401,767.27	$59,014.58
Mar-36	$7.516.00	$7,269.77	$246.24	$402,013.51	$51,744.81
Apr-36	$7,516.00	$7,300.10	$215.91	$402,229.41	$44,444.71
May-36	$7.516.00	$7,330.56	$185.45	$402.414.86	$37,114.15
Juu 2036	$7.516.00	$7,361.15	$154.86	$402,569.72	$29,753.01
Jul-36	$7,516.00	$7,391.86	$124.14	$402,693.86	$22,361.15
Aug-36	$7,516.00	$7,422.70	$93.30	$402,787.16	$14,938.45
Sep-36	$7,516.00	$7.453.67	$62.33	$402,849.50	$7,484.77
Oct-36	$7.516.00	$7.484.77	$31.23	$402.880.73	$0.00

At this point, we must consider the EU APR, and the nominal and effective interest rates, in case of both monthly compound interest rate and monthly simple interest amortization plan.

Using a finance calculator, the above-evidenced amortization plan determines the following interest rates:

Nominal annual interest rate in compound interest: 5.006992%, which is our expected result.
Nominal interest rate in simple interest: 9.685151%%
Effective annual interest rate in compound interest: 5.123509%
Global nominal annual interest rate in compound interest: 5.175907%
EU APR 5.300477%
Global nominal annual interest rate in simple interest, or EU APR in simple interest: 10.327683%

Case 2: without the Substitute Tax deducted from the loan (NO CAPITALIZATION OF TAX)

First, we need to "clean" the cost deriving from the Substitute Tax that, in our example, corresponds to be of $ 2,375.00. Therefore, we must consider in our example a loan for $ 947,625.00 and not for, anymore, $ 950,000.00 ($ 950,000.00 - $ 2,375.00).

In this case, using the same finance calculator, monthly payment would be $ 7,516.00 as follows:

Monthly Payments

$ 7,497.21

Total Principal Paid	$947,625
Total Interest Paid	$401,873.52

The amortization plan would be the following, if in monthly compound interest rate:

Payment Date	Payment	Principal	Interest	Total Interest	Balance
Nov 2021	$7,497.21	53.543.25	$3,953.97	$3,953.97	$944,081.75
Dec 2021	$7.497.21	$3.558.03	$3.939.18	$7,893.15	$940,523.72
Jan 2022	$7,497.21	$3,572.88	$3,924.34	$11,817.48	$936.950.84
Feb 2022	$7.497.21	$3.587.79	$3.909.43	$15,726.91	$933.363.05
Mar 2022	$7.497.21	$3.602.76	$3.894.46	$19.621.37	$929.760.30
Apr 2022	$7.497.21	$3.617.79	$3,879.42	$23,500.79	$926.142.51
May 2022	$7.497.21	$3.632.88	$3,864.33	$27,365.12	$922.509.62
Jun 2022	$7.497.21	$3.648.04	$3.849.17	$31,214.29	$918.861.58
Jul 2022	$7.497.21	$3.663.26	$3,833.95	$35,048.24	$915.198.32
Aug 2022	$7,497.21	$3,678.55	$3,818.66	$38,866.91	$911,519.77
Sep 2022	$7,497.21	$3,693.90	$3,803.32	$42,670.22	$907,825.87
Oct 2022	$7,497.21	$3.109.31	$3,787.90	$46,458.13	$904.116.56
Nov 2022	$7,497.21	$3,724.79	$3,772.43	$50,230.55	$900,391.77
Dec 2022	$7,497.21	$3,740.33	$3,756.88	$53,987.44	$896,651.44
Jan 2023	$7,497.21	$3,755.94	$3,741.28	$57,728.72	$892,895.51

Feb 2023	$7,497.21	$3,771.61	$3,725.61	$61.454.32	$889.123.90
Mar 2023	$7,497.21	$3,787.34	$3,709.87	$65,164.19	$885,336.55
Apr 2023	$7,497.21	$3.803.15	$3,694.07	$68,858.26	$881,533.41
May 2023	$7,497.21	$3,819.02	$3,678.20	$72,536.46	$877,714.39
Jun 2023	$7,497.21	$3.834.95	$3,662.26	$76.198.72	$873,879.44
Jul 2023	$7,497.21	$3,850.95	$3,646.26	$79.844.98	$870,028.49
Aug 2023	$7,497.21	$3,867.02	$3,630.19	$33.475.18	$866,161.47
Sep 2023	$7,497.21	$3,883.16	$3,614.06	$87,089.23	$862,278.31
Oct 2023	$7,497.21	$3,899.36	$3,597.86	$90,687.09	$858,378.95
Nov 2023	$7,497.21	$3.915.63	$3,581.59	$94,268.68	$854.463.33
Dec 2023	$7.497.21	$3,931.97	$3,565.25	$97,833.93	$850.531.36
Jan 2024	$7,497.21	$3.948.37	$3.548.84	$101,382.77	$846,582.99
Feb 2024	$7,497.21	$3,964.85	$3,532.37	$104,915.13	$842,618.14
Mar 2024	$7,497.21	$3.981.39	$3,515.82	$108.430.96	$838,636.75
Apr 2024	$7.497.21	$3.998.00	$3.499.2	$111.930.17	$834,638.75
May 2024	$7,497.21	$4,014.68	$3,482.53	$115,412.70	$830,624.07
Jun 2024	$7,497.21	$4,031.44	$3,465.78	$118,878.48	$826,592.63
Jul 2024	$7,497.21	$4,048.26	$3,448.96	$122,327.44	$822,544.38
Aug 2024	$7.497.21	$4,065.15	$3,432.07	$125.759.50	$818,479.23
Sep 2024	$7,497.21	$4.082.11	$3.415.10	$129.174.61	$814,397.12
Oct 2024	$7,497.21	$4,099.14	$3,398.07	$132,572.68	$810.297.98
Nov 2024	$7,497.21	$4.116.25	$3,380.97	$135,953.65	$806,181.73
Dec 2024	$7,497.21	$4.133.42	$3,363.79	$139,317.44	$802,048.31
Jan 2025	$7,497.21	$4,150.67	$3,346.55	$142.663.99	$797,897.64
Feb 2025	$7,497.21	$4,167.99	$3,329.23	$145,993.22	$793.729.66
Mar 2025	$7,497.21	$4,185.38	$3.311.84	$149,305.05	$789,544.28
Apr 2025	$7,497.21	$4,202.84	$3,294.37	$152,599.43	$735.341.44
May 2025	$7.497.21	$4.220.38	$3,276.84	$155,876.26	$781,121.06
Jun 2025	$7,497.21	$4,237.99	$3,259.23	$159,135.49	$776,883.08
Jul 2025	$7,497.21	$4,255.67	$3,241.54	$162,377.04	$772,627.41
Aug 2025	$7,497.21	$4,273.43	$3,223.79	$165,600.82	$768,353.98
Sep 2025	$7,497.21	$4,291.26	$3,205.96	$168,806.78	$764,062.72
Oct 2025	$7,497.21	$4,309.16	$3.188 .05	$171,994.83	$759,753.56
Nov 2025	$7,497.21	$4,327.14	$3.170 .07	$175.164.91	$755.426.42
Dec 2025	$7,497.21	$4.345.20	$3.152 .02	$178.316.92	$751.081.22
Jan 2026	$7.497.21	$4,363.33	$3.133 .89	$181,450.81	$746.717.89
Feb 2026	$7A97.21	$4,381.53	$3.115. 68	$184.566.49	$742,336.36
Mar 2026	$7,497.21	$4,399.82	$3,097.40	$187,663.89	$737,936.54
Apr 2026	$7,497.21	$4.418.17	$3,079.04	$190,742.93	$733,518.37
May 2026	$7,497.21	$4.436.61	$3,060.61	$193.803.53	$729,081.76
Jun 2026	$7,497.21	$4.455.12	$3,042.09	$196,845.63	$724,626.64
Jul 2026	$7,497.21	$4.473.71	$3,023.50	$199,869.13	$720,152.93
Aug 2026	$7,497.21	$4,492.38	$3,004.84	$202,873.97	$715,660.56

Sep 2026	$7,497.21	$4.511.12	$2,986.09	$205,860.06	$711,149.44
Oct 2026	$7,497.21	$4.529.94	$2,967.27	$208.827.33	$706.619.49
Nov 2026	$7,497.21	$4.548.84	$2,948.37	$211,775.70	$702,070.65
Dec 2026	$7,497.21	$4.567.82	$2,929.39	$214.705.09	$697.502.82
Jan 2027	S7.497.21	5-4.586.88	$2,910.33	$217,615.42	$692,915.94
Feb 2027	$7,497.21	$4.606.02	$2,891.19	$220,506.62	$688,309.92
Mar 2027	$7,497.21	$4,625.24	$2,871.97	$223,378.59	$683,684.68
Apr 2027	$7,497.21	$4.644.54	$2,852.67	$226.231.26	$679,040.14
May 2027	$7.497.21	$4.663.92	$2.833.29	$229,064.56	$674.376.22
Jun 2027	$7,497.21	$4.683.38	$2,813.83	$231.878.39	$669.692.84
Jul 2027	$7,497.21	$4.702.92	$2,794.29	$234,672.69	$664,989.92
Aug 2027	$7,497.21	$4,722.54	$2,774.67	$237,447.36	$660,267.38
Sep 2027	$7,497.21	$4.742.25	$2,754.97	$240,202.32	$655,525.13
Oct 2027	$7,497.21	$4,762.04	$2,735.18	$242,937.50	$650,763.09
Nov 2027	$7,497.21	$4,781.91	$2,715.31	$245,652.81	$645.981.19
Dec 2027	$7,497.21	$4,801.86	$2,695.36	$248.348.17	$641.179.33
Jan 2028	$7.497.21	$4.821.89	$2,675.32	$251,023.49	$636,357.44
Feb 2028	$7,497.21	$4.842.01	$2,655.20	$253.678.69	$631,515.42
Mar 2028	$7,497.21	$4.862.22	$2,635.00	$256,313.69	$626.653.21
Apr 2028	$7,497.21	$4.882.50	$2,614.71	$258,928.40	$621,770.70
May 2028	$7.497.21	$4,902.88	$2.594.34	$261,522.74	$616.867.83
Jun 2028	$7,497.21	$4.923.33	$2,573.88	$264.096.62	$611.944.49
Jul 2028	$7,497.21	$4,943.88	$2,553.34	$266,649.95	$607.000.62
Aug 2028	$7.497.21	$4.964.50	$2,532.71	$269.182.66	$602,036.12
Sep 2028	$7,497.21	$4.985.22	$2,512.00	$271,694.66	$597,050.90
Oct 2028	$7,497.21	$5,006.02	$2,491.19	$274,185.86	$592,044.88
Nov 2028	$7,497.21	$5,026.91	$2,470.31	$276,656.16	$587,017.97
Dec 2028	$7,497.21	$5,047.88	$2,449.33	$279,105.50	$581.970.09
Jan 2029	$7,497.21	$5,068.94	$2,428.27	$281,533.77	$576,901.15
Feb 2029	$7,49721	$5,090.09	$2,407.12	$283,940.89	$571,811.05
Mar 2029	57.497.21	$5.111.33	$2.385.88	$286,326.77	$566.699.72
Apr 2029	$7,497.21	$5.132.66	$2.364.55	$288,691.32	$561,567.06
May 2029	$7,497.21	$5.154.08	$2.343.14	$291.034.46	$556.412.98
Jun 2029	$7.497.21	$5.175.58	$2,321.63	$293.356.09	$551,237.40
Jul 2029	$7.497.21	$5.197.18	$2,300.04	$295.656.13	$546,040.23
Aug 2029	$7.497.21	$5.218.86	$2,278.35	$297,934.48	$540.821.37
Sep 2029	S7.497.21	$5.240.64	$2,256.58	$300,191.06	$535.580.73
Oct 2029	$7,497.21	$5.262.50	$2.234.71	$302,425.77	$530.318.23
Nov 2029	$7.497.21	$5.284.46	$2.212.75	$304,638.52	$525,033.76
Dec 2029	$7,497.21	$5,306.51	$2.190.70	$306,829.23	$519,727.25
Jan 2030	$7,497.21	$5,328.65	$2.168.56	$308,997.79	$514,398.60
Feb 2030	$7,497.21	$5,350.39	$2.146.33	$311,144.12	$509,047.72
Mar 2030	$7.497.21	$5,373.21	$2,124.00	$313.268.12	$503.674.50

Apr 2030	$7.497.21	$5,395.63	$2,101.58	$315,369.70	$498,278.87
May 2030	$7,497.21	$5,418.15	$2,079.07	$317,448.77	$492.860.73
Jun 2030	$7,497.21	$5.440.75	$2.056.46	$319.505.23	$487,419.97
Jul 2030	$7.497.21	$5.463.45	$2.033.76	$321.538.99	$481.956.52
Aug 2030	$7,497.21	$5,486.25	$2.010.96	$323,549.96	$476,470.27
Sep 2030	$7,497.21	$5,509.14	$1,988.07	$325,538.03	$470,961.13
Oct 2030	$7.497.21	$5,532.13	$1.965.09	$327.503.11	$465.429.00
Nov 2030	$7.497.21	$5,555.21	$1.942.00	$329.445.12	$459.873.79
Dec 2030	$7.497.21	$5,578.39	$1,918.82	$331,363.94	$454,295.40
Jan 2031	$7,497.21	$5,601.67	$1,895.55	$333,259.49	$448.693.73
Feb 2031	$7.497.21	$5.625.04	$1,872.17	$335,131.66	$443,068.69
Mar 2031	$7,497.21	$5,648.51	$1,848.70	$336,980.36	$437,420.18
Apr 2031	$7,497.21	$5,672.08	$1,825.14	$338,805.50	$431,748.10
May 2031	$7,497.21	$5,695.75	$1,801.47	$340,606.97	$426,052.36
Jun 2031	$7.497.21	$5.719.51	$1.777.70	$342.384.67	$420,332.85
Jul 2031	$7,497.21	$5,743.38	$1,753.84	$344,138.51	$414,589.47
Aug 2031	$7.497.21	$5.767.34	$1.729.87	$345.868.39	$408.822.13
Sep 2031	$7.497.21	$5.791.40	$1.705.81	$347,574.20	$403.030.73
Oct 2031	$7.497.21	$5.815.57	$1.681.65	$349.255.84	$397,215.16
Nov 2031	$7,497.21	$5,839.83	$1,657.38	$350,913.22	$391,375.33
Dec 2031	$7,497.21	$5.864.20	$1,633.01	$352,546.24	$385,511.13
Jan 2032	$7.497.21	$5,888.67	$1.608.55	$354.154.78	$379,622.46
Feb 2032	$7.497.21	$5,913.24	$1.583.97	$355.738.76	$373,709.22
Mar 2032	$7,497.21	$5,937.91	$1,559.30	$357,298.06	$367,771.30
Apr 2032	$7.497.21	$5.962.69	$1.534.53	$358.832.58	$361.808.62
May 2032	$7.497.21	$5.987.57	$1.509.65	$360.342.23	$355,821.05
Jun 2032	$7.497.21	$6,012.55	$1.484.66	$361.826.89	$349.808.50
Jul 2032	$7,497.21	$6,037.64	$1,459.58	$363,286.47	$343,770.86
Aug 2032	$7,497.21	$6,062.83	$1 .434.38	$364,720.85	$337,708.03
Sep 2032	$7,497.21	$6,088.13	$1 .409.09	$366,129.94	$331,619.90
Oct 2032	$7,497.21	$6,113.53	$1 .383.68	$367,513.62	$325,506.37
Nov 2032	$7,497.21	$6,139.04	$1 .358.18	$368,871.80	$319,367.33
Dec 2032	$7,497.21	$6.164.65	$1 .332.56	$370,204.36	$313,202.68
Jan 2033	$7,497.21	$6,190.38	$1 .306.84	$371.511.20	$307,012.30
Feb 2033	$7,497.21	$6,216.21	$1 .281.01	$372.792.21	$300.796.10
Mar 2033	$7,497.21	$6,242.14	$1 .255.07	$374,047.28	$294.553.96
Apr 2033	$7,497.21	$6.268.19	$1 .229.03	$375.276.30	$288.285.77
May 2033	$7,497.21	$6.294.34	$1 .202.87	$376,479.18	$281,991.43
Jun 2033	$7,497.21	$6,320.60	$1.176.61	$377,655.79	$275,670.82
Jul 2033	$7,497.21	$6,346.98	$1,150.24	$378,806.02	$269,323.85
Aug 2033	$7.497.21	$6,373.46	$1,123.75	$379,929.78	$262,950.39
Sep 2033	$7.497.21	$6,400.05	$1,097.16	$381,026.94	$256,550.33
Oct 2033	$7,497.21	$6,426.76	$1,070.46	$382,097.39	$250,123.57

Nov 2033	$7.497.21	$6.453.57	$1,043.64	$383.141.03	$243,670.00
Dec 2033	$7,497.21	$6,480.50	$1,016.71	$384,157.75	$237,189.50
Jan 2034	$7,497.21	$6,507.54	$989.67	$385,147.42	$230.681.96
Feb 2034	$7,497.21	$6,334.69	$962.52	$386,109.94	$224,147.27
Mar 2034	$7,497.21	$6,561.96	$935.25	$387.045.19	$217,585.31
Apr 2034	$7,497.21	$6.589.34	$907.87	$387,953.07	$210,995.97
May 2034	$7.497.21	$6.616.83	$880.38	$388,833.45	$204,379.13
Jun 2034	$7,497.21	$6,644.44	$852.77	$389,686.22	$197,734.69
Jul 2034	$7,497.21	$6,672.17	$825.05	$390,511.27	$191,062.52
Aug 2034	$7.497.21	$6.700.01	$797.21	$391.308.48	$184.362.52
Sep 2034	$7,497.21	$6,727.96	$769.25	$392,077.73	$177.634.56
Oct 2034	$7,497.21	$6,756.03	$741.18	$392,818.91	$170,878.52
Nov 2034	$7,497.21	$6,784.22	$712.99	$393,531.90	$164,094.30
Dec 2034	$7,497.21	$6.812.53	$684.68	$394,216.59	$157,281.77
Jan 2035	$7,497.21	$6,840.96	$656.26	$394,872.84	$150,440.81
Feb 2035	$7,497.21	$6,869.50	$627.71	$395,500.56	$143,571.31
Mar 2035	$7,497.21	$6,898.16	$599.05	$396,099.61	$136,673.15
Apr 2035	$7,497.21	$6.926.95	$570.27	$396,669.88	$129,746.21
May 2035	$7,497.21	$6.926.95	$570.27	$396,669.88	$129,746.21
Jun 2035	$7.497.21	$6.955.85	$541.37	$397.211.24	$122,790.36
Jul 2035	$7,497.21	$6,984.87	$512.34	$397,723.59	$115.805.49
Aug 2035	$7,497.21	$7,014.02	$483.20	$398,206.79	$108,791.47
Sep 2035	$7,497.21	$7,043.28	$453.93	$398.660.72	$101,748.19
Oct 2035	$7,497.21	$7,072.67	$424.54	$399,085.26	$94,675.52
Nov 2035	$7,497.21	$7,102.18	$395.03	$399,480.30	$87,573.34
Dec 2035	$7,497.21	$7,131.81	$365.40	$399,845.70	$80,441.53
Jan 2036	$7,497.21	$7,161.57	$335.64	$400,181.34	$73,279.95
Feb 2036	$7,497.21	$7,221.46	$275.75	$400,762.85	$58,867.04
Mar 2036	$7,497.21	$7,251.59	$245.62	$401,008.48	$51,615.45
Apr 2036	$7,497.21	$7,281.85	$215.37	$401,223.84	$44,333.60
May 2036	$7,497.21	$7,312.23	$184.98	$401,408.82	$37,021.37
Jun 2036	$7,497.21	$7,342.74	$154.47	$401,563.29	$29,678.63
Jul 2036	$7,497.21	$7,373.38	$123.83	$401,687.13	$22,305.25
Aug 2036	$7,497.21	$7,404.15	$93.07	$401,780.20	$14,901.10
Sep 2036	$7,497.21	$7.435.04	$62.17	$401,842.37	$7,466.06
Oct 2036	$7,497.21	$7,466.06	$31.15	$401,873.52	$0.00

Once again, at this point, we must consider the EU APR, and the nominal and effective interest rates, in case of both monthly compound interest rate and monthly simple interest amortization plan, but without the Substitute Tax in it. Therefore, we must deduct from the $ 10,485.00 the Substitute Tax of 0.25% of the credit that was corresponding to be $ 2,375.00 ($ 950,000.00 * 0.25%).

The calculus comes to be $ 10,485.00 - $ 2,375.00 = $ 8,110.00 “new” upfront costs deducted from the financed loan of $ 950,000.00.

Using a finance calculator, the above-evidenced amortization plan determines the following interest rates:

Nominal annual interest rate in compound interest: 4. 969006%
Nominal interest rate in simple interest: 9.545955%%
Effective annual interest rate in compound interest: 5.083749%
Global nominal annual interest rate in compound interest: 5.099283%
EU APR 5.220166%
Global nominal annual interest rate in simple interest, or EU APR in simple interest: 10.031335%

Case 3: Without the Substitute Tax Deducted From the Loan (NO CAPITALIZATION OF TAX), and Either From the Financed Loan

In this third hypothesis, we need to "clean" the cost deriving from the Substitute Tax that, in our example, corresponds to be of $ 2,375.00 but just from the upfront costs, and not from the loan. Therefore, we must consider in our example a loan for $ 950,000.00 and not for, anymore, $ 947,625.00 ($ 950,000.00 - $ 2,375.00).

In this case, using the same finance calculator, monthly payment would be $ 7,516.00 with identical amortization plan as for Case#1 of this paper, but things change for EU APR and interest rates. In fact, contrary to Case#1 before illustrated, the financed loan remains the same, but the upfront costs diminish of the amount for the Substitute Tax.

Using a finance calculator, this third scenario (Case#3) determines the following interest rates:

Nominal annual interest rate in compound interest: 5.006992%
Nominal interest rate in simple interest: 9.685151%%
Effective annual interest rate in compound interest: 5.123509%
Global nominal annual interest rate in compound interest: 5.137410%
EU APR 5.260120%
Global nominal annual interest rate in simple interest, or EU APR in simple interest: 10.177756%

Let compare all these results among themselves:

WITH SUBSTITUTE TAX CAPITALIZED (Case 1)			
RATE	INTEREST RATE	INTEREST DUE	EU APR
$ 7.516,00	5,006992%	$ 402.880,73	5,30047700%

WITHOUT SUBSTITUTE TAX CAPITALIZED (Case 2)			
RATE	INTEREST RATE	INTEREST DUE	EU APR
$ 7.497,21	4,969006%	$ 401.873,52	5,22016600%

WITHOUT SUBSTITUTE TAX CAPITALIZED (Case 3)			
RATE	INTEREST RATE	INTEREST DUE	EU APR
$ 7.516,00	5,006992%	$ 402.880,73	5,26012000%

The **Case #1** is the standard and most common one: the bank provides the borrower of the loan, fully amortized; it deducts the Substitute Tax together with the other upfront costs and the borrower effectively receives a minor amount then the credit original granted.

The **Case #2** considers the case that the bank keeps the same economic conditions of the Case #1, but does not debit the Substitute Tax together with the other upfront costs. Moreover, the bank excludes this tax even from the credit granted; therefore, the borrower effectively receives the full credit granted, without the Substitute Tax in it, and pays the upfront costs, again without the Substitute Tax in them.

The **Case #3** considers the case the bank keeps the same conditions of the Case #1, but does not debit the Substitute Tax together with the other upfront costs, and keeps the original amount of credit granted and then lent.

These three scenarios create the following differences:

DIFFERENCE Case 1 - Case 2)			
RATE	INTEREST RATE	INTEREST DUE	EU APR
$ 18,79	0,037986%	$ 1.007,21	0,08031100%

Consequence:

- If the bank does not capitalize the Substitute Tax, it loses $ 18.79 of interest per each rate, for a total of $ 1,007.21 for the whole life span of the loan. This extra profit made over the Substitute Tax, as per capitalization effect, in a constant mortgage with monthly instalment, corresponds to be in +0.080311 EU APR percentage points.

DIFFERENCE Case 1 - Case 3)			
RATE	INTEREST RATE	INTEREST DUE	EU APR
$ -	0,000000%	$ -	0,04035700%

Consequence:

- If the bank does not capitalize the Substitute Tax, it does not reduce the amount of the loan lent, but just reduces the upfront costs deducted of the Substitute Tax at the time of the contract, at parity of monthly instalments and total interest due, the EU APR percentage points decrease of 0.040357.

Conclusions on the Above-Evidenced Empiric Examples

About Profits

Without any doubts, the highest profits for the banks come from capitalizing the Substitute Tax keeping it from the loan lent at the time of the contract, together with the other upfront costs (**Case #1** of the empiric example).

In this hypothetical ranking, the second best scenario for the banks is the **Case #3**, where the bank keeps the full amount of the loan and deducted of just the upfront costs, without the Substitute Tax into them and asking the borrower to pay the Substitute Tax with his own money, separately, to the lender.

At the end, the worst of these three scenarios for the bank is the **Case #2**, in which the borrower receives the amount of the loan at the net of the upfront costs, for a minor amount not including the Substitute Tax that the borrower will pay, separately, with his own money.

In fact, the Case #2 represents the case in which the bank grants a credit for the exact amount the borrower needs, not financing to him the Substitute Tax, paid separately and with his own money.

This Case #2 means, for the bank, less credit lent / less interests and costs into it, and, at parity of Substitute Tax received in payment by the borrower, less general profit.

About the Consumer Credit Market

A secondary consequence is on to the consumer credit market. In fact, the EU APR was born to assure the consumers to receive a proper preliminary information about the loan conditions ruling the consumer credit contracts. In fact, it appears clear that computing the Substitute Tax among the costs inside the EU APR indicator, in its general formula, would influence the entire credit consumer market.

This because the Substitute Tax makes the EU APR higher of its cost and of the interest matured on it, asked together with the other physiological interest prescribed into the consumer credit contracts. Therefore, the Substitute Tax in the EU APR formula misleads the indicator that becomes not anymore a real benchmark for the consumer credit market and not any more reliable as it should be, in theory.

About the Property of the Substitute Tax Money and Its Interests and Profits

Another aspect to take into consideration about the Substitute Tax on EU APR and loans is the ownership and property of the money coming from the Substitute Tax. In fact, this tax is clearly a State income, and not a bank income; therefore, it cannot become a source of profit for the lenders, most over because the Substitute Tax is a tax itself, and belongs to the State although cashed by the banks and due by them to the Administration. In other words, when banks collect the Substitute Tax, effectively, they cash money not belonging to them, but belonging to the State that means the Substitute Tax is not money of the banks, but money of the State.

The problems come up when we state that is legit to cash borrowers' money due to the State by the lender switched on borrowers' shoulders and recognizing that this money, anyway, belongs to the State, because it is a tax due by the lenders of the credit. Actually, we front here the problem of banks financing the borrowers of one tax due to the Administration by the banks. It appears clear that it should be not legal for anybody to pretend money not his or hers from somebody, because – for the case of the Substitute Tax – the Administration is the unique and real owner of this money, and not anybody else.

Banks pretend borrowers to pay the Substitute Tax on behalf of the banks and this is possible and lawful, but only if borrowers pay to the Administration directly this tax due by banks, and not paying this tax due by banks to lenders instead. In fact, otherwise, it happens a different mechanism behind this operation that allows banks to cash money that banks own to the Administration, and should revert to it by law.

A second problem is the interest bearing on this money. In fact, banks not only cash the Substitute Tax from borrowers, but also pretend the maturing of interests at the interest rate of the contract, for the whole life span of the loan, as previously demonstrated with the three Cases in example. Therefore, we should front the problem of banks – through capitalization of the Substitute Tax – profiting interests on money not belonging to them, but belonging to the Administration that it seems not legal and legit.

We would have a third problem and this time it is connected with the "time" of these operations regarding the Substitute Tax management by banks. In fact, banks grant credits continuously, without interruption and they operate in the credit market without stopping, while the payments of the taxes to the Administration have fixed terms and deadlines. In Italy, for example, this tax is due, partially, on April 30 of the following year of the contract (45% of the 95% of the total amount due for this tax) and on October 30 of the following year of the contract for the residue.

These financial evidences apparently provide several main extra advantages to the banks:

1) The first advantage is to cash interest accrued on this tax, because it becomes part of the amortization plan;
2) The second advantage is to cash compound interest accrued on this tax, because of the very nature of the constant mortgage amortization plan method;
3) The third advantage is related to the time "t", both as "t_0", both as "t_m", because the banks cash this tax from the borrower at the time of the contract [*rectus*: capital transfer] and not when this tax is due to the State from the banks;
4) The fourth advantage is related to the property of the money (even in the lawsuit meaning of the term, such as "rights" deriving from the property of the money), because the Substitute Tax belong to the Administration and not to the banks;
5) The fifth advantage is deriving by the kind of amortization plan used for the reimbursement plan of the loan. Both this point #5 and point #3 of this list depend on the principles related to the time value of money.
6) The sixth advantage is the intrinsic propriety of discount factors applied on the Substitute Tax as well, like for all the other components of money of the loan, and the principle of time value of money.

This last point of the above-evidenced list needs a specification.

In fact, we have to assume that the Substitute Tax cash flows, both inflows and/or outflows, managed by the banks, can become like a sort of perpetuity, because the banks continuously issue into the credit market field loans always hit by this tax. Therefore, we need to assign to these Substitute Tax cash flows a value, in theory similar to the one given to the infinity sequence of cash flows. In other words, the principle is that \$1 today worth more than \$1 tomorrow, which is worth more than \$1 a year from now, which is worth more than \$1 two years from now, and so on. Therefore, a value of a dollar paid out in the future declines at a rate for which we can actually figure out what the value is today. Using the same principle of discounting for the cash flows, it is possible, in theory, to recreate a sequence of cash flows even for the Substitute Tax, discounting them by 1 plus *r*, 1 plus *r* squared, and so on, forever.

We do need to consider another further aspect that directly connects the kind of amortization plan prescribed for the loan as per contract with the Substitute Tax cashflows. In fact, we could consider two different variables: the variable amount of the cash flow (*CF*) changing every time, in perpetuity, and in growth, and the variable of the rate of growth (*g*) that corresponds to be a perpetual growth of *CF* with progression 1 plus *g*. With this mechanism, we can assume that in the progression sequence of 1 plus *g* there is a component coming from the Substitute Tax amount, becoming part of the whole process of capitalization, perpetually, in compounding factor.

To understand better the influence of the kind of amortization plan applied for the reimbursement of a loan onto the Substitute Tax management, it is mandatory to focus the attention on the principles about the time value of money.

We do not need to explain what is meant by "time value of money" and all the concepts of simple and compound interest, neither to explain how to calculate present value of an ordinary annuity, or to determine periodic ordinary annuity payments. We invite the readers to learn how interest rates are determined in the financial markets, how to calculate future and present values when time intervals are less than one year, and how to consider the problems involving compounding or discounting to determine present values.

All these basilar concepts could be easily studied in any students financial books at any University, for which the author is inviting the readers to get knowledge and consider all these aspects here entirely absorbed.

Cleared all the preambles, what we need to consider in this positioning paper is the effect of the financial technique of reimbursement. In fact, it has influence on the Substitute Tax dealing by banks in their loans.

As well known, the most common tool of financial loan in use, also known as "mortgage constant", technically built under the principles of the compound interest methods.

This technique of amortization clearly shows that every single rate is constant and composed by the sum of the principal part plus the accrued interest. The rates of this kind of amortization plan result in an inversely proportional scheme: higher interest reimbursed at the beginning of the amortization plan, increasing rate by rate.

Consequently, this amortization plan has two main general conditions:

1. The interest rate must be calculated and applied on the ending balance of the standing principal each time updated of the payments done as per amortization schedule.
2. Every single stand-alone rate of the amortization plan is constant.

For what of our interest here in this paper, this mechanism determines the higher profit on the Substitute Tax at the very beginning of the life span of the loan rather than the last periodical installments paid by the borrowers. If we take into consideration the financial effects of the principles related to the time value of money, it results clear that through a mortgage constant the banks cash out the highest profits from the Substitute Tax capitalization at the first periodical payments of the rate, and this is a clear extra benefits for the lenders. Before in this paper, we showed the difference between the interest outcome in three different cases and we demonstrated the influence of the Substitute Tax capitalization on the total interest due at the end of the life span of the loan. Now, we need considering the incidence of the different quantity of interests cashed from the Substitute Tax capitalization in a typical mortgage constant structure.

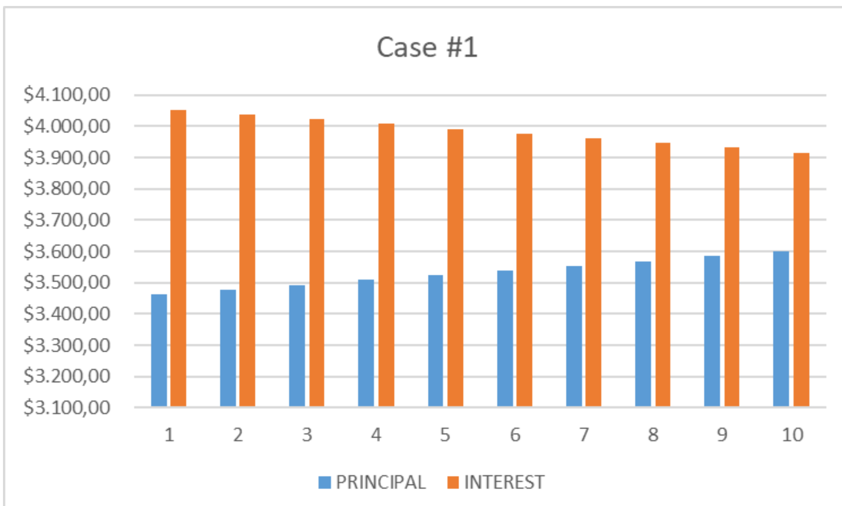
For this reason, we keep on using the same previous example of a \$ 950,000.00 with related Substitute Tax of \$ 2,375.00 and total upfront costs of \$ 10,485.00 deducted from the credit and including the \$ 2,375.00 of the Substitute Tax, 180 monthly installments for a total of 15 years, at the nominal annual interest rate of 5.007%.

This cleared - we can compare, for instance, the first ten monthly installments of the two cases in which the banks capitalizes and does not capitalize the Substitute Tax in the loan. The result is the following:

Case #1: The Bank Capitalizes the Substitute Tax in the Loan as an Upfront Cost

PAYMENT #	PAYMENT	PRINCIPAL	INTEREST	TOTAL INTEREST	BALANCE
1	$ 7.516,00	$ 3.463,63	$ 4.052,37	$ 4.052,37	$ 936.051,37
2	$ 7.516,00	$ 3.478,57	$ 4.037,43	$ 8.089,80	$ 932.572,80
3	$ 7.516,00	$ 3.493,58	$ 4.022,42	$ 12.112,22	$ 929.079,22
4	$ 7.516,00	$ 3.508,64	$ 4.007,36	$ 16.119,58	$ 925.570,58
5	$ 7.516,00	$ 3.523,78	$ 3.992,22	$ 20.111,80	$ 922.046,80
6	$ 7.516,00	$ 3.538,98	$ 3.977,02	$ 24.088,82	$ 918.507,82
7	$ 7.516,00	$ 3.554,24	$ 3.961,76	$ 28.050,58	$ 914.953,58
8	$ 7.516,00	$ 3.569,57	$ 3.946,43	$ 31.997,01	$ 911.384,01
9	$ 7.516,00	$ 3.584,97	$ 3.931,03	$ 35.928,04	$ 907.799,04
10	$ 7.516,00	$ 3.600,43	$ 3.915,57	$ 39.843,61	$ 904.198,61

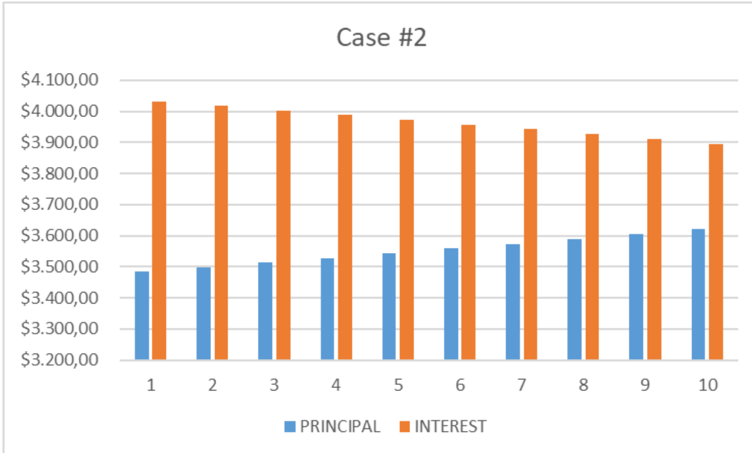
From which we can evidence the interest and capital reimbursement trends as follows:



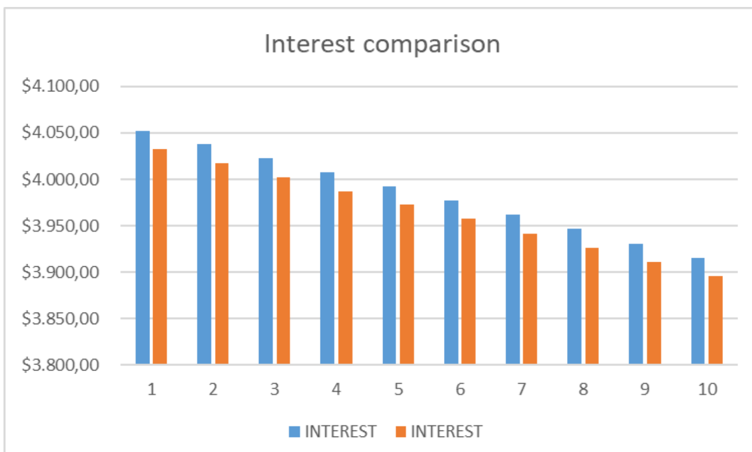
Case #2: The Bank Does Not Capitalize the Substitute Tax in the Loan as an Upfront Cost

PAYMENT #	PAYMENT	PRINCIPAL	INTEREST	TOTAL INTEREST	BALANCE
1	$ 7.516,00	$ 3.483,60	$ 4.032,40	$ 4.032,40	$ 938.406,40
2	$ 7.516,00	$ 3.498,52	$ 4.017,48	$ 8.049,88	$ 934.907,88
3	$ 7.516,00	$ 3.513,50	$ 4.002,50	$ 12.052,38	$ 931.394,38
4	$ 7.516,00	$ 3.528,54	$ 3.987,46	$ 16.039,84	$ 927.865,84
5	$ 7.516,00	$ 3.543,64	$ 3.972,36	$ 20.012,20	$ 924.322,20
6	$ 7.516,00	$ 3.558,82	$ 3.957,18	$ 23.969,38	$ 920.763,38
7	$ 7.516,00	$ 3.574,05	$ 3.941,95	$ 27.911,33	$ 917.189,33
8	$ 7.516,00	$ 3.589,35	$ 3.926,65	$ 31.837,98	$ 913.599,98
9	$ 7.516,00	$ 3.604,72	$ 3.911,28	$ 35.749,26	$ 909.995,26
10	$ 7.516,00	$ 3.620,15	$ 3.895,85	$ 39.645,11	$ 906.375,11

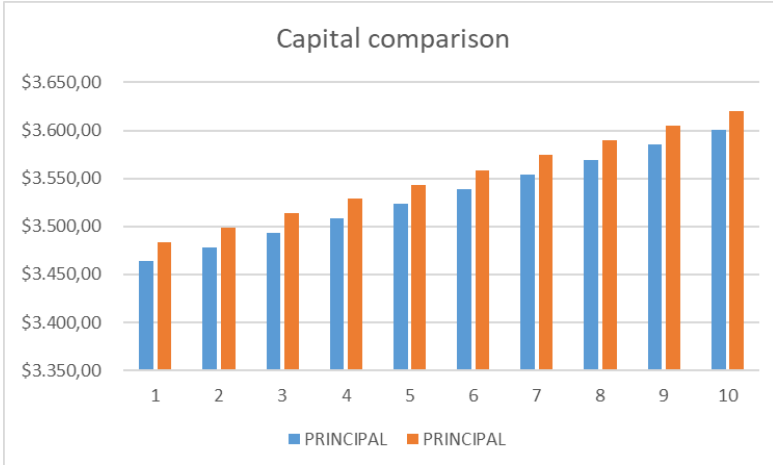
From which we can evidence the interest and capital reimbursement trends as follows:



What is appearing from these graphs is that in Case #1 the banks obtain much more profit than Case #2 not just under the profile of the total cash flows, but also for the trend of volume of these cash flows. This evidence is much clearer with the following chart, in which the accrued interest of the Case #1 (in blue colour) is placed side by side with the accrued interest of the Case #2 (in orange colour), as follows:



The amount of accrued interest in Case #1 is always higher than in Case #2, as same as for the capital lot, as follows, where the capital reimbursed in each instalment is in blue colour for the Case #1 and in orange colour for the Case #2:



The advantages for the banks after such financial operations are almost endless. In fact, not the above-evidenced empiric examples and representations still consider another time value of the money aspect that is the time of the effective (in theory) payment to the Administration of the Substitute Tax from the banks.

The best scenario for banks is the case in which the lender keeps the Substitute Tax as an upfront cost to the borrower at the time of the contract and benefits of this original keeping for one year, until the moment of the tax deadline on favour of the Administration.

Therefore, this is the case where the bank is able to benefit of the right of use of the Substitute Tax, in theory, for the longest possible span, before been obliged to transfer this tax to the rightful owner and beneficiary that is the Administration. For this reason, we should consider the future value (FV) of the Substitute Tax at the time of transferring to the Administration that would correspond to be the extra "benefit" of the lender in keeping it available and at its disposal for a year time.

The future value formula for lump sum cash flows is the following:

$$FV = PV \times (1+i)^n$$

where

FV is the future value
PV is the present value
i is the discount rate
n is the number of periods

In general, for our example, we should use the following calculus:

$$FV = 2{,}375.00 \times (1 + 0.05007)^{12} = \$\ 2{,}496.68$$

If we would consider the thesis that the Substitute Tax is due one a year and the bank keeps it for an entire year at its disposal before transferring it to the Administration through a monthly instalment mortgage constant, $ 2,375.00 would value after a year $ 2,496.68. The idea is that if the bank knows in advance that it could benefit of the availability of these sums of money deriving from the Substitute Tax capitalization and deduct from the loans granted, the bank can manage too these sums until the time of the transfer to the Administration, such as a year after, in the best theoretical scenario.

In other words, the bank could get two extra advantages from this situation:

1) Making business and profit using the Substitute Tax amount for a year period, because the bank will have the availability of this sum until the date of its payment to the Administration;
2) Selling, making business and profiting of the right of use of this sum of money deriving from the Substitute Tax for a year time until the date of its payment to the Administration.

In finance, a sum of money has a financial and economic value, the rights, and the titles and the privileges deriving from this availability of money has a secondary financial and economic value. This means that the FV could not be anymore – in this case - a solid indicator of value. Instead, it could be useful to start understanding the financial mechanism behind the authorization, even if implicit, to the banks to use the Substitute Tax as a vehicle to manage these sums of money deriving from the deduction of this tax from the money lent.

a) About the influence of the Substitute Tax money and its interests and profits on loanable funds and on pricing

Another aspect to take into consideration about the Substitute Tax on EU APR and on loans is the possible influence of this mechanism observed in this paper on loanable funds.

We usually define the loanable funds like the amount of money made available by lenders to borrowers. The mechanism behind the loanable funds is simple: borrowers demand funds from lenders to invest these funds so to earn a satisfactory return above the cost of the funds. On the other side, lenders supply funds to borrowers as long as lenders earn a satisfactory return on their loans.

As a logic consequence, we can define the interest rate like the price of loanable funds in the market that, exactly like for any common markets, meets the demand for the funds and their counterpart supply. This logic moves constantly behind the financial market, depicting a constant determination of an equilibrium between the demand for funds and their supply, maturing an equilibrium interest rate, considered satisfactory for both borrowers and lenders.

In such general scenario, we should consider the possibility that the Substitute Tax could interfere with the equilibrium interest rate making it move from the “natural” one, to a new level as result of a market “shock”.

In fact, we must remember that the Substitute Tax, if capitalized by banks, can constitute extra funds at bank’s disposal to supply in the loanable funds for both consumers and for the bank/banks itself/themselves. In other words, if extra funds become available because “created” by the Administration, automatically, we must consider this weight in the financial market. For example, these extra funds coming from the Substitute Tax cash flows might result of an unanticipated change or shift on the demand for, or supply, of loanable funds.

This unanticipated change could clear the market by bringing the demand by borrowers for funds in equilibrium with the supply by lenders of the funds because, in theory, giving this demand a larger quantity of loanable funds, borrowers could accept to pay higher interest rates, or vice versa.

For example, lenders could decrease the interest rates to make sure that these extra funds coming from the Substitute Tax investment would finish in the financial market through the consumer credit loans, or, eventually, influence the borrowers’ feelings about the real interest rate market, may be through inflation handling. Therefore, the funds deriving from the Substitute Tax on loans might create an unnatural shift in supply curves determines an increase in the equilibrium interest rate, or, vice versa, a possible opposite phenomenon, where the interest rates will downward to equals the lower demand for loanable funds to reach the new equilibrium interest level.

The banks could profit of all these “shocks” in the market they themselves create from the very beginning to make higher profits, rigging the interest rate levels and the quantity of loanable funds deciding if supplying the Substitute Tax at their disposal, and when, in the financial market, during any period.

Therefore, the banks can use even the Substitute Tax like a channel to influence the loanable funds theory and, consequently, all the sectors of the economy, the borrowers debts and their current savings, so that the Substitute Tax could become a factor itself to affect the supply of and demand for loanable funds.

A further variable that influences the interest rate market and the loanable funds quantity is the pricing. In fact, the extra funds deriving from the Substitute Tax capitalization that generates an extra profit for the banks can push the lenders to rebate the “natural” interest rates. Hence, if the lenders have extra funds coming from the Substitute Tax use for their loans, the banks may offer loan spreads lower than the level needed to compensate bank’s equity holders for bearing risks. Effectively, this manoeuvre lets the banks to use a sum of capital that does not belong to them, but to the Administration, so that the banks can purge part or their loan risk smearing it *pro quota* even on the Substitute Tax. Thus, the banks will profit even in terms of risk, because they could reduce the level of the risk of their exposures with the borrowers, thanks to the fact that the banks would detain the Substitute Tax from the *traditio* of the capital, and reducing the risk that this sum would not later reimbursed by the borrowers.

A compression of the level of the risk on the loans corresponds to a very possible decrease in spreads on new loans and a low bank profitability could show up because of a possible under-pricing activity of the lenders over the loans. A secondary effect that could derive from this supposed scenario is that under-priced loans could bring to downward reduction of the interest rates due to the competition among intermediaries and this could lead to fragilities in the financial system, most over if the credit risk may increase suddenly due to a market shock.

A downward projection of the interest rate quotation could be due to the availability of the Substitute Tax, because it could be an extra capital at disposal for the loanable funds in favour of the banks. A lower expected returns than fairly priced loans could lean the banks to offer the credit differently, especially towards to those firms with high credit risk score. This scenario would bring to two possible different consequences:

a) First, banks' profitability and their ability to generate internal capital may generally decrease on one side, while,
b) Second, the under-pricing of loans to specific firms could suggest that these banks would not consider of interest in investing on these firms and on their markets, suggesting an under-pricing due to lack of profitability.

In other words, an eventual under-pricing on loans, for the compression of the level of the risk deriving from the dilution into the *pro quota* capital lent through the Substitute Tax, could influence the expected returns and the expected net interest and fee incomes of loans and the expected credit , operating costs and cost of funding.

About the Influence of the Substitute Tax on Mortgage Interest Rates, Ratios and Household Markets Default

Another influence of the capitalisation of the Substitute Tax and its watering-down in the loanable funds is on the mortgages interest rates, especially for the ARM. In fact, we know that, basically, higher interest rates increase required mortgage payments on ARMs, (Adjusted-rate Mortgages) tightening borrowing constraints and more often triggering defaults. On the other side, an under-pricing of the interest rate because of the Substitute Tax volume thrown into the consumer credit market, can affect even the default behaviour and other parameterization, such as the mortgage *premia*. This hypothetical scenario effects the endogenously chosen level of savings, directly connected with the variance of the loanable funds volume, and even the mortgage market, built on ARM and FRM primarily.

When the bank grants a mortgage to the borrower, the lender must respect some parameters: some of these parameters come from the local laws, some others from international and common laws and rules. For example, in Italy we have the art. 38 paragraph #2 of the Consolidated Law on Banking (in Italian, "*Testo Unico Bancario*", or a.k.a. for its acronym "*T.U.B.*"), that is the Legislative Decree nbr. 385/1993. This law it indicates an LTV (Loan-to-value) limit fixed for any kinds of household mortgages at 80% maximum, regardless if ARM or FRM. This law is even remarked by the Interministerial Committee for Credit and Savings (C.I.C.R.), which is the Ministry of Economy and Finance and the Bank of Italy (see C.I.C.R. ruling of April 22, 1995). In other countries, regulator ban high LTV ratios in effort to control the incidence of mortgage control, or, eventually, to impose thresholds on the mortgage affordability ratios LTI and MTI, either in the form of guidelines or strict limits.

These laws concern about LTV, as said, but if the banks include in the amount financed of the mortgages even the Substitute Tax, this tax finishes inside the LTV parameter, and the first consequence it would be that the banks, effectively, would finance less for the value of the property, and more for the effective amount transferred to the borrowers. Anyway, LTV is just one of the ratios affected by the capitalization of the Substitute Tax on mortgages, because we must remember that the reduction of the effective loan because of this cost deducted by the *traditio*, at the time of the loan origination, causes consequences even on the interest rates and on the household markets as well. For example, on one hand, the capitalisation of the Substitute Tax influences (negatively) the LTV ratio, but, on the other hand, it influences (positively) the Loan-to-income (LTI) and the Mortgage-payments-to-income (MTI) ratios. In fact, the LTV ratio measures the household's initial equity stake, while LTI and MTI are measures of initial mortgage affordability. A clear understanding of the relation between these ratios and the Substitute Tax is particular interesting and important, because if the bank finances the Substitute Tax among the other costs and the capital itself, it means that the bank evaluated the borrower's liability and capability to return back the whole finance. If the Substitute Tax is part of the loan, as *pro quota* capitalized cost, this means that the bank considers the borrower being affordable to return even the extra interests maturing on the Substitute Tax, and not only on the capital lent.

The last element affected by the capitalisation of the Substitute Tax is, again, the PD (Probability of Default). In fact, this mechanism follows the variances of the interest rates pricing, as before already explained in this study. In fact, the LTI ratio, which is partially influenced by the capitalisation of the Substitute Tax, affects default probabilities through a different channel. A higher initial LTI ratio does not increase the probability of negative equity; however, it reduces mortgages affordability making borrowing constraints more likely to blind. The level of negative home equity that triggers default becomes less negative, and default probability accordingly increase. This paper does not want to attempt to solve for the housing market equilibrium, neither to offer an equilibrium model about interest rate risk or about mortgage probability of default, but here it is necessary to generally mention this factors that are, anyway, crucial. About these last mentioned factors (interest rate risk and PD), we emphasize the influence of the Substitute Tax on realized and expected return and expected possibility of default of the borrower, that will be obliterated of the Substitute Tax reimbursement through periodic instalments increased of the interest rate of the contract, even on the Substitute Tax *pro quota* periodic payments. Another aspect to consider is that the Substitute Tax comes to assume the role of a sort of down-payment itself, since this tax is deducted from the *traditio* of the loan to the borrower. About this aspect, we find the model of mortgage default of John J. Campbell and Joao F. Cocco ("*A Model of Mortgage Default*", 2014 version). In their paper, the authors found correct formulas about mortgage contracts (see paragraph #2.1.5 of their study), through some steps. The authors assumed in their paper that the household is not allowed to borrow against future labor income. Furthermore, the maximum loan amount is equal to the value of the house less a down-payment. Therefore initial loan amount (D_{i1}):

$$D_{i1} \leq (1 - d_i) P_1 P_1^H H_i$$

where d_i is the required down-payment. The authors used a subscript *i* for the required down-payment to allow for the possibility that it differs across households and the authors simplified the model by assuming that the household would have financed the initial purchase of the house of size H_i with previously accumulated savings and a nominal mortgage loan equal to the maximum allowed, of $(1 - d_i) * H_i$.

Therefore, for the authors in comment, the LTV and LTI ratios at mortgage origination would be given by:

$$LTV_i = (1 - d_i)$$
$$LTI_i = \frac{(1 - d_i) H_i}{L_{i1}}$$

where L_{i1} denotes the level of household labor income at the initial date. Required mortgage payments depend on the type of mortgage. The authors considered several alternative types, including FRMs, ARMs, and ARMs with a teaser rate. Let $Y_T^{i,\,FRM}$ be the interest rate that household *i* pays on a FRM with maturity *T*. It is equal to the expected interest rate over the life of the loan, or the yield on a long-term bond, plus an interest rate premium which depends on loan and borrower characteristics. The date *t* real mortgage payment, M_{it}^{FRM}, is given by the standard annuity formula:

$$M_{it}^{FRM} = \frac{(1-d_i)H_i\left[\left(Y_T^{i,FRM}\right)^{-1} - \left(Y_T^{i,FRM}\left(1+Y_T^{i,FRM}\right)^T\right)^{-1}\right]^{-1}}{P_t}$$

A distinctive feature of the US mortgage market is that FRMs come with a refinancing option that the authors modelled. More precisely, if households take out FRMs when interest rates are high, and rates subsequently decline, then households who have the required level of positive home equity, d_i, may decide to refinance the loan and take advantage of the lower interest rates. The authors assumed that refinancing costs are equal to a proportion c_r of loan amount. The authors, in their paper, also assumed that households refinance into a FRM with remaining maturity $T - t_r + 1$, where t_r denotes the period of the refinancing. More precisely, the authors assumed that households refinance into the contract and the borrowing position that they would have been in period t_r, had the interest rates at the time that the loan began been lower.

Let $Y_{1t}^{i,ARM}$ be the one-period nominal interest rate on an ARM taken out by household i, and let D_{it}^{ARM} be the nominal principal amount outstanding at date t. The date t real mortgage payment, M_{it}^{ARM} is given by:

$$M_{it}^{ARM} = \frac{Y_{1t}^{i,ARM} D_{it}^{ARM} + \Delta D_{i,t+1}^{ARM}}{P_t}$$

where $\Delta D_{i,t+1}^{ARM}$ is the component of the mortgage payment at date t that goes to pay down principal rather than pay interest. The authors assumed that for the ARM the principal loan repayments, $\Delta D_{i,t+1}^{ARM}$, equal those that occur for the FRM. This assumption simplifies the solution of the model since the outstanding mortgage balance is not a state variable. The date t nominal interest rate for the ARM is equal to the short rate plus a constant premium:

$$Y_{1t}^{i,ARM} = Y_{1t} + \psi^{i,ARM}$$

where the mortgage premium $\psi^{i,ARM}$ compensates the lender for the prepayment and default risk of borrower i. In the case of an ARM with a teaser rate, the mortgage premium is set to zero for one initial period. For a FRM the interest rate is fixed and equals the average interest rate over the loan maturity (the average zero-coupon bond yield for that maturity under the expectations hypothesis of the term structure) plus a premium $\psi^{i,ARM}$. In addition to prepayment and default risk, the FRM premium compensates the lender for the interest rate refinancing option that borrowers receive. At times when the one-year yield is low (high), the term structure is upward (downward) sloping, and long-term rates are higher (lower) than short-term rates. The study of Cocco and Campbell here recalled demonstrated through different models that default even depends by low resources given to the borrowers. Even the Substitute Tax generates this reducing of resources; since it is deducted from the capital lent at the time of the loan origin, representing an operation similar to an initial down-payment, and, therefore, could influence the probability of default like a “dual triggers” factor. This reduction on the capital lent due to the Substitute Tax has different influences on the probability of default up to the kind of mortgage, if ARM or FRM, and up to the eventual options of the borrowers to change the kind of mortgage interest rate, from adjustable to fixed, or vice versa.

ARE IRR AND APR EQUIVALENT? THE THINGS ARE NOT THAT SIMPLE

When talking about loanable funds theory, the first immediately consequence of this matter is the inflation which leads lenders to require a higher rate of interest, because, as previously evidenced, the loanable funds should reflect the equilibrium interest rate given by the matching of the demand and the supply of funds. Since the equilibrium interest rate depicts how interest rates are determined, the interest rate is reflecting also the inflation, the inflation influences the interest rates vice versa, and the Substitute

Tax is a component of the loanable funds, we cannot ignore inflation rate from this positioning paper in its influencing aspects into it, or, better to say, the inflation premium.

In fact, the real rate of interest can differ from the nominal interest rate due to some extra factors that might shift the demand for loanable funds in all the sectors of the economy, such as the inflation. In other words, in addition to supply-and-demand relationships, a various number of specific factors could determine the interest rates, such as the inflation premium that is the additional expected return to compensate for anticipated inflation over the life span of loans. As already previously said in this paper, the inflation represents another impacting factor over the loanable funds quantity, over the loanable funds expectations and over the expected return to compensate the lenders of their credits lent. In this field, another principle already mentioned in this positioning paper about US and EU APR is the discount factor that means the conversions of future payments to today, using a discount rate, very often equalized to an expected inflation rate. We do not want to deal here with the scholar concepts about the Net Present Value (NPV), for which we address the reader to study, but we consider more interesting to consider the common instrument existing between the NPV and the US and EU APR: discount factors of future cashflows to today.

In fact, US APR and EU APR formulas remind, somehow, the discount factors principles concerning future cashflows that, for APR, applies the principle of discounting future payments due by borrowers to evaluate the future costs for the borrower to today. This theoretical common aspect brought some people, especially in Italy, to consider EU APR equivalents to NPV equivalents at zero value. In fact, many senior expert witnesses in Italy on banking consider the EU APR to the Internal Rate of Return (IRR) for a cash flow. Stated that IRR is the discount rate at which the NPV is zero, whoever assumes that the IRR is the EU APR makes the things very simple and easy, which is not, at all.

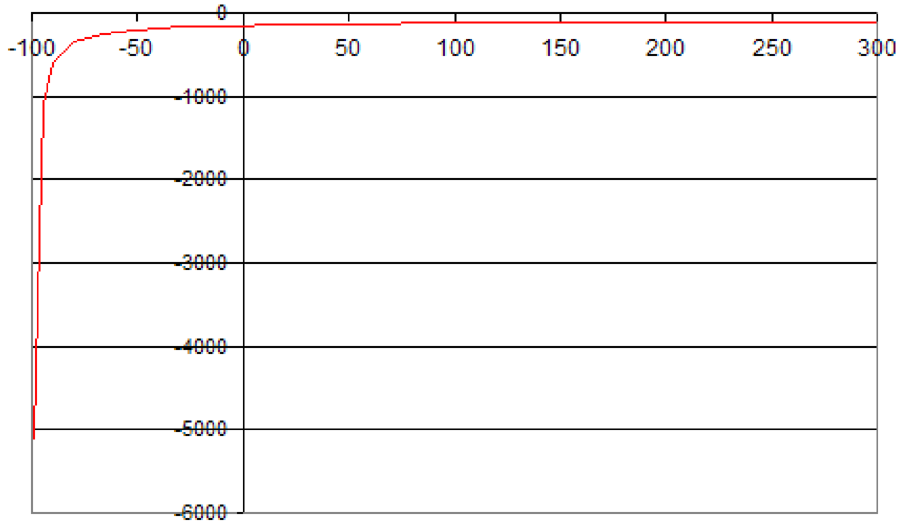
The first main difference between the IRR and the APR (bot US and EU) is the exactness: while the APR could have just only one value, deriving from its formula, the IRR could have many, different or even uncertain. One of the reasons of this uncertainty is because we have not guarantee that for any arbitrary cash flow NPV will ever reach zero value. Furthermore, if it does, we have not guarantee that this will happen at only one point.

Before proceeding, we must remember the NPV equation:

$$\text{NPV} = f_0 + \frac{f_1}{1+d} + \cdots + \frac{f_3}{(1+d)^3}$$

NPV	net present value
f_0	initial cash flow
f_1	cash flow 1
f_3	cash flow 3
d	discount rate

The simplest example of a case when NPV never reaches zero value is when the money flows in only one direction, for example, when we have just a series of negative cash flows because we suppose that we will never receive anything back, i.e. -$ 100.00 and, then, -$ 50. For this example, then NPV at 5.00% discount rate comes to be -$ 147.50 but there is not IRR, because NPV would never record a zero value in its track, as follows:



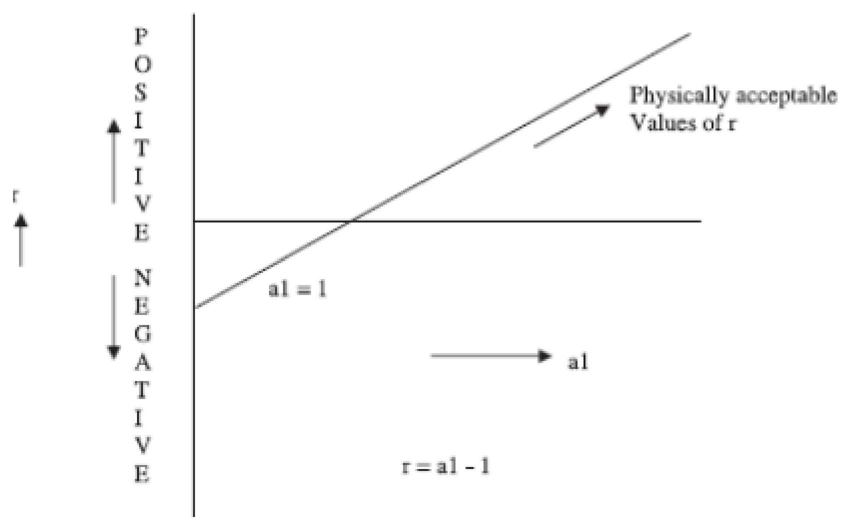
This graph depicts the discount rate that only influences the second loss of $ 50 we recorded: the higher is the negative discount rate the higher would be our loss. On the contrary, the higher is the discount rate the higher is the approximate reduction to zero value of the NPV (+∞)= -100.00 because our initial loss of $ 100.00 would never be erased.

Another example of non-existence of IRR is the following project:

	CF_0	CF_1	CF_2
Project1	–105	250	–150

In fact, the sum of all the cash flows after the initial cash outflow is not enough to cover the original outflow (250 – 150 = 100, which is less than 105), that means that to have a positive and real IRR we need a cash inflow at the end of one year exceeding the magnitude of the initial cash outflow.

Further, this solution is unique because there is only one solution for *r*.



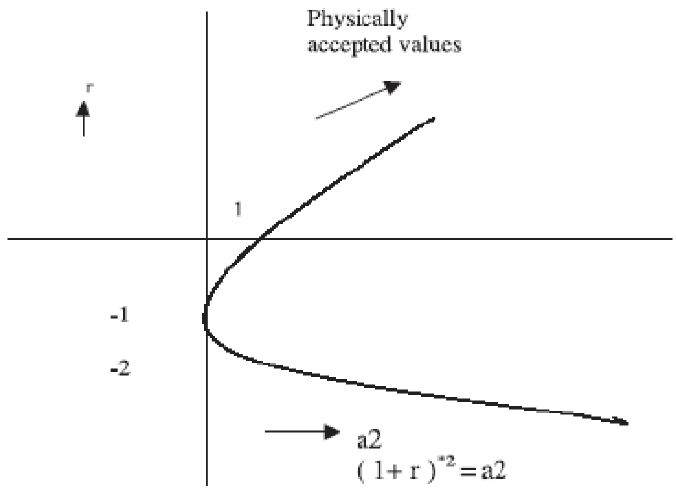
Let suppose a project with an initial cash outflow P_O followed by a sing cash inflow A_1 of after one year, so to say that P_O (1+r) = A_1 where r is the IRR, or (1+r) = a_1 where $a_1 = \frac{A_1}{P_0}$. Otherwise, $r = -1 + a_1$, so a real, positive solution for r can exist only if $a_1 > 1$, for example, $A_1 > P_O$.

Instead, if there is only one cash inflow of A_2 at the end of two years following an initial cash outflow of P_O, then we have two solutions for r:

a) $r = -1 + \sqrt{a_2}$
b) $r = -1 - \sqrt{a_2}$

where $a_2 = \frac{A_2}{P_0}$ and $\sqrt{a_2}$ represents the positive square root of a_2.

With the first possible solution, there would not be any IRR because there would not be any positive and real value. For the second solution, instead, there would be a positive and real value of IRR, just only if $a_2 > 1$, for example $A_2 > P_O$, that means that there would be a possible real and positive value if and only if $A_2 > P_O$.



Now let consider a different scenario where the initial outflow P_0 is followed by only one inflow A_3 at the end of year three. In such a situation, $P_0\,(1+r)^3 = A3$. Or, $(1+r)^3 = a_3$, where $a_3 = \frac{A_3}{P_0}$. The Internal Rate of Return will thus be determined by the equation $(1+r)^3 = a_3$. We will have a real and positive value only if $a_3 > 1$, for example $A_3 > P_0$. The value of the inflow must therefore exceed the amount of the initial outflow. If α, β and γ are the roots of the above equation, then $(r - \alpha)(r - \beta)(r - \gamma) = 0$ or, $r^3 - r^2(\alpha + \beta + \gamma) + r(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$.

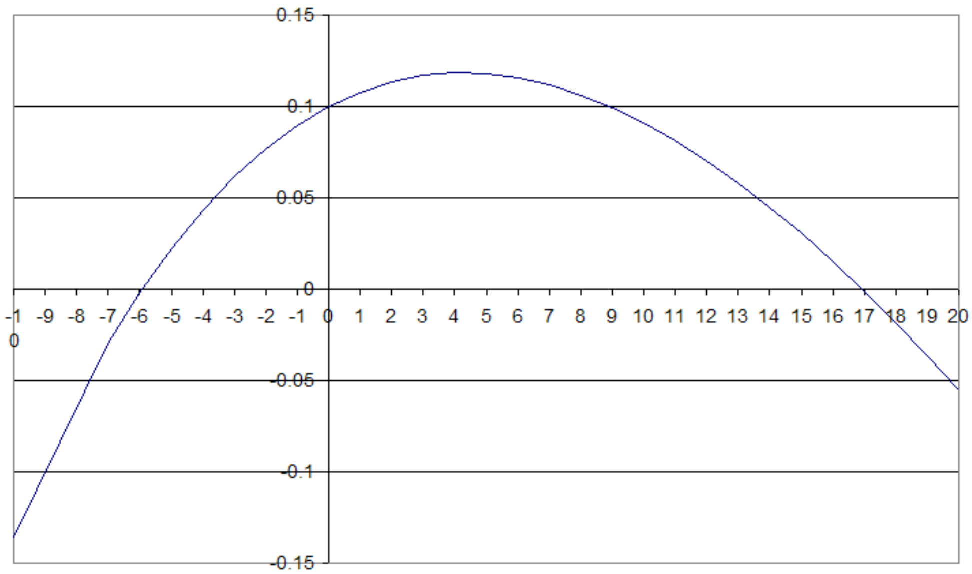
Comparing with the equation $r^3 + 3r^2 + 3r + 1 = a_3$, or $r^3 + 3r^2 + 3r + (1 - a_3) = 0$, we can write down: $\alpha + \beta + \gamma = -3$; $\alpha\beta + \beta\gamma + \gamma\alpha = 3$; and $\alpha\beta\gamma = a_3 - 1$. Since $\alpha + \beta + \gamma = -$ ve, all three roots cannot obviously be positive, but $\alpha\beta\gamma = +$ ve (for $a_3 > 1$); hence there would be either two negative roots or two complex (complex conjugate of each other) roots. Therefore, there is only one physically acceptable solution for r that is, practically, interesting. In other words, a unique IRR exists in this case if and only if $a_3 > 1$ for example $A_3 > P_0$. The other two roots for r are either real, negative or complex (complex conjugate of each other) and need to be ignored. For $a_3 < 1$, however, either no IRR exists or two IRRs exist. Each of the three cases discussed above has a unique, physically acceptable solution for IRR under a given condition. The condition, basically, warrants that the subsequent inflow, only one in number in each of the above cases, must exceed the initial outflow.

The above-evidenced example finds another good one published in Internet by Ivan Krivyakov together with the previous here mentioned, with whom the author demonstrate the possibility of existence of two (or more) NPV values at the same time.

We offer the reader the Krivyakov' example on an irregular cash flow. Let, for example, assume this cash flow:

-10; 21.1; -11.

NPV graph for this cash flow looks as follows:



We drop here Krivyakov' comments on this graph: "*The graph crosses zero at two points: near -6% and near 17%. This means that we could claim that we are losing around 6% a year on average* **or** *that we are gaining around 17% a year at the same time!*

This happens because the net value of our cash flow is almost zero, and the discount rate allows the second and the third payment to play against each other. With low discount rate the second payment gains the upper hand and overall NPV is positive. As discount rate increases, the importance of the second and third year decreases and at some point they fail to compensate for the initial loss of $10. At the discount rate of positive infinity, our NPV would be exactly -10.

On the other hand, if we slide into the negative discount rate territory, this increases the importance of the last year loss. At some point it overtakes the second year gain and will drive NPV further and further down, approaching negative infinity as the discount rate approaches -100%.

This shows that the concept of IRR, although intuitively obvious, may sometimes lead to surprising results, where you gain and lose money at the same time".

The equation to calculate the IRR is the following:

$0 = f_0 + \frac{f_1}{1+\mathrm{IRR}} + \cdots + \frac{f_3}{(1+\mathrm{IRR})^3}$	
IRR	internal rate of return
f_0	initial cash flow
f_1	cash flow 1
f_3	cash flow 3

The "*near 17%*" mentioned by Krivyakov comes from the following calculus:

internal rate of return	
initial cash flow	– €10.00 (euros)
cash flow 1	€21.10 (euros)
cash flow 2	– €11.00 (euros)
cash flow 3	€0 (euros)
internal rate of return	16.91% = 0.1691

That confirms the result.

After this empiric example, with the possibility of a NPV never equals to zero, we can assume that there would be the possibility of the inexistence of the IRR as well, since it is the NPV equals to zero.

On the contrary, although the IRR on this example would not exist, the APR would find a value anyway. Let use as source the material made available by MIT to proceed on the following examples and schemes.

We can schematize the IRR for a generic Treasury bond, perhaps:

$$B_1 = \{x_0, x_1\}/\{t_0, t_1\} = \{-C_0, C_1\}/\{0, 1/4\}.$$

(In this example, we have considered a three-month Treasury bond, with C_0 as issue price, and C_1 as face value).

Stated that B_1 is our NPV and i is the annual interest rate, the NPV is the following:

$$A(0, B_1) = -C_0 + C_1(1 + i)^{-1/4},$$

That it nulls with

$$i = \left(\frac{C_1}{C_0}\right)^4 - 1$$

Let now consider a three-year Treasury par bond, at the strike price C equals to the face value e with annual coupon I:

$$B_2 = \{-C, I, I, I, I, C+I\}/\{0, 1/2, 1, 3/2, 2, 5/2, 3\}.$$

Let call $i^* = i_{1/2}$ the relative six-monthly interest rate. In this case, the NPV is the following:

$$A(0, B_2) = -C_0 + I(1 + i^*)^{-1} + I(1 + i^*)^{-2} + I(1 + i^*)^{-3} + I(1 + i^*)^{-4} + I(1 + i^*)^{-5} + I(1 + i^*)^{-6} =$$

$$= I\,\frac{1-(1+i)^{-6}}{i} - C\,(1-(1+i)^{-6}) = (1-(1+i)^{-6})\left(\frac{I}{i^*} - C\right)$$

That it nulls only with

$$i^* = \frac{I}{C}$$

Therefore, the IRR for a par bond Treasury equals to the relationship between a fixed annual coupon bonds and its NPV. We make an empiric example: let suppose two six-monthly zero coupon bonds for which we want to calculate their IRRs and the relative spread between the two, as follows:

$$ZCB_1 = \{-97.532,\ 100\}/\{0,\ 1/2\}.$$

$$ZCB_2 = \{-96.111,\ 100\}/\{0,\ 1/2\}.$$

Let calculate the two IRRs separately as follows:

$$A(0, ZCB_1) = -97.532 + 100\,(1 + i)^{-1/2} = 0 \leftrightarrow i_1\ \left(\frac{100}{97.532}\right)^2 - 1 = 5.12\%$$

$$A(0, ZCB_2) = -96.111 + 100\,(1 + i)^{-1/2} = 0 \leftrightarrow i_2\ \left(\frac{100}{96.111}\right)^2 - 1 = 8.25\%$$

Therefore, the spread between ZCB_2 and ZCB_1 is the following:

8.25 – 5.12 = 3.13% that means 313 bps

Let consider this loan example:

$$F = \underline{\mathbf{x}} / \underline{\mathbf{t}} = \{100,\ -10,\ -10,\ -110\}/\{0,\ 1,\ 2,\ 3\}.$$

We want to calculate the IRR of this investment, if it exists.

We need to use the NPV formula keeping i^* as variable.

$$NPV\,(i^*, F) = 100 + (-10)\,(1 + i^*)^{-1} + (-10)\,(1 + i^*)^{-2} + (-10)\,(1 + i^*)^{-3} = 0$$

and through $v = (1 + i^*)^{-1}$ we obtain the following equation:

$$11v^3 + v^2 + v - 10 = 0$$
$$11v^3 + v^2 + v - 10 = 10v^3 + v^3 + v^2 + v - 10 = 10\,(v^3 - 1) + v\,(v^2 + v + 1) =$$
$$= 10\,(v - 1)\,(v^2 + v + 1) + v\,(v^2 + v + 1) =$$
$$= [\,10\,(v - 1) + v\,]\ (v^2 + v + 1) = (11v - 10)\,(v^2 + v + 1) = 0$$

That admits just one solution, from which we obtain the IRR:

$$v = \frac{11}{10} \leftrightarrow 1 + i^* = \frac{11}{10} \leftrightarrow i^* = \frac{1}{10} = 10.00\%$$

For example, in other cases when the duration is longer, we must use an iteration function. Let make another example as follows, in which we want to calculate the IRR at three digits approximation:

$$O = \underline{\mathbf{x}} / \underline{\mathbf{t}} = \{-100,\ 20,\ 30,\ 40,\ 50\}/\{0,\ 1,\ 2,\ 3,\ 4\}.$$

Let write, directly, the equation of the variable:

$v = (1 + i^*)^{-1}$ that represents the discount factor from which we reduce the problem to the determination of a zero of the following function:

$$F\,(v) = 50v^4 + 40v^3 + 30v^2 + 20v - 100 = 0$$

Let apply the approximation method subsequent the interval (0, 1):

Since F (0) = −100, F (1) = 40, we calculate $F = \frac{1}{2}$ = −74.375

Hence, let consider the interval ($\frac{1}{2}$, 1) and let calculate the value of $F(v)$ in its average point of $\frac{3}{4}$ = 0.75

Through iteration process, with decimal numbers, we will have:

F (0, 875) = −3, 425,

F (0, 90625) = 6, 261,

F (0, 8828125) = −1, 071,

F (0, 9375) = 16, 699,

F (0, 890625) = 1, 326,

F (0, 88671875) = 0, 121;

At this stage, we know exactly that the solution is between 0, 8828125 and 0, 88671875 and therefore we have already two entire values after the decimal point.

Let split in twice the interval to find the third value:

F (0, 884765625) = −0, 476,

F (0, 8857421875) = −0, 177,

F (0, 88623046875) = −0, 028,

Then, at the third decimal number, the value v^* in which F (v^*) = 0 it is roundly 0.886 that means an annual interest rate $i^* = \frac{1}{v*} - 1$= 12.86% that is the IRR of the investment O.

Therefore, we can state that we have an IRR when the NPV polynomial equation in v admits a solution between zero (0) and one (1).

The most popular result is of Carl J. Norstrøm in 1972: stated that

$$O = \underline{\mathbf{x}} / \underline{\mathbf{t}} = \{x_1, \ldots, x_m\}/\{t_1, \ldots, t_m\}$$

If these hypothesis:

- $X_1 < 0$,
- $X_k > 0$, per $k = 2, \ldots, m$,
- $X_1 + X_2 + \ldots + X_m > 0$,

Therefore, O have a positive IRR. One of the targets of this paper is to demonstrate that IRR and APR have some common aspects, but they cannot be equal to each other. As previously stated, if the APR is always determined, IRR is not, so that we cannot assume that IRR and APR are the same. To let the reader had better understand this point, it is useful to demonstrate some cases in which while APR is just one possible solution, IRR could have various and different value, or none at all!

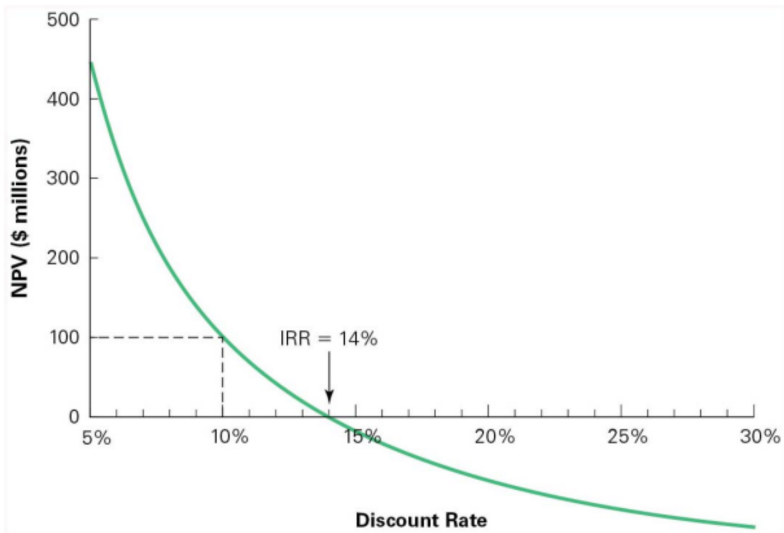
We found in Pearson Education publishing some good examples for this evidence, specifically in the textbook Corporate Finance, fifth edition, Berk / De Marzo in its chapter 7, that we quote here below. In fact, Berk and De Marzo discussed about the controversy between NPV and IRR that will confirm in this paper, automatically, that the IRR cannot be the APR. The authors in their book evidenced some cases with empiric calculus demonstrating the exactness of IRR respect to NPV, and vice versa. The first example the author choose is a take-it-or-leave-it investment decision involving a single stand-alone project for Fredrick's Feed and Farm (FFF). They supposed a project cost of $250 million and expected it to generate cash flows for $35 million per year, starting yet the end of the first year and lasting forever. Berk and De Marzo proceeded as follows:

The NPV of the project is

$$NPV = -\,250 + \frac{35}{r}$$

The NPV is dependent on the discount rate.

FIGURE 7.1 OF THE BOOK



If FFF's cost of capital is 10.00%, the NPV is $100 million, and they should undertake the investment. There are some alternative investment rules that may give the same answer as the NPV rule but at other times, they may disagree. In case of rules conflict, the NPV decision rule is the one to follow. An alternative rule is the IRR, as already said in this paper, that often, but not always, corresponds to be the NPV result. "*In general, the IRR rule works for a stand-alone project if all of the project's negative cash flows precede its positive cash flows. In Figure 7.1, whenever the cost of capital is below the IRR of 14%, the project has a positive NPV, and you should undertake the investment. In other cases, the IRR rule may disagree with the N P V rule and thus be incorrect. – Situations where the IRR rule and NPV rule may be in conflict: ♣ Delayed Investments*

- ♣ *Non-existent IRR*
- ♣ *Multiple IRRs*

Assume you have just retired as the C E O of a successful company. A major publisher has offered you a book deal. The publisher will pay you $1 million upfront if you agree to write a book about your experiences. You estimate that it will take three years to write the book. The time you spend writing will cause you to give up speaking engagements amounting to $500,000 per year. You estimate your opportunity cost to be 10%.

Should you accept the deal?

– Calculate the IRR

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		1,000,000	−500,000	0	
Solve for 1		23.38%				=RATE(3,500000,1000000,0)

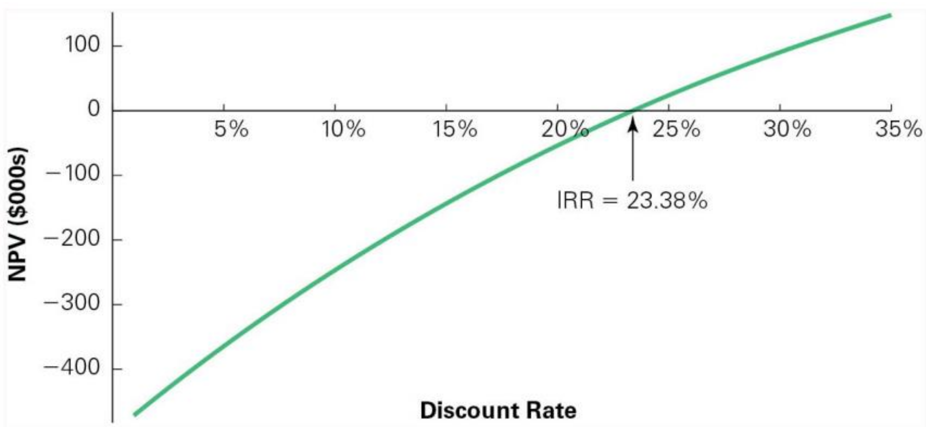
– *The IRR is greater than the cost of capital*
– *Thus, the IRR rule indicates you should accept the deal*

Should you accept the deal?

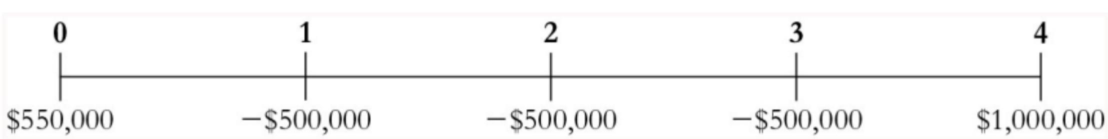
$$\text{NPV} = 1{,}000{,}000.00 - \frac{500{,}000.00}{1.1} - \frac{500{,}000.00}{1.1^2} - \frac{500{,}000.00}{1.1^3} = \$\ 243{,}426.00$$

Since the NPV is negative, the NPV rule indicates you should reject the deal.

FIGURE 7.2 OF THE BOOK



When the benefits of an investment occur before the costs, the NPV is an increasing function of the discount rate. Suppose Star informs the publisher that it needs to sweeten the deal before he will accept it. The publisher offers $550,000 advance and $1,000,000 in four years when the book is published. Should he accept or reject the new offer? The cash flows would now look like

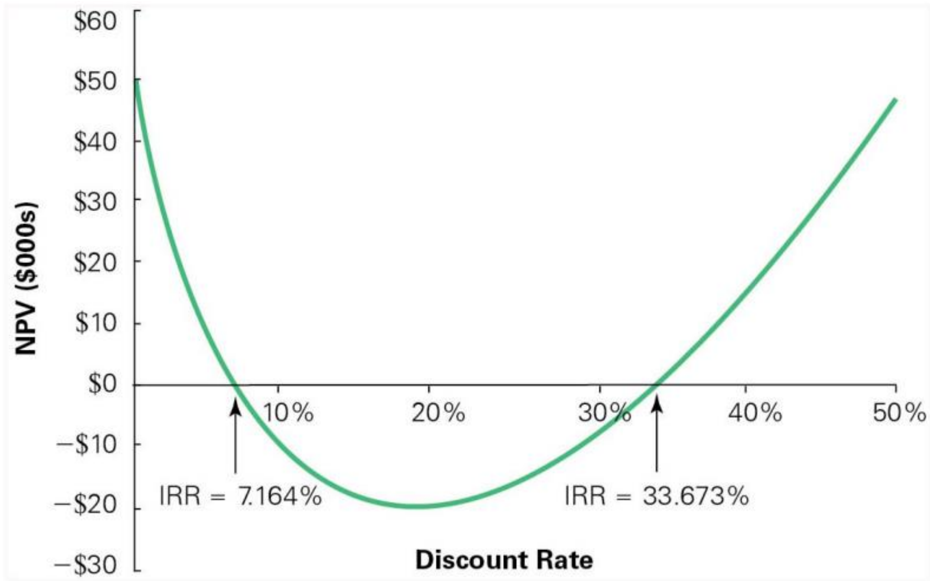


The NPV is calculated as

$$\text{NPV} = 550{,}000.00 - \frac{500{,}000.00}{(1+r)} - \frac{500{,}000.00}{(1+r)^2} - \frac{500{,}000.00}{(1+r)^3} - \frac{1{,}000{,}000.00}{(1+r)^4}$$

By setting the NPV equal to zero and solving for r, we find the IRR. In this case, there are two IRRs: 7.164% and 33.673%. Because there is more than one IRR, the IRR rule cannot be applied.

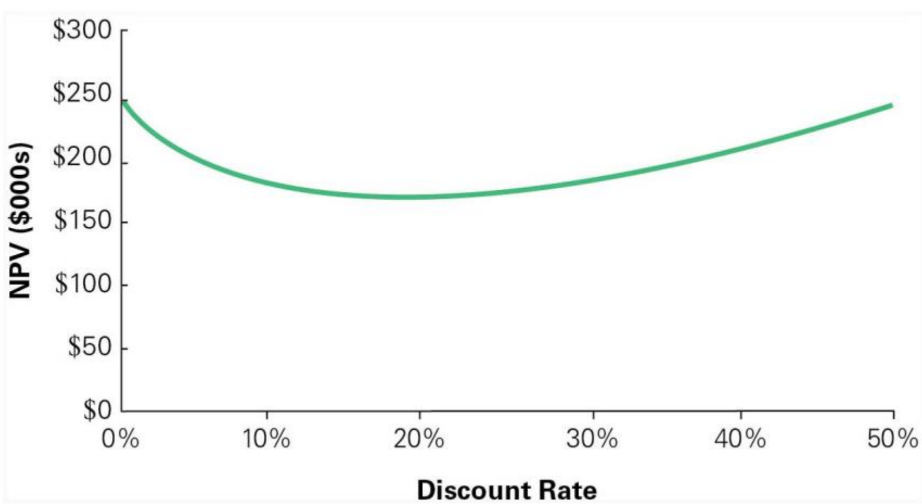
FIGURE 7.3 OF THE BOOK



As seen in Figure 7.3 [the above attached , *between 7.164% and 33.673%, the book deal has a negative N P V . • Since your opportunity cost of capital is 10%, you should reject the deal.*

Finally, Star is able to get the publisher to increase his advance to $750,000, in addition to the $1 million when the book is published in four years. With these cash flows, no IRR exists; there is no discount rate that makes NPV equal to zero.

FIGURE 7.4 OF THE BOOK



No IRR exists because the NPV is positive for all values of the discount rate. Thus the IRR rule cannot be used. IRR Versus the IRR Rule

– While the IRR rule has shortcomings for making investment decisions, the IRR itself remains useful. IRR measures the average return of the investment and the sensitivity of the NPV to any estimation error in the cost of capital.

Problem With the IRR Rule

Problem [Example #7.1 of the book]

Consider projects with the following cash flows:

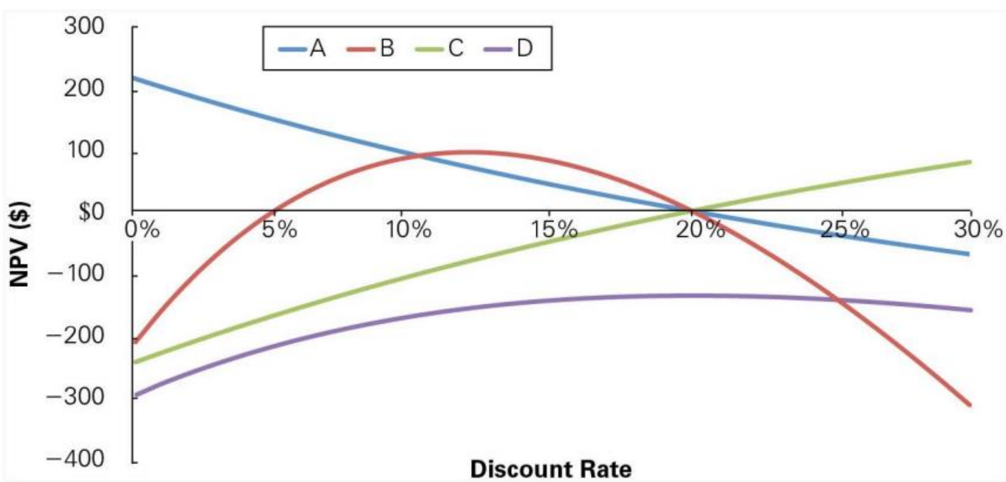
Project	0	1	2
A	−375	−300	900
B	−22,222	50,000	−28,000
C	400	400	−1,056
D	−4,300	10,000	−6,000

Which of these projects have an IRR close to 20%? For which of these projects is the IRR rule valid?

Solution

We plot the NPV profile for each project in Figure 7.5 from the NPV profiles, we can see that projects A, B, and C each have an IRR of approximately 20%, which project D has no IRR. Note also that project B has another IRR of 5%. The IRR rule is valid only if the project has a positive N P V for every discount rate below the IRR. Thus, the IRR rule is only valid for project A. this project is the only one for which all the negative cash flows precede the positive ones.

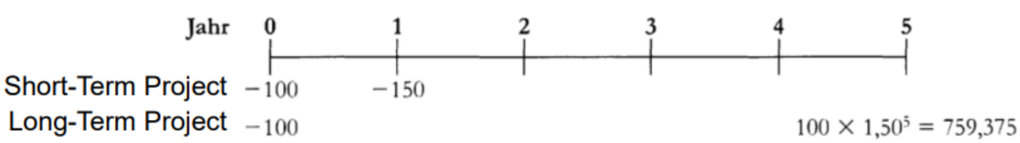
While the IRR Rule works for project A, it fails for each of the other projects.



(…)

Another problem with the IRR is that it can be affected by changing the timing of the cash flows, even when the scale is the same.

– IRR is a return, but the dollar value of earning a given return depends on how long the return is earned. Consider two projects. Both have the same initial scale but different horizon. Both have same IRR.



$$\text{WACC} = 10\%$$

$$NPV_{Short-Term} = -100 + \frac{150}{1.1} = \$36.36$$

$$NPV_{Long-Term} = -100 + \frac{759.375}{1.1^5} = \$371.51$$

$$Short-Term: -100 + \frac{150}{1+IRR} = 0 \Rightarrow IRR = 50\%$$

$$Long-Term: -100 + \frac{759.375}{(1+IRR)^5} = 0$$

$$\frac{759.375}{100} = (1+IRR)^5$$

$$\sqrt[5]{7.59375} - 1 = IRR \Rightarrow IRR = 50\%$$

Timing of Cash Flows – Another problem with the IRR is that it can be affected by changing the timing of the cash flows, even when the scale is the same.

♣ IRR is a return, but the dollar value of earning a given return depends on how long the return is earned.

(…)

Problem [Example #7.4 of the book]

Your firm is considering overhauling its production plant. The engineering team has come up with two proposals, one for a minor overhaul and one for a major overhaul. The two options have the following cash flows (in millions of dollars):

Proposal	0	1	2	3
Major overhaul	−10	6	6	6
Minor overhaul	−50	25	25	25

What is the IRR of each proposal? What is the incremental IRR? If the cost of capital for both of these projects is 12%, what should your firm do?

Solution

We can compute the I R R of each proposal using the annuity calculator. For the minor overhaul, the IRR is 36.3%:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		−10	6	0	
Solve for rate		**36.3%**				=RATE(3,6,−10,0)

For the major overhaul, the IRR is 23.4%:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		−50	25	0	
Solve for rate		**23.4%**				=RATE(3,25,−50,0)

Which project is best? Because the projects have different scales, we cannot compare their IRRs directly. To compute the incremental IRR of switching from the minor overhaul to the major overhaul, we first compute the incremental cash flows:

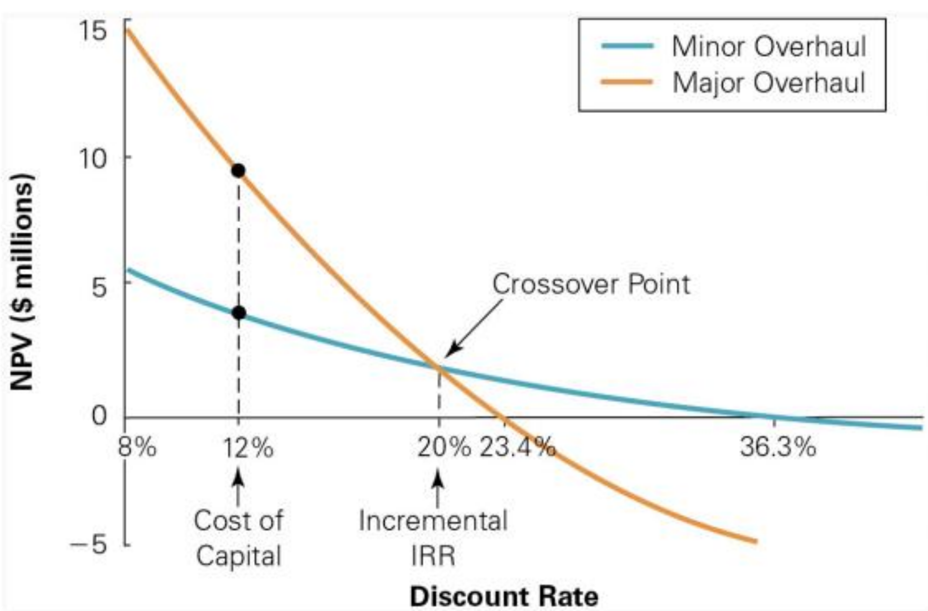
Proposal	0	1	2	3
Major overhaul	−50	25	25	25
Less:Minor overhaul	−(−10)	−6	−6	−6
Incremental cash flow	−40	19	19	19

These cash flows have an IRR of 20.0%

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		−40	19	0	
Solve for Rate		20.0%				=RATE(3,19,−40,0)

Because the incremental IRR exceeds the 12% cost of capital, switching to the major overhaul looks attractive (i.e., its larger scale is sufficient to make up for its lower IRR). We can check this result using Figure 7.5, which shows the N P V profiles for each project. At the 12% cost of capital, the NPV of the major overhaul does indeed exceed that of the minor overhaul, despite its lower IRR. Note also that the incremental IRR determines the crossover point of the NPV profiles, the discount rate for which the best project choice switches from the major overhaul to minor one.

FIGURE 7.5



In Example7.4, we can see that despite its lower IRR, the major overhaul has a higher NPV at the cost of capital of 12%. Note also that the incremental IRR of 20% determines the crossover point or discount rate at which the optimal decision changes. Shortcomings of the Incremental IRR Rule

- *The incremental IRR may not exist.*
- *Multiple incremental IRR scould exist.*
- *The fact that the IRR exceeds the cost of capital for both projects does not imply that either project has a positive NPV*
- *When individual projects have different costs of capital, it is not obvious which cost of capital the incremental IRR should be compared to.*

The above-copied examples operated by Berk /De Marzo in their book shows consistent results pointing out some pitfalls arising out of non-existing or multiples IRRs in certain situations. We shall explore more this scenario for what is concerning the theory of equivalency between IRR and APR (US / EU, indistinctly), because US and/or EU APR is uniqueness while, apparently, IRR could be sometimes various and erroneous, although it is so popular among practicing managers. As repeatedly here said, IRR is the rate of interest at which NPV = zero. For a project with net cash flows, F_j the IRR i^* is given by

$$PV(i^*) = \sum_{j=0}^{N} \frac{F_j}{(1+i^*)^j} = 0$$

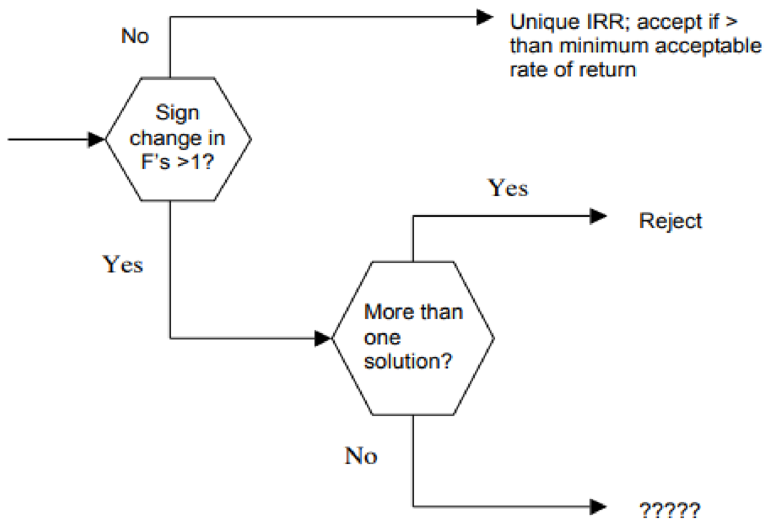
The decision criterion of the project is that if the minimum required rate of return < i^*, we will accept the project, otherwise, if the minimum required rate of return > i^*, we will reject it.

We need to remember that the equation for the IRR is an N^{th} order polynomial in i^*. There will in general be more than one root, and we might have a problem to resolve if we have more than one positive and real roots as possible valid solution. For such a scenario, we need to recall to Descartes' Rule of Sign: that says that for an N-*th* degree polynomial with real coefficients, the number of real, positive roots is never greater than the number of sign changes in the sequence of coefficients.

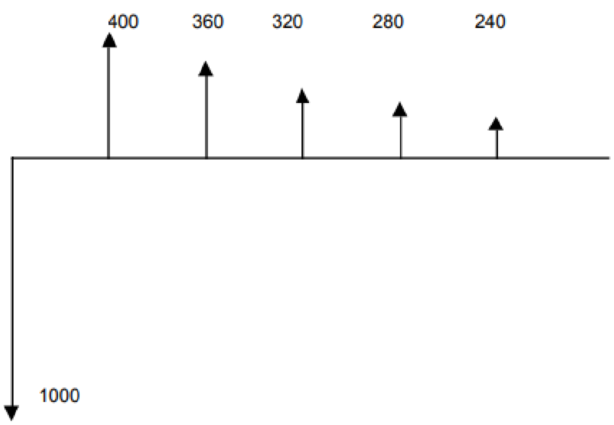
If we write $1/(1+ i^*) = X$, we can rewrite the IRR equation as

$$F_0 + F_1X + F_2X^2 + \ldots + F_N X^N = 0$$

Therefore, if there is only one change of sign in the sequence of cash flows, there will be only one real, positive solution for the IRR. The project that begins with cash outflows, and ends with cash inflows is the typical example of this kind of sequence, with one real and positive IRR. On the other hand, if there is more than one change of sign in the sequence of cash flows, there may be more than one real and positive root; moreover, even if there is only one real and positive root, the result we would obtain may not be meaningful.



We need to specify another concept related to this analysis: the project balance $PB_{(t)}$, that is the amount of money linked to the project at the a given point in time *t* and that the investor would cash out, anytime, during the investment. We could define this amount of money at the point of time *t* as *Vj*, such as the outstanding investment at the given point of time *t*. Let make an empiric example as follows:



Let assume a minimum acceptable rate of return for this project of 10%/yr.

$$PB(10\%)_0 = -\,1{,}000$$

$$PB(10\%)_1 = -\,1{,}000\,(1 + 0.01) + 400$$

$$PB(10\%)_2 = -\,1{,}100\,(1 + 0.01) + 400\,(1 + 0.01) + 360$$

In general,

$$PB(i\,\%)_n = F_0\,(1 + i)^n + F_1\,(1 + i)^{n\text{-}1} + \ldots + F_n$$

and

$$PB(i\,\%)_N = FV(i\,\%)$$

For i = 10%/yr, PB $(10\%)_5$ = + 241.84

In general, if $PB(i)_N > 0$ then the project recovers its initial investment plus the "interest owned" and makes additional profit. Otherwise, if $PB(i)_N = 0$ then the project makes just enough to pay back the initial investment and the "interest owned". **This is the IRR of the project**, but we still need to make a distinction:

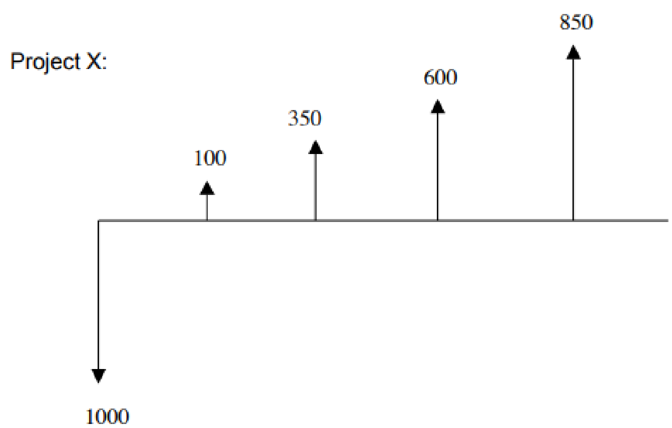
a) Projects for which $PB(i^*)_n \leq 0$ for all $n < N$ called "*Pure investment*" projects;
b) Projects for which $PB(i^*)_n > 0$ for some n called "*Mixed investment*" projects.

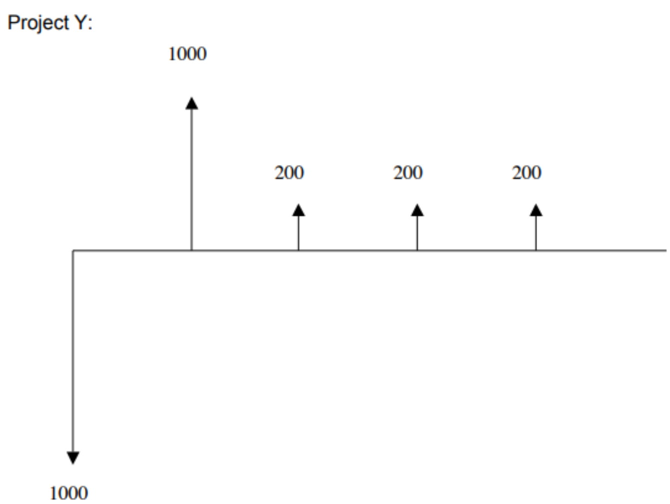
For "*pure investment*" projects, the owner is always a "lender" to the project, and the IRR is the interest rate earned on the committed project balance of the investment, *id est*, the "internal earning rate" of the project. Instead, for "*mixed investment*" projects, the owner operates as a "borrower" to the project at certain times (for example, when PB>0). During these periods, effectively, the owner takes a "loan" out of the project. Therefore, the overall return on the project will depend on the external interest rate that the investor will earn on the surplus, and so, in this case, IRR is not determined and this because IRR method has a meaning for pure investments only.

However, there are situations in which, even for pure investment problems, the IRR approach and the PV approach will lead to apparently contradictory results.

Example (from Riggs and West, p. 134):

Suppose we have two projects, X and Y, and we are trying to decide between them:





Let assume a minimum acceptable rate of return for this project of 10%/yr.

$$PV(10\%)_X = -1{,}000 + 100\,(P/A,10\%,4) + 250\,(P/G,10\%,4) = \$\ 411.56$$

$$PV(10\%)_Y = -1{,}000 + 800\ (P/F,10\%,1) + 200\ (P/A,10\%,4) = \$\ 361.27$$

Hence, we should be inclined to choose towards the Project X.

Instead, let use the IRR method, as follows:

For Project X:

$$-1{,}000 + 100\ (P/A,\ i^*,\ 4) + 250\ (P/G,\ i^*,\ 4) = 0$$

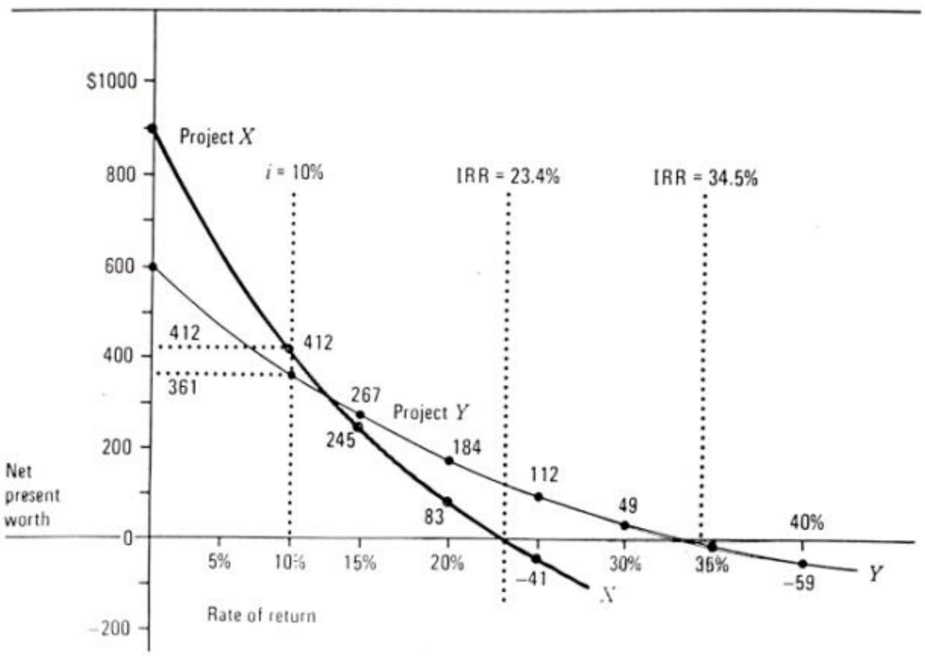
Through iteration process, we obtain $i^* =$ **23.40%**

For Project Y:

$$-1{,}000 + 800\ (P/F,\ i^*,\ 1) + 200\ (P/A,\ i^*,\ 4) = 0$$

Through iteration process, we obtain $i^* =$ **34.50%**

Hence, we should be inclined to choose towards the Project Y as per IRR method, and not, anymore, for the Project X! How to explain this apparent contradiction? The following chart shows how the NPVs of the two projects vary as a function of the interest rate.



Therefore, we can state that:

If MARR < 13.00%, we should choose the Project X

If 13.00% < MARR < 35.00%, we should choose Project Y

If MARR > 35.00%, we should choose neither

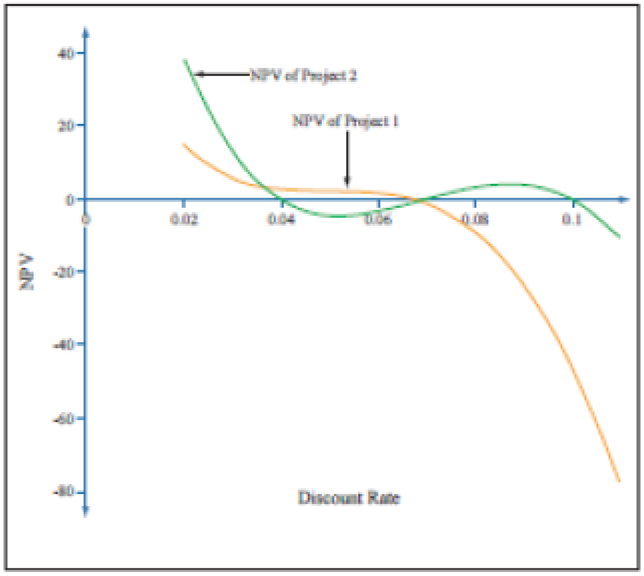
Conclusion on This Point

The IRR method is more complex and more easily misinterpreted than the other methods. If we know the cost of money, it is not necessary to use it and the PV method reaches the right decision in this case, and it is more straightforward than the IRR method.

Most the examples cited have two IRRs, but, in rare cases, three IRRs can exist too. Let take the following projects, as follows, for example:

	CF_0	CF_1	CF_2	CF_3
Project1	–500,000	1,575,000	–1,653,750	578,815
Project2	–500,000	1,605,000	–1,716,900	612,040

If we would calculate the IRR through the IRR method, we will obtain the following graph:



IRR (1) = 7.00%
IRR (2) = 4.00%; 7.00%; 10.00%.

Under rare circumstances, such a slight adjustment of the cash flow pattern, can lead to a sudden switchover from a single IRR to two additional IRRs equidistant from the original solution. This is akin to certain non-linear dynamics (problems leading to bifurcations). Since we expect that such switchovers are quite rare, practicing managers hardly have any reason to be worried about the same.

Anyway, the possibility to have several valid IRRs valid at the same time, for the same project is not a very rare option. While the APR cannot be more than one, IRR can be vary. This problem cannot be resolved even through the known studies and theorems of Norstrøm (1972), De Faro (1973), Aucamp and Eckardt (1976) and Bernard (1979). In fact, all these researchers offered criteria, which represent sufficient conditions to evidence the existence of a unique IRR, of course, only if we meet the criteria. Otherwise (and this is the main problem for this paper), we might have several IRRS, perhaps one, perhaps more. It is true that, very often, it is possible, anyway, to establish the exact number of IRRs and their locations, but this confirms, instead, that it is impossible to consider identical the IRR to the APR. As repeatedly evidenced in this positioning paper, by the time of TILA, or even before it, the goal of the Legislator was to disclosure to consumers, in a clearly manner, the real costs applied to the consumer credit market, in particular for loans and mortgages. The whole disclosure process founds itself on the idea that consumers have the right of the awareness of the consumer credit contracts they are going to subscribe. This because of the responsibilities, in terms of contract meanings and debit exposure, the consumers will be under and will front for the whole life span of the loan received. In several good textbooks, we are led to believe that counting the sign changes in cash flow series is enough to apply Descartes' rule, as well as Norstrøm theorem. To tell the truth, the cash flow series may have more sign changes, but the polynomial expression derived could reveal only one change in sign, and making possible to determine at least one IRR, regardless the number of change of signs in the cash flow series.

For example, the series of expected cash flows [A_t] = [-10, +5, -2, +10, -1] has the following NPV equation:

$$NPV = \frac{-10}{(1+r)^0} + \frac{5}{(1+r)^1} - \frac{2}{(1+r)^2} + \frac{10}{(1+r)^3} - \frac{1}{(1+r)^4}$$

For this example, we have four changes in signs, but the polynomial expression reveals only one change with only one result, that is the following:

internal rate of return	
initial cash flow	– $10.00 (US dollars)
cash flow 1	$5.00 (US dollars)
cash flow 2	– $2.00 (US dollars)
cash flow 3	$10.00 (US dollars)
cash flow 4	– $1.00 (US dollars)

In this case, we recall Budan and Sturm theorems that would help to find at least one root existing even when there are multiple sign changes in the cash flow series, as per Descartes' rule. It is not the goal of this paper to confirm the possibility to find an existing root when, in theory, we could have several IRRs, more than four, and not all possible of determination, but to discuss about the idea that IRR and APR would be equivalent, and they are not.

Another factor between IRR and APR is the currency that must not undervalued. In fact, there might be the possibility to have a loan built under the principle of an embedded derivative with different currencies as underlying. The question is if it is possible that one or more currencies as underlying an investing project could influence the Net Present Value (NPV) of this project, with so much magnitude to make changing the result, from positive to negative, or vice versa.

First, we need to remember that NPV and IRR as well, lays on the world of certainty: this means that we consider sure all the expected future cash flows, risk-free based. In other words, when we calculate the NPV (or IRR, as well) the positive value we come out for it must be considered confirmed without uncertainty. With these preambles, when we have a sequence of expected future cash flows, if these cash flows are denominated in dollars and/or other currency, we should not see any influence on the cash flows of the sequence. In theory, assuming that the exchange rates of the underlying currencies do not change over the time (and it is an important assumption, by the way) there should be not different result in term of the sign of the NPV by changing the exchange rate. In fact, presumably, if we multiply the cash flows by the same number, either positive of one number, or positive of another number, both the currencies and exchange rates will be always positive, because if you multiply a sequence by a positive number, you can anyway factor it out.

Therefore, in theory, we should agree that no matter what we multiply it by, as long as it is a positive number, it could not change the sign, so the currency should not matter. This is in theory, but in practice, we should consider that for the exchange rate bring the actuals at different times. If the exchange rate is the same over time, then when we multiply by one number, it is the same number for every cash flow and we can factor it: if it is positive, it stays positive; if it is negative, it stays negative. This evidence would state that this process is fixed, but it does not mean that it is certain as well: this is a subtlety. In fact, if we assume that the exchange rate is fixed and known, but going up over the time, whatever currency we have, it stays fixed and this makes anyway a difference. For example, there is this possibility if we change the underlying currencies over an embedded derivative, such as an adjusted-rate mortgage (ARM) with variable interests hedged in a foreign currency. In this case, we would have a counter currency over the ARM that would influence the value of the cash inflows during the life span of the loan, especially if the currency rapidly appreciates or rapidly depreciates, and the magnitude of this appreciation or depreciation might change the NPV path. Of course, in such scenario we would take into consideration another main factor that is the risk management world, since the currency fluctuation would be another component of risk to weight for this particular kind of loans hedged in more than one currency.

Even excluding the case of ARM in foreign underlying currency, the supposed equivalency of the IRR method to the EU and/or US APR is not correct.

Let make an empiric example as follows:

- ✓ Mortgage: $ 1,000,000.00;
- ✓ Kind of loan: ARM – constant payment mortgage (CPM)
- ✓ Fully amortisation loan;
- ✓ Monthly payments over 30 years, 360 instalments
- ✓ Annual interest rate: 8.00%.

360 = N, 8% = I/YR, 1000000 = PV, 0 = FV, Compute: PMT = 7337.65.

Solve for "*r*":

$$0 = -\ \$\ 1{,}000{,}000.00 + \sum_{t=1}^{360} \frac{\$\ 7{,}337.65}{(1+r)^t}$$

Obviously, $r = 0.667\%$, → $i = r * m = (0.667\%) * 12 =$ **8.00%** = YTM

Here, YTM = "***contract interest rate***"

Now, let suppose the following empiric example, instead, where the loan has 1.00% origination fee, also known as "prepaid interest", or "discount points", or disbursement discount").

Then PV ≠ L

Where "*L*" is the loan amount

Therefore, borrower receive only $ 990,000.00 – that is lender's disbursement.

Thus: $r = 0.6755\%$, $i = r * m = (0.6755\%) * 12 =$ **8.11%** = YTM

The difference above-evidenced expressed in percentage points derives from the origination fee point and from the mortgage market valuation changes over time. As interest change (or default risk in loan changes) in an ARM, the "secondary market" for loans will place different values on the loan, reflecting the need of investors to earn a different "*going-in*" IRR when they invest in the loan. The market's YTM for the loan is similar to the market's required "*going-in*" IRR for investing in the loan.

In such scenario, APR comes to be a caveat of the bank (lender) from borrower's perspective, at time of loan origination, and so the APR might differ across lenders, because lowest effective cost to borrower may not be from lender with lowest official APR.

Therefore, for an ARM, the **APR represents an expected yield** (*ex ante*) at the time of loan origination, based on the contractual terms of the loan, where the contract does not pre-determine the future interest rate in the loan. Hence, we can assume that APR (both US and/or EU) of an ARM bases on a forecast of future market interest rate (the "index" governing the ARM's applicable rate). We can even state that government regulations require that the "official" APR reported for ARMs bases on a flat forecast of market interest rates (*id est*, the APR is calculated assuming the index rate remains constant at its current level for the life of the loan).

This is a reasonable assumption when the yield curve has its "normal" slightly upward-sloping shape (*id est*, when the shape is due to purely to interest rate risk and preferred habitat). It is a poor assumption for other shapes of the yield curve (*id est*, when bond market expectations imply that future short-term rate are likely to differ from current short-term rates). This brings to another conclusion: YTM differs from the expected returns, that is the mortgage investor's expected total return (the "*going-in*" IRR for mortgage investor), and borrower's "cost of capital", *E[r]*.

The difference between YTM and *E[r]* comes from two reasons:

a) YTM is based on contractual cash flows, ignoring probability of default;

b) YTM assumes loan remains to maturity, even if loan has payment clause.

This is another focal difference between IRR (YTM) and APR (*E[r]*) that could be explicit like a sort of comparison between the expected return and the stated yields that will consider even the measuring of the impact of default risk. Traditionally, we define the expected return (also known as "*expected yield*" or "*ex ante yield*") the probability distribution of future total return on the bond or mortgage investment. Due to its nature, the expected return is the measure that has more fundamental for mortgage investors, when they have to make investment decision. On the other hand, the stated yield (also known as "*contractual yield*") corresponds to be the Yield-To-Maturity based on contractual obligation, and, generally, they are useful in mortgage design and evaluation.

The first main difference between the expected return and the stated yields is the impact of default risk in *ex ante* return investors cares about. We must recall, here, the shortfalls to the lender (mortgage investor) because of default and foreclosures, that is called "*credit losses*", and what the lender (investor) actually receives ("our" IRR), that is called "*realized yield*". Then, we must consider the impact of credit losses on the lender's realized yield as compared to the contractual yield (expressed in IRR units) that is called "*Yield degradation*". The yield degradation ("*YDEGR*") corresponds to be the lender's losses measured as a multi-period lifetime return on the original investment (IRR impact).

Therefore, we can resume these concepts as follows:

Contractual Yield

− **Yield Degradation** ⬅ **Due to *Credit Loss***

= Realized Yield

Let make an empiric and numerical example of Yield Degradation:

- ✓ $ 100.00 loan;
- ✓ 3 years, annual payments in arrears;
- ✓ 10.00% interest rate;
- ✓ Interest-only loan.

Here are the contractual terms of the loan as an NPV equation:

$$0 = -\$\,100.00 + \frac{\$\,10.00}{1+(0.10)} + \frac{\$\,10.00}{(1+(0.10))^2} + \frac{\$\,110.00}{(1+(0.10))^3}$$

Contractual YTM = 10.00%

Let suppose that the loan defaults in 3rd year, that the bank takes the property and sells in foreclosure, but that the bank only gets the 70% of OLB ($ 77.00).

- $ 33.00 = "Credit Losses"
- 70% = "Recovery Rate"
- 30% = "Loss Severity"

Here are the realized cash flows of the loan as an NPV equation:

$$0 = -\ \$\ 100.00 + \frac{\$\ 10.00}{1+(-0.0112)} + \frac{\$\ 10.00}{\left(1+(-0.0112)\right)^2} + \frac{\$\ 110.00}{\left(1+(-0.0112)\right)^3}$$

Realized IRR = - 1.12%
Yield Degradation = 11.12%
Contract YTM – Yield Degradation = Realized Yield:
10.00% - 11.12% = -1.12%

From an ex ante perspective, this 11.12% yield degradation is a "conditional" yield degradation, because it is the yield degradation that will occur if the loan defaults in the third year, and if the lender gets 70% of the OLB at that time, beside the fact that even the 70% is a conditional recovery rate.

Let suppose the default occurred in the 2^{nd} year instead of the 3^{rd}:

$$0 = -\ \$\ 100.00 + \frac{\$\ 10.00}{1+(-0.0711)} + \frac{\$\ 77.00}{\left(1+(-0.0711)\right)^2}$$

Yield Degradation = -17.11%

Keeping the other things equals, in particular the conditional recovery rate, we can state that the conditional yield degradation is greater, the earlier the default occurs in the loan life. From a loan lifetime performance perspective, lenders are hit worse when default occurs early in the life of a mortgage.

<u>*Note*</u>: "*YDEGR*" as defined in the previous example, was the reduction of the IRR (Yield-To-Maturity) below the contract rate, and it was the conditional on default occurring (in the 3^{rd} year), and based on a specific conditional recovery rate (or loss severity) in the event that default occurs.

$$YDEGR_t = YTM - YLD|DEF_t = YTM - IRR\ (loss\ severity_t)\ DEF_t$$

For example, if the loss severity were 20% instead of 30%, then the conditional yield degradation would be 7.13% instead of 11.12%, as follows:

$$0 = -\ \$\ 100.00 + \frac{\$\ 10.00}{1+(0.0287)} + \frac{\$\ 10.00}{\left(1+(0.0287)\right)^2} + \frac{\$\ 88.00}{\left(1+(0.0287)\right)^3}$$

$YDEGR_3$ = 10% - 2.87% = 7.13% that is the expected result.

There are, anyway, some relations among contract yield, conditional yield degradation and the expected return on the mortgage, that could be a reason why, incorrectly, many Italian court experts think that APR and IRR are equivalent.

For example, we must say that the expected return is an *ex ante* measure, for which we must specify, to compute it, the *ex ante* probability of default and the conditional recovery rate (or the conditional loss severity) that will occur in the event of default.

Let suppose that at the time the mortgage is issued, there is:

- ✓ 10% chance of default in 3^{rd} year;
- ✓ 70% conditional recovery rate for such default;
- ✓ No chance of any other default event.

Then at the time of mortgage issuance, the expected return is:

$$E[r] \quad = 8.89\% \quad = (0.9)\ 10.00\% + (0.1)\ (-1.12\%)$$
$$= (0.9)\ 10.00\% + (0.1)\ (10.00\% - 11.12\%)$$

$$= 10.00\% - (0.1)\ (11.12\%) = 8.89\%$$

Therefore, we can state that, in general:

*Expected return = Contract Yield – Probability of Default * Yield Degradation*

$$E[r] = YTM – (PrDEF)\ (YDEGR)$$

Let make other verifications. For example, what would be the expected return if the *ex ante* default probability and conditional credit loss expectations were:

- ✓ 80% chance of no default;
- ✓ 10% chance of default in 2nd year with 70% conditional recovery;
- ✓ 10% chance of default in 3rd year with 70% conditional recovery.

The answer would be:

$$E[r] = YTM - \sum (PrDEF)\ (YDEGR)$$

$$E[r] =10\% - (.1)\ ((11.12\%) – (.1)\ (17.11\%) = 10\% - 2.82\% = 7.18\%.$$

Note: The probabilities we were working with the previous example:

- ✓ 80% chance of no default;
- ✓ 10% chance of default in 2nd year;
- ✓ 10% chance of default in 3rd year.

Were "*unconditional probabilities*" as of the time of mortgage issuance:

- They did not depend on any pre-conditioning event;
- They describe an exhaustive and mutually-exclusive set of possible outcomes for the mortgage, *id est*
- The probabilities sum to 100% across all the eventualities.

More realistic and detailed analysis of mortgage (or bond) default probability (and the resulting impact of credit losses on expected returns, and, consequently, about the hypothetical equivalence of IRR with APR) usually works with conditional probabilities of default, known as "***hazard function***".

The hazard function tells the conditional probability of default at each point in time given that default has not already occurred before then.

For example, let suppose that this is the hazard function for the previous 3-year loan:

Year	Hazard
1	1%
2	2%
3	3%

Id est, there is:

- 1% chance loan will default in the 1st year (i.e., at the time of the first payment);
- 2% chance loan will default in the 2nd year if it has not already defaulted in the 1st year, and
- 3% chance loan will default in the 3rd year given that it has not already defaulted by then.

Given the hazard function for a mortgage, we can compute the cumulative and unconditional default and survival probabilities.

Example:

Let suppose this is the hazard function for the previous 3-year loan:

Year	Hazard
1	1%
2	2%
3	3%

Then the table below computes the unconditional and cumulative default probabilities for this loan as follows:

Year	Hazard	Conditional Survivor	Cumulative Survivor	Unconditional PrDEF	Cumulative PrDEF
1	0.01	1-.01=0.9900	0.99*1.0000=0.9900	.01*1.0000=0.0100	0.0100
2	0.02	1-.02=0.9800	0.98*0.9900=0.9702	.02*0.9900=0.0198	.0100+.0198=0.0298
3	0.03	1-.03=0.9700	0.97*0.9702=0.9411	.03*0.9702=0.0291	.0298+.0291=0.0589

Hence, we can assume that:

- "*Conditional Survival Probability*" (for year *t*) = 1 – *Hazard* for year *t*
- "*Cumulative Survival Probability*" (for year *t*) = Probability loan survives through that year
- "*Unconditional Default Probability*" (for year *t*) = Probability (as of time of loan origination) that loan will default in the given year (*t*) = *Hazard * Cumulative Survival (t-1) = Cumulative Survival (t) – Cumulative Survival (t-1)*.
- "*Cumulative Default Probability" (yr.t)* = Probability (as of time of loan origination) that loan will default any time up through year *t*.

In this case, the unconditional probability (as of time of loan origination) that this loan will default (at some point in its life) is equal to **5.89%** given by 5.89% = 1.00% + 1.98% + 2.91% = 1 – 0.9411.

For each year in the life of the loan, a conditional yield degradation can be computed, conditional on default occurring in that year, and given an assumption about the conditional recovery date in that year.

For example, we saw that with prevision 3-year loan the conditional yield degradation was 11.12% if default occurs in year 3, and 17.11% if default occurred in year 2, in both cases assuming a 70% recovery rate. Similar calculations reveal that the conditional yield degradation would be 22.00% if default occurs in year 1 with an 80% recovery rate. Defaults in each year of a loan's life and no default at all in the life of the loan represent mutually-exclusive events that together exhaust all of the possible default timing occurrences for any loan.

For example, with the three-year loan, borrower will either default in year 1, year 2, year 3, or never. Thus, the expected return on the loan can be computed as the contractual yield minus the sum across all the years of the products of the unconditional default probabilities time the conditional yield degradation.

$$E[r] = YTM - \sum_{t=1}^{T} (PrDEF_t)(YDEGR_t)$$

Example:

- Given previous hazard function (1%, 2%, and 3% for the successive years);
- Given conditional recovery rates (80%, 70%, and 70% for the successive years);
- Expected return on the 3-year 10% mortgage at the time it is issued, it would be:

$$E[r] = 10.00\% - ((.0100)(22.00\%) + (.0198)(17.11\%) + (.0291)(11.12\%))$$
$$= 10.00\% - 0.88\%$$
$$= 9.12\%$$

The 88 basis-point shortfall of the expected return below the contractual yield is the "ex ante yield degradation" (also known as "*unconditional yield degradation*") and it reflects the *ex ante* credit loss expectation in the mortgage as of the time of its issuance. There are two other alternative ways to compute the expected return:

Method 1 – "*Return based*" *E[IRR (CF)]* that takes the expectation over the conditional returns. This is the most commonly used method.

$$E[r] = YTM - \sum_{t=1}^{T} (PrDEF_t)(YDEGR_t)$$
$$= YTM - \sum_{t=1}^{T} (PrDEF_t)(YTM - YLD|DEF_t)$$
$$= (PrNODEF)\, YTM + \sum_{t=1}^{T} (PrDEF_t)(YTM - YLD|DEF_t)$$
$$= \sum_{i=1}^{N} (PrSCEN_i)(YLD_i) = \sum_{i=1}^{N} (PrSCEN_i)(IRR(CF_i))$$

Makes sense if investor preferences are based on the return achieved.

Method 2 – "*Expected CF-based*", or "*Pooled CF-based*" *IRR (E[CF])* that takes the expectation over the conditional cash flows and then compute the return on the expected cash flow stream:

$$E[r] = IRR\left(\sum_{i=1}^{N} (PrSCEN_i)(CF_i)\right)$$

Makes sense if investor preferences are based on the cash flows achieved.

Let take the previous example to apply in case of prepayment after 10-year time, we would have:

$$0 = -\$\,990{,}000.00 + \sum_{t=1}^{120} \frac{\$\,7{,}337.65}{(1+r)^t} + \frac{\$\,877{,}247.00}{(1+r)^{120}}$$

$r = 0.6795\%$, → $E[r]/yr = (0.6795\%) * 12 =$ **8.15%** = YTM

Therefore, we can assume that the shorter is the prepayment horizon, the greater the effect of any disbursement discount on the realistic yield (expected return) on the mortgage.

In addition, prepayment penalties cause effect on similar (slightly smaller) loan yield, as below depicted:

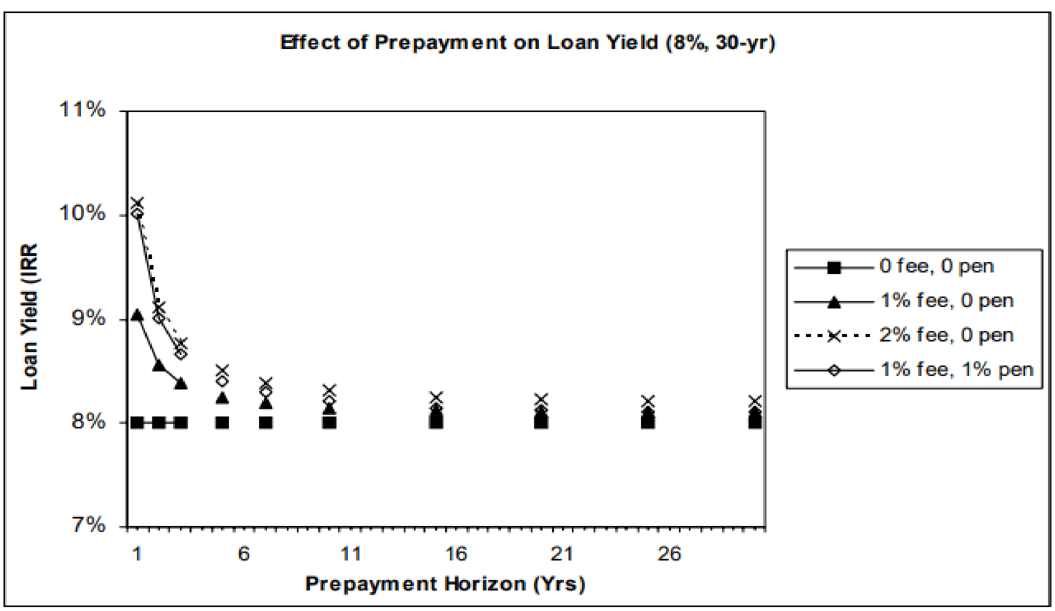


Exhibit 17-2b: Yield (IRR) on 8%, 30-yr CP-FRM:							
	Prepayment Horizon (Yrs)						
Loan Terms:	1	2	3	5	10	20	30
0 fee, 0 pen	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
1% fee, 0 pen	9.05%	8.55%	8.38%	8.25%	8.15%	8.11%	8.11%
2% fee, 0 pen	10.12%	9.11%	8.77%	8.50%	8.31%	8.23%	8.21%
1% fee, 1% pen	10.01%	9.01%	8.67%	8.41%	8.21%	8.13%	8.11%

Notes:

(a) The holding period over which we wish to calculate the yield may not equal to maturity of the loan (for example, if the loan will be paid off early, so *N* may not be the original maturity of the loan;

(b) The actual time-zero present cash flow of the loan may not equal the initial contract principal of the loan (for example, if there are "points" or other closing costs that cause the cash flow disbursed by the lender and/or the cash flow received by the borrower to not equal the contract principal on the loan, *P*);

(c) The actual liquidating payment that pays off the loan at the end of the presumed holding period may not exactly equal the outstanding loan balance at that time (for example, if there is a "prepayment penalty" for paying off the loan early, then the borrower must pay more than the loan balance, so *FV* is then different from *OLB*).

For these reasons, we must always make sure that the amounts in the *N*, *PV* and *FV* registers reflect the actual cash flows.

Let make another example, still adapting the previous case used for examples here, with same loan conditions. Therefore, we need to compute 10-year yield on 8.00%, 30-years, CP-FRM with 1.00-point discount and 1.00-point prepayment penalty.

We need to make the following calculations steps:

1) Calculation of the PMT that comes to be = −.00734;
2) Changing of N to reflect actual expected holding period to compute OLB at the end: FV = −.87725;
3) Adding prepayment penalty to OLB to reflect actual cash flow at that time and enter that amount into FV register: −.87725 x 1.01 = −.88602 → FV;
4) Removing discount points from amt in PV register to reflect actual CF_0: RCL PV 1 x .99 =.99 → PV;
5) Computing interest (yield) of the actual loan cash flows for the 10-years hold now reflect in registers: *CPT I/yr* = **8.21%**

This method is used to value mortgages because the market yield is similar to the expected return ("*going-in*") required by investors in the mortgage market, both in the primary and the secondary mortgage markets.

Let make another example:

- ✓ Mortgage: $ 1,000,000.00;
- ✓ Kind of loan: ARM – constant payment mortgage (CPM)
- ✓ Fully amortisation loan;
- ✓ Monthly payments over 30 years, 360 instalments
- ✓ Annual interest rate: 8.00%.
- ✓ Balloon: 10-year

We want to know how much is this loan worth if the market yield would be currently 7.50% (=7.50/12) = =0.6125%/mo) MEY (*id est*, 7.62% CEY yld in bond mkt).

Answer: $ 1,033,509

$$\$\ 1{,}033{,}509.00 = \sum_{t=1}^{120} \frac{\$\ 7{,}337.65}{(1.00625)^{t}} + \frac{\$\ 877{,}247.00}{(1.00625)^{120}}$$

That is just the inverse of the previous yield computation problem. If you know the required loan amount (from borrower) and the required yield (from mortgage market), then you can compute required PMTs, hence, required contract INT and Points.

Let take the previous example and let consider a market yield of 8.50%, instead of 7.50%, and suppose that we want to know how many Points the lender must charge on 8.00% loan to avoid NPV < 0.

$$\$\ 967{,}888.00 = \sum_{t=1}^{120} \underbrace{\frac{\$\ 7{,}337.65}{(1.0070833)^{t}}}_{=\ 8.50\%/\text{yr}} + \frac{\$\ 877{,}247.00}{(1.0070833)^{120}}$$

Another aspect to consider is the prepayment option that is when the borrower can choose to pay off early, and this decision in option for the borrower influences the IRR evaluation and the relative calculus. Let see how to evaluate this decision in option for the borrower. First, we need to compare two hypothetical loans:

a) The “existing” loan, let say the “old” one;
b) The “new” loan that would replace the “existing” loan.

Traditionally, we would make this comparison using the common DCF (and NPV) methodology. The traditional refinancing calculation is the following, in our example:

- ✓ Mortgage: $ 1,000,000.00;
- ✓ Kind of loan: ARM – constant payment mortgage (CPM)
- ✓ Fully amortisation loan;
- ✓ Monthly payments over 30 years, 360 instalments
- ✓ Annual interest rate: 8.00%.
- ✓ Maturity loan: 10-year
- ✓ Taken out four years ago;
- ✓ 2 points prepayment penalty
- ✓ Expected to be prepaid after another 6 years (at maturity).

$$0 = -\ \$\ 1{,}000{,}000.00 + \sum_{t=1}^{120} \frac{\$\ 7{,}337.65}{\left(1+\frac{.08}{12}\right)^{t}} + \frac{\$\ 877{,}247.00}{\left(1+\frac{.08}{12}\right)^{120}}$$

New loan:

Available at 7.00% interest, 6-year maturity, 30-year amortisation, 1.00-point fee upfront.

a) Step One: Compute Current OCC (based on new loan terms). → = 7.21%, as new 30-year amortisation, 6-year maturity, 7.00%, 1-point loan per $ of loan amt, gives IRR = 7.21%:

PMT = [.07/12, 24*12, .006653] = FV [.07/12, 6*12, .006653] = .926916

$$0 = -\ \$\ 0.99 + \sum_{t=1}^{72} \frac{\$\ 0.006653}{\left(1+\frac{.0721}{12}\right)^{t}} + \frac{\$\ 0.926916}{\left(1+\frac{.0721}{12}\right)^{72}}$$

1.00-point fee upfront

b) Step two: Compute Old Loan Liquidating Payment (= OLB + PPMT Penalty): → = $ 981,434.00 = 1.02 x $ 962,190.00, where:

$$\$\ 962{,}190.00 = \sum_{t=1}^{72} \frac{\$\ 7{,}337.65}{\left(1+\frac{.08}{12}\right)^{t}} + \frac{\$\ 877{,}247.00}{\left(1+\frac{.08}{12}\right)^{72}}$$

c) Step Three: Compute Present Value of Old Loan Liability = $997,654.00, as:

$$\$\ 997{,}654.00 = \sum_{t=1}^{72} \frac{\$\ 7{,}337.65}{\left(1+\frac{.0721}{12}\right)^{t}} + \frac{\$\ 877{,}247.00}{\left(1+\frac{.0721}{12}\right)^{72}}$$

d) Step Four: Compute the NPV of Refinancing

NPV = $ 997,654.00 NPV = $ 997,654.00 - $ 981,434.00 = + $ 16,220.00

- (1.02) 962190 = $ 981,434.00 = Old Loan Liquidating Pmt Amt (incl. penalty).
- 981434 / 0.99 = $ 991,348.00 = New Loan Amt.
- → PMT [.07/12, 30*12, 991348] = $ 6,595.46 / mo.
- → PV [.07/12, 24*12, 6595.46] = FV [.07/12, 6*12, 6595.46] = $ 918,896.00 balloon.

$$\$\ 981{,}434.00 = \sum_{t=1}^{72} \frac{\$\ 6{,}595.46}{\left(1+\frac{.0721}{12}\right)^{t}} + \frac{\$\ 918{,}896.00}{\left(1+\frac{.0721}{12}\right)^{72}}$$

From Previous Step (2)

NPV = $ 997,654.00 - $ 981,434.00 = + $ 16,220.00

From Previous Step (3)

The traditional calculation leaves out the prepayment option value. Let suppose a refinancing transaction cost of $ 10,000.00 (variable X), then according to traditional DCF calculation, we should have:

NPV = $ 16,220.00 - $ 10,000.00 = + $ 6,220.00 → should refinance

Anyway, we should point out that we still leave out some important considerations, such as:

a) Old Loan includes prepayment option;
b) This option has value to borrower;
c) Borrower gives up (loses) the option when she exercises it (prepays the old loan);
d) Hence, Loss of the value of this option is an opportunity cost of refinancing, for the borrower;
e) *Id est*, instead of refinancing today, the borrower could wait and refinance next month, or next year, because it might be better.

In the previous example, the current interest rate is 7.00%. Let suppose that the interest rate of the next year could be either 5.00% (50% of probability) or 9.00% (50% of probability): can either refinance today or wait 1 year.

With 5.00% interest rate New Loan (30-year amortisation, 5-year balloon) → 5.24% yld.

1-year from now Old Loan will have 5-years left (60 month horizon), and OLB^{OLD} = $ 950,699.00, → X1.02 = $ 969,713.00 Liq.Pmt.

➔ PV (CF^{OLD}) = $ 1,062,160.00.

- ➔ NPV (next year, @5.00%) = 1062160 – 969713 – 10000 = + $ 82,448.00.

Similarly, if interest rate next year is 9.00%:

- ➔ NPV (next year, @9.00%) = - $ 75,079.00. Thus, would not prepay: → NPV = 0.
- ➔ Expected Value (as of today) of Prepayment Option next year:
  = (50.00%) $ 82448 + (50.00%) 0= $ 41,224.00.

This option may be quite risky. Suppose it requires an OCC = 30.00%, then:

- ➔ PV (today) of Prepayment Option = 41224/1.30 = $ 31,711.00.
- ➔ NPV (Refinancing today, including opportunity cost of option) = + $ 6,220.00 - $ 31,711.00 < 0:
- ➔ ***Do not refinance today***

The prepayment option value is included in the Market Value of the Old Loan.

Let "*D (Old)*" = Market Value of Old Loan and "*C (Prepay)*" = Market Value of Prepayment Option

$$D\ (Old) = PV\ (CF^{OLD}) - C\ (Prepay)$$

Thus, we can observe the Market Value of Old Loan, than we can compute correct NPV of refinancing as:

$$NPV\ (Prepay) = D\ (Old) - OLB^{OLD} - X$$

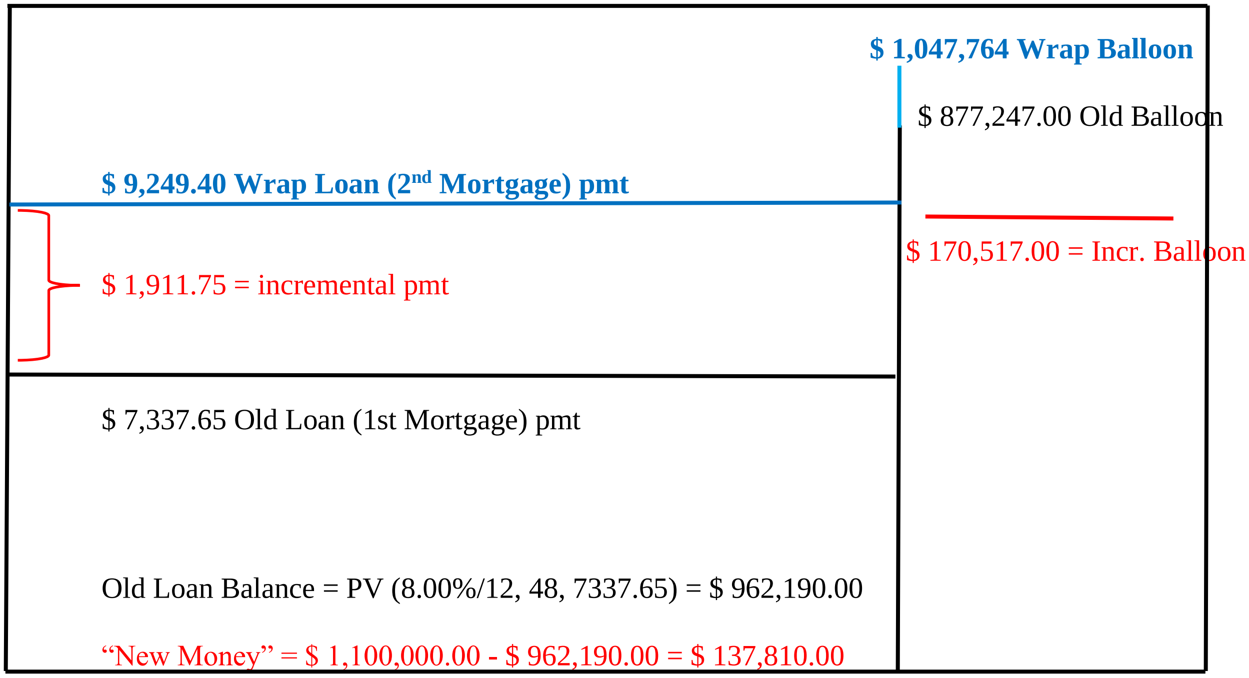
Very often, real estate mortgages are illiquid and this corresponds to be a very difficult possibility to observe their market value. Moreover, many borrowers may not be accounting for the option cost, and/or the effect of a possibility short holding horizon for the old loan due to possibility of a house move, creating, therefore, a concrete hypothesis of too much residential refinancing. Let consider, again, the previous example:

$ 1,000,000.00; 30-years amortisation; 8.00%; 10-years maturity loan. Taken out 4-years ago. Expected to be prepaid after another 6 years (at maturity):

$$0 = -\ \$\ 1{,}000{,}000.00 + \sum_{t=1}^{120} \frac{\$\ 7{,}337.65}{\left(1+\frac{.08}{12}\right)^{t}} + \frac{\$\ 877{,}247.00}{\left(1+\frac{.08}{12}\right)^{120}}$$

Now, let suppose interest rates have gone up instead of down, such that a new 6 years first mortgage would be available at 10.00% interest, 6-year maturity and 30-years amortisation.

Let suppose, now, that the original borrower wants to sell the property, but they hate to lose the value of the below-market-interest old loan, and suppose that the old loan is not "assumable", but has no "due on sale" clause. The seller (the original borrower) could offer buyer a "wraparound" second mortgage at, say, 9.50% (below market rate), and use this to cash out her value in the below her value in the below-market-rate old loan, and help sell the property. Let suppose value of the building is now $ 1,500,000.00, and buyer would want to finance purchase with a $ 1,100,000.00 mortgage. Let suppose wrap has 30-year amortisation, 6-year balloon.

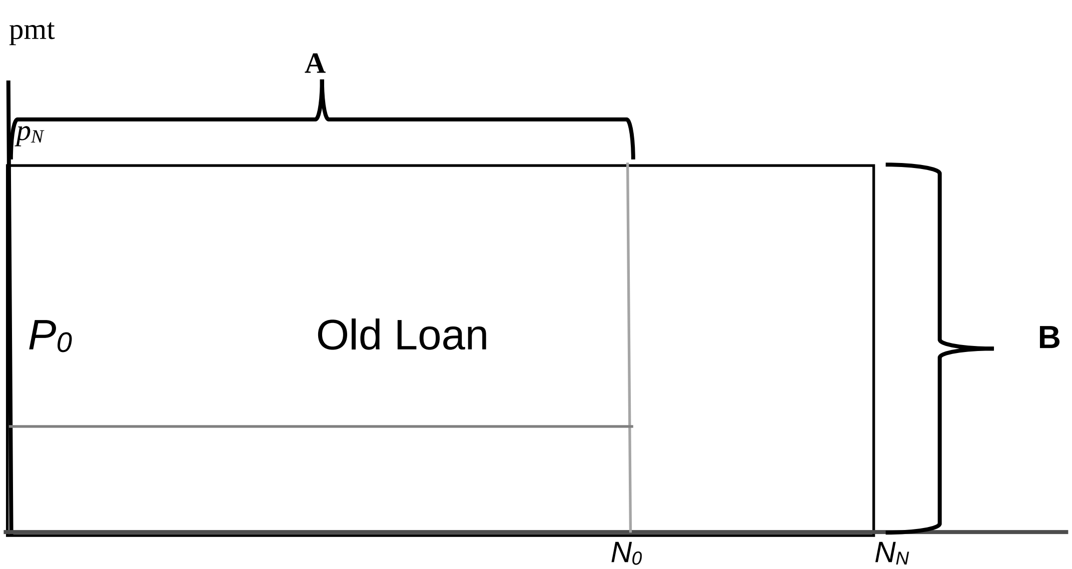


Wrap yield = Rate (72, 1911.75, -137810, 170517) = **18.81%**

The 18.81% wrap yield is a "super-normal" yield (above the OCC of the new money investment), reflecting the positive NPV of the old loan's below-market interest rate, realized by the old loan borrower via the wrap transaction.

At this point, we can enucleate the general wrap loan mechanics as follows:

L_0 = OLD on old loan;
L_N = Contractual initial principal on wrap loan;
p_0 = Pmt on old loan;
p_N = Pmt on wrap loan;
N_0 = Periods left on old loan;
N_N = Periods in wrap loan;
r_N = IRR of wrap loan to wrap lender



"New Money" = $L_N – L_0 = P_V (A @ r_N) + PV (B @ r_N)$

$$L_N – L_0 = (p_N – p_0) \left[\frac{1-1/(1+r_N)^{N_0}}{r_N} \right] + p_N \left[\frac{1-1/(1+r_N)^{N_N-N_0}}{r_N} \right] \left(\frac{1}{(1+r_N)^{N_0}} \right)$$

Solve this equation algebraically for L_N or p_N, given the other variables, or solve it numerically (in calculator or spreadsheet) for r_N given the other variables

Recalling the general formula $\frac{a}{(1+r)} + \frac{a}{(1+r)^2} + \ldots + \frac{a}{(1+r)^N} = a \left(\frac{1-1/(1+r\ )^N}{r} \right)$

Example:

Old loan was originally \$ 1,000,000.00 for 20 years (amortizing) @ 6.00%, taken out 15 years ago, with current OLB = L_O = \$ 370,578.00; pmt = p_0 = \$ 7,164.31/mo.

New (wrap) loan would be for \$ 1,000,000.00 with 20-year amortisation and 10-year balloon, @ 8.00%. The question to answer is to find the yield (IRR) on the new money.

240 = N, 8 = I, 1000000 = PV, 0 = FV; → pmt = \$ 8,364.40 = p_N.
➔ $p_N – p_0$ = 8364.40 – 7164.31 = \$ 1,200.09/mo; N_0 = 240 - 180 = 60;
120 = N; → FV = \$ 689,406.00 = new loan balloon month 120 = N_N.
689406 + 8364.40 = \$ 697,770.00 = last month's CF (month 120).
New Money = \$ 1,000,000.00 - \$ 370,578.00 = \$ 629,422.00 = $L_N – L_0$.
Now if we go to CF keys of calculator:
-629422 = CF0, 1200.09 = CF1, 60 = N1, 8364.4 = CF2, 59 = N2, 697770 = CF3;
→ **IRR** = **8.33%** = y_N.

Another step for this paper is about the comparison of the rates of bonds and mortgages. First, traditionally, bonds pay interests semi-annually (twice per year) and the bond interest rates (and yields) are quoted in nominal annual terms (ENAR), assuming semi-annual compounding (m = 2). This is often called "bond-equivalent yield" (BEY), or "coupon-equivalent yield" (CEY). Thus:

$$EAR = (1 + BEY/2)^2 - 1$$

On the contrary, traditionally, mortgages pay interests monthly and mortgage interest rates (and yields) are quoted in nominal annual terms (ENAR) assuming monthly compounding (m = 12). This is often called "mortgage-equivalent yield" (MEY). Thus:

$$EAR = (1 + MEY/12)^{12} - 1$$

Example #1:

Yields in the bond market are currently 8.00% (CEY). What interest rate must you charge on a mortgage (MEY) if you want to sell it at par value in the bond market?

Answer: **7.8698%**

$$EAR = (1 + BEY/2)^2 – 1 = (1.04)^2 – 1 = 0.0816$$

$$MEY = 12 [(1 + EAR)^{1/12} – 1] = 12 [(1.0816)^{1/12} – 1] = 0.078698$$

Which is the expected result.

Example #2:

We have just issued a mortgage with 10.00% contract interest rate (MEY). We must answer to the following question: how high can yields be in the bond market (BEY) such you can still sell this mortgage at par value in the bond market?

Answer: **10.21%**

$$EAR = (1 + MEY/12)^{12} – 1= (1.00833)^{12} – 1 = 0.1047$$

$$BEY =2\ [(1 + EAR)^{1/2} – 1]= 2\ [(1.1047)^{1/2} – 1]= 0.1021$$

Which is the expected result.

There are many other factors that might influence both IRR and APR, but not at the same magnitude. One of this factor is, for example, the embedded options over a mortgage and/or other kind of loans. In fact, we must recall that an option is the right but not the obligation to do something. For instance, in finance, an option is about the possibility to buy or sell a security at a pre-set price, or, eventually, the possibility of prepayment. Let consider the option regarding the caps and floor rates: with caps, we put a ceiling on the floating rate paid, while with floors we put a lower bound on the floating rate paid, that corresponds to be, respectively, a maximum and a minimum interest rate on the loan. This means fixing a maximum and a minimum guaranteed profit for the lenders, and, consequently, we can assume that even the cash inflows will be directly affected by these options. If for the APR these options have an influence, for the IRR these same options, instead, have a bigger magnitude on the IRR value, because of the floating cash inflows determined directly by these embedded options.

In other words, with the presence of a floor rate clause in the contract, the lender gets protection against low revenues when rates fall, while, on the contrary, with caps rate clause in the contract, the borrower gets the protection from possible very high rates. All these options van be considered like triggered rates, that, eventually, take place to react to high increasing rate levels, or against very rapidly decrease of the interest rates. These kind of options stay together with other, with the same consideration, for what is concerning the cash inflows, such as the deferral, the forbearance, the income-based repayment, the consolidation and the default that affect the timing and/or the size of cash flows to the benefit of borrowers.

CONCLUSIONS

This paper has indicated the differences in the educational level in Europe rather than in U.S.A., about the knowledge of the existence of the APR commonly used in consumer credit market field. In fact, as discussed in this paper, there are many differences between the U.S. APR and the EU APR, not only on technical and financial aspects, but also on level of APR awareness among borrowers. It is notable in these results that differences in the relative impacts on the EU Commission law are more apparent among borrowers, regardless the credit classifications and/or particular types of credit. These differences, anyway, seem to be more relevant in Italy than in other EU locations, where APR awareness is still at a very poor level, especially among professionals, than show, generally, a very low education on finance.

There cannot be surprise if Italy is after Zambia in finance culture and banking awareness, as stated in the recent worldly rank published by Standard's & Poor 2020 survey that stated that Italy is the 63^{rd} country in the world on this classification. In 2021, even the OECD (Organisation for Economic Co-operation and Development) released a survey over the financial culture levels in 26 country members of the OECD. The result of this recent survey demonstrates that Italy is 25^{th} on this rank. While the OECD range evaluation was 13, Italy got only 11.2, much less than the average level of the other countries.

If these evidences mean already a huge gap between Italy and the other countries in the world about financial culture, EU Commission did not help the situation, because of the apparently misleading of Prof. Seckelmann EU APR formula, as he apparently stated by himself, repeatedly.

APR and IRR differ even under the expected returns aspects, since the lender's perspective does not match with the borrower's perspective, reflecting that original "conflict" exposed previously in this positioning paper.

One of the aspects that clearly evidences this other difference between APR and IRR is the YTM in an ARM and the prepayment factors eventually occurring during the life span of the mortgage, with higher magnitude if it is not a FRM.

The list of the factors compromising the identity between the IRR and the APR is very long. In this paper, we have evidenced just few of them, such as the different perspective between lender and borrower regarding the evaluation of the loan, under the IRRs and NPVs aspects on bonds and mortgages evaluation during their lifetimes.

In this paper we showed the differences occurring in the IRRs and NPVs evaluations and calculus when, at parity of bonds, mortgages or scenarios, we might consider other conditions, such as prepayment, refinance, several sign changes in the cash flow series, wrapping. In these cases, we just introduced some basilar concepts in finance that bring to the conclusion that IRRs and NPVs are differently solid indicators by themselves, but often none of them strong, and they both need of further investigations, through other to implement in the preliminary decision making phase and even for the following steps of monitoring and supervisory.

We did not discuss at all of the risk management strongly influencing all the apparatus regarding the decision-making on investment and, consequently, nothing have been reasoned about the component of the risk management models, the risk factors, the risk appetite, and their frameworks. Equally, we have not considered the surely influences on IRRs and NPVs of the companies policy operating in the consumer credit market, the CFO, CRO, CLO, CEO, COO Areas of each lender and how they mutually cooperate and relate inside the company who lend the money.

All these aspects should be separately treated in a specific other paper, weighting the new EU rules and laws that in the recent past had been constantly growing and developing more complex structure to better monitoring and testing the internal frameworks of banks and other financial institutions.

One of the several goals of this positioning paper is to underline the typical structure of a constant mortgage amortisation plan that is build on the compound interest mechanism, but under different perspective. In fact, although worldwide knows that this well-known mechanism has intrinsic compound interest mathematical formula, its lawsuit appears in the only Italian Courts under a different vest. This problem is keeping busy all the Italian Courts with controversy and doubtful results that got the reward of the total confusion.

As already previously explained, when the financial culture is missing, the abnormal case law follows. Without any doubts, actually, Italy is the record holder in this "special" rank, since its jurisprudence is abounding of "exotic" rulings about compound interest. In if in the world, compound interest is a financial mechanism through which interests create other interest (a.k.a. "*anatocism*"), in Italy, it is not, or it is sometimes, but not always, or it is, but it is not, as well, at the same time. In fact, we have judgments of Italian Courts in which they state that although compound interest mechanism is on compound interest, this compounding interest mechanism does not create interest over interest!

In reality, this problem comes from the specific and local laws in use in Italy, in which, it is illegal, the presence of anatocism, regardless the mechanism the lender and the borrower agree on loans, while the mathematics and international laws say different on this matter, where compound interest is lawful. The EU Commission enrolled Prof. Seckelmann to determine a general APR formula that could satisfy all the single EU Members accordingly to their local institutions and rules. Prof. Seckelmann reached the actual EU APR formula, still in use in all EU, over the almost last 25 years, pleasing the instruction received by the EU Commission. Although Prof. Seckelmann had correctly accomplished this task, EU Commission apparently mislead his EU APR formula that, anyway, is more similar the US APY rather than a standard APR. This mislead evidences that APR formula is intrinsic in compound interest, so with anatocism, but on yearly basis. Therefore, the anatocism shows up just after 1-year time, and not before, to safe the Italian rule that does not permit anatocism, generally. The problem sorted, anyway, when banks asked an interim annual interest rate, which, automatically, compound the interest rate upon the periodic interim instalments agreed in the contract of the loan; in other words, what you wanted to be out of the door came in through the window.

This will bring us to another topic of this paper: **uncertainty**. IRRs and NPVs work without considering risks and this means that uncertainty should be off from them completely. On the other side, even APR (both U.S. and/or EU) should not be uncertain and always identified. Unfortunately, IRR, NPV and APR could be uncertain, with the substantial difference that IRR and NPV could be uncertain because of mathematics, while APR could be uncertain because of lack in disclosure. In fact, here in the paper, we demonstrated empirically, mathematically and with theorems that IRR and NPV could be inexact, sometimes, while APR has not this inexactness but eventually just misleading or cunning. For example, if we have 2.00% monthly interest rate, we could have an APR that can vary from 24.00% up to 27.10%, depending from the compounding period, from annually to continuously. The compounding period becomes essential for the APR that, in Italy, we incredibly considering it still like on simple interest and not compounded, even over the 1-year lifetime.

We offer another empiric example. Let imagine a checking account that offer a flat rate of 2.00% APR monthly. This means that the annual APR is the following:

$$\left(1+\frac{2}{100}\right)^{12} = 26.82\%$$

If the bank still would quote a monthly rate of interest at 2.00%, but would add the interest to overdrawn accounts quarterly, this is what it would happen to the APR:

$$\left(1+\frac{6}{100}\right)^{4} = 26.25\%$$

This comes because the bank, on this example, would have compounded quarterly the interests and that means three time 2.00% (that is 6.00%) four times. For instance, if, in the same scenario, the interest rate had compounded continuously, the APR were equal to 27.10% as follow:

$$\lim_{n\to\infty}\left(1+\frac{24}{100n}\right)^{n} = exp\ (0.24) = 1.271 = 27.10\%$$

This above-evidenced example show either that if the disclosure to the consumers is not efficient, frequently, you will find that there is insufficient information to do the calculation, or that you will get a different APR value from the one quoted in the contract because a vital piece of information is missing.

Hence, if we consider the originating meaning of the APR, starting by the U.S. APR, and arriving to the offshoot EU APR, if we consider the EU APR misleading by EU Commission and, especially, by the Italian Courts, the final result is shocking. In this situation, not only banks do not benefit from this chaos, but also they could be the first real victims, because the laws, asymmetrically, increase the gap between theory and practice of the economy rules, especially in the credit consumer market, where the consumers, if not correctly aware of the contracts, could damage and shock the credit market. It is a perfect example, for instance, the case of mortgages that offer the option to the borrower to extinguish the loan earlier, or to refinance it. In fact, as discussed here in this paper, the effect of this option falls on the IRR and the NPV, but not with the same magnitude in the APR, even considering its possible worst scenario.

Effectively, IRR and NPV consider the options, the rights and the titles connected to the investment, while APR consider just the cost of credit and, eventually, the costs reflected by the options, the rights and the title connected to the loan: different positions, like in front of a mirror, but inversed. This to say that the APR mandatory needs the information to be determined, while the IRR and the NPV need the cash flow series known. For the IRR and for the NPV, the options, the rights and the title of the investment influence them through the cash flows. For the APR, instead, the options, the rights and the titles influence the APR only if these options, these rights and these titles produce an identifying cost of credit, added among the other costs of credit indicated in the contracts.

On top of all these evidences, the Substitute Tax on loans constitutes another factor to consider. As discussed in this paper, the Substitute Tax effects on the interest rates, on the expected return and even on the consumer credit market, not only for its cost, but also for the volume of capital "produced" by this tax, directly and indirectly. In Italy, the capitalisation of the Substitute Tax corresponds to be an illegal activity by which banks make profits form money that do not belong to them and they profit by using this extra money for their own business. If we consider that in the Italian consumer credit market this tax weight 0.25% of the total amount of the loan, multiplying this money by the number of contracts, and considering the interest rate applied onto it, the volume of business and the volume of money is very huge. Every year, banks obtains from the 0.25% (if not a 2% for some specific cases) of total loans granted, extra interests at the interest rate of the contracts in circulation. It is not finished about this aspect: in fact, through the Substitute Tax, banks, substantially, "create" *ex nihilo* an extra 0.25% (if not a 2% for some specific cases) of loanable funds, that finish together with the other credit circulating in the consumer credit market, interfering with inflation, interest rate, consumer credit market equilibrium, and so on.

At last, but not least, the kind of amortisation plan influence all the previous aspects discussed in this paper, because in case of constant mortgage, the prominence of the interest pro-quota of each instalment creates an extra advantage for the lender. Of Course, this advantage for the lender becomes another extra financial value to be considered for the IRR, for the NPV and, obviously, for the APR.

At the end of this paper, we can assume that the constant mortgage is on compound interest method, that it influences the effective interest rate. The cost of credit deriving from this kind of amortisation plan influences the US and EU APR formula. As well, the Substitute Tax influence the EU APR value, the consumer credit market, inflation and the interest rates, but even the probability of default of the borrowers. The EU APR is on compound interest method, but on yearly base, while the US APR is better similar to an APY, instead of an APR. The reason of this difference between U.S. and EU APR is, principally, due to its main ideological justification: saving and not cost of credit. Hence, the IRR cannot be the APR, and the APR is not equivalent of the IRR.

In Italy, these discrepancies constitute a gigantic gap in all sectors:

(a) From the financial theory to the financial practice;
(b) From the law to the ruling;
(c) From the financial culture to the said financial experts;
(d) From the financial literacy to the theorems;
(e) From the reality and the truth to the financial sci-fi that is possible to read in the poor quality of Italian said financial experts.

A still very long way to go.