When the Rising Tide Lifts All Boats Differently: Income Distribution Matters

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In 2018, Alonzi, Drougas, and Condon ("ADC") developed a simple macroeconomic model to analyze the effect of a less equal income distribution. This paper builds upon that paper by constructing a model incorporating a rising absolute income in both the high and low groups while at the same time allowing a higher proportion of income to go to the high-income group, but a lower proportion go to the low-income group. Notably, we find that the qualitative results of the "Reverse Robin Hood" case remain in the "Rising Tide" case but there are quantitative differences.

Keywords: income distribution, aggregate demand, multiplier, macroeconomics

INTRODUCTION

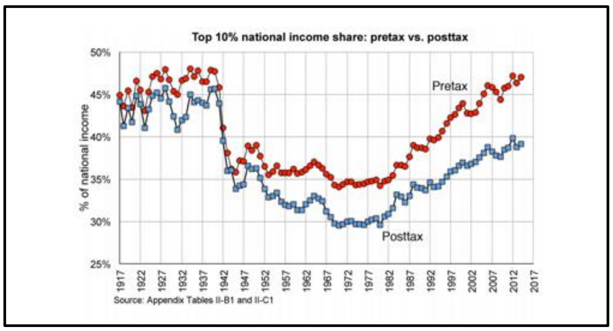
Understanding the impact of income distribution on the Macroeconomy rises in importance as the proportion of income going to the top 10% surges. Blinder (1975) raised the question analyzing it theoretically and empirically without firm conclusions. Stiglitz laments "the distribution of income is seldom mentioned in macroeconomics, and that's exactly the point." (Stiglitz, 2013, pp 298-99). In 2018, Alonzi, Drougas, Condon ("ADC") used a simple macro model to analyze the effect of a less equal income distribution. They considered a change in income distribution due to a simple *mechanical* transfer of income from the lower income group to the higher income group. They dubbed this the "*Reverse Robin Hood*" income redistribution. They concluded that the "*Reverse Robin Hood*" redistribution leads to a less price-level elastic aggregate demand curve as well as to smaller aggregate demand shifts in response to changes in exogenous variables such as autonomous investment, government spending, and money supply. The key to these findings is the size of the expenditure multiplier. The expenditure multiplier is reduced as the composite MPC falls when a greater proportion of income goes to the high-income group.

ADC's (2018) research on the effect of "*Reverse Robin Hood"* income redistribution considered a *mechanical* change in income distribution; that is, a change brought about by taking income from the low-income group and giving it to the high-income group. In this *mechanical* approach both the <u>absolute amount</u>

and the <u>proportion</u> of income received by high income group increased while simultaneously both the <u>absolute amount</u> and <u>proportion</u> of income received by the low-income group decreased. Since the 1970's however the change in the United States's income distribution has been more complex than ADC's (2018) research approach, which is a *mechanical*, zero sum approach. The change has been *organic*.

Specifically, in the *organic* change in income distribution while the <u>proportion</u> of income going to the <u>high income group increases</u> and the <u>proportion</u> going to the <u>low income group decreases</u> (see Figure 1 below), simultaneously <u>both</u> the high and the low income groups each <u>receive more income in absolute terms</u>. Significantly Figure 1 reveals that the increase of inequality persists even when all welfare transfers are taken into account, i.e. the Post-tax case.[1] Both groups can receive more income ***<u>because</u>*** the gross national income (GNI) in both real and nominal terms have increased. See the chart in endnote two illustrating this organic change in income distribution.[2]

FIGURE 1
TOP 10% NATIONAL INCOME SHARE



(Piketty et al., 2018).

We name this complex, *organic* change in income distribution the "Rising Tide Lifts All Boats Differently" change. Both the *"Rising Tide"* and the *"Reverse Robin Hood"* cases exam the effect of a more unequal income distribution. But in the "*Reverse Robin Hood*" case the inequality is in ***<u>absolute</u>*** terms as well as relative terms whereas in the "*Rising Tide*" case the inequality is only in ***<u>relative</u>*** terms.

This difference between the two ways income distribution changes raises the question "Can the way income distribution changes matter?" Matter even though the low-income group finds its absolute level of income rising? Or alternately, do the results of the "*Reverse Robin Hood*" case continue to hold in the "*Rising Tide*" case"?

This paper addresses these questions by constructing a model incorporating a rising absolute income in both the high and low groups while simultaneously allowing a higher proportion of income to go to the high-income group, but a lower proportion go to the low-income group. Notably, we find that the qualitative results of the "*Reverse Robin Hood*" case remain in the "*Rising Tide*" case but there are quantitative differences.

The remainder of this paper presents the results of this model in two sections. The first section revises the ADC (2018) model to allow for the relative change of income distribution even when income rises for each group. Analysis of this revised model yields the results presented in the second section. The paper ends with a brief conclusion and directions for further research.

Revised Model for the Case of "*Rising Tide Lifts All Boats Differently"*

The analysis of ADC's model (2018) used a "*Reverse Robin Hood*" change in given income distribution. In that approach two income groups were considered: a high-income group and a low-income group. The division of total income (y) between the two groups was captured by introducing the parameter α where $1 > \alpha > 0$. The high-income group received the proportion of income α while the proportion of income going to the low-income group $(1-\alpha)$. In their "*Reverse Robin Hood*" approach ADC (2018) considered an increase in α, the proportion going to the high-income group, to analyze the effect of a change in the income distribution. Essentially, this mechanical approach changed the income distribution by taking income from the low-income group and giving it to the high-income group as represented by an increase in α. Consequently, not only was the proportion of income received by high income group increased and that of the low-income group reduced. But simultaneously the absolute amount of income received by the high-income group was increased and the amount of income received by the low-income group was reduced.

Since the 1970's however the change in the income distribution has been more complex than this *mechanical*, zero sum game <u>*because*</u> the gross national income (GNI) in both real and nominal terms have increased. This rising income level allows a *less mechanical, organic* change of income distribution dubbed the "*Rising Tide*" change of income distribution. In this "*Rising Tide*" change each group receives more absolute income while simultaneously the proportion of income going to each group goes in opposite directions: the low-income group receiving a smaller proportion while the high-income group receives a higher proportion.

To represent this "*Rising Tide*" case in this paper we revise the model of ADC by incorporating a second parameter of income distribution ρ, while retaining the ADC parameter α. In tandem these two parameters allow the for a relative change in income distribution via changes in ρ while simultaneously also increasing the income going to each group. To do this we first break the current level of income y_2 into two parts y_1 and Δy where $\Delta y \equiv y_2 - y_1$. Here y_1 is some fixed, base level of income. Of this base level, the high-income group again receives the proportion α with $1 > \alpha > 0$ and the low-income group again receives the $(1-\alpha)$ proportion. Additionally, we assume the high-income group receives the ρ proportion of Δy while the low-income group receives the remaining portion $(1-\rho)$ of Δy. Notably, we assume $\rho > \alpha$. Combining these ρ and α proportions the high-income group (denoted by an h subscript) receives total income of $y_h = \alpha y_1 + \rho\Delta y$ and the low-income group (denoted by an l subscript) receives $y_l = (1-\alpha)y_1 + (1-\rho)\Delta y$. In total the two groups combine to receive all of y_2 as $y_h + y_l = y_2$. (Please see Appendix 1A). Whenever $\Delta y > 0$ each group receives no less than its α proportion of y_1 even though the high income group receives a larger proportion due to $\rho > \alpha$ of the change in income Δy and simultaneously the low-income group receives a smaller proportion $(1-\rho) < (1-\alpha)$ of the change in income Δy, thus reducing its proportion of total income.

Household consumption

We denote c_h the consumption of the high-income group and c_l the consumption of the low-income group. Summing these we obtain total consumption as

$$c = c_h + c_l. \tag{1}$$

Incorporating these assumptions and using the definitions of the proportions ρ and α given above we obtain c_h for the high income group as (3) and c_l for the low income group as (5):

High Income group

$$c_h = c_{0h} + MPW_h w_h + MPC_h(\alpha y_1 + \rho\Delta y - t_h) \tag{2}$$

where: $c_{0h} > 0$ & $1 > MPW_h$ & $MPC_h > 0$, $t_h = t_{0h} + (\alpha y_1 + \rho\Delta y)t_{1h}$, t_{0h} = lump sum, $1 > t_{1h} > 0$

$$c_h = c_{0h} + MPW_h w_h - MPC_h t_{0h} + MPC_h (\alpha y_1 + \rho\Delta y)(1-t_{1h}) \tag{3}$$

where MPW is the marginal propensity to consume out of wealth w, $1 > MPW > 0$.

Low Income Group

$$c_l = c_{0l} + MPW_l w_l + MPC_l [(1-\alpha)y_1 + (1-\rho)\Delta y - t_l] \tag{4}$$

where: $c_{0l} > 0$ & $1 > MPW_l$ & $MPC_l > 0$, $t_l = t_{0l} + ((1-\alpha)y_1 + (1-\rho)\Delta y)t_{1l}$, t_{0l} = lump sum, $1 > t_{1l} > 0$

$$c_l = c_{0l} + MPW_l w_l - MPC_l t_{0l} + MPC_l [(1-\alpha)y_1 + (1-\rho) \Delta y] (1-t_{1l}) \quad (5)$$

With the subscript h denoting the high income group and l the low income group.
Recall the standing assumptions: $1 > MPC_l \geq MPC_h > 0$ and $0 < t_{1l} < t_{1h} < 1$
And the new assumption $1 > \rho > \alpha > 0$.

In our model of a closed economy, in addition to consumption aggregate expenditures include investment (6) and government spending (7):

$$i = i_0 - jR \qquad i_0 > 0 \ \& \ j > 0 \quad (6)$$

Where i is total real investment
i_0 is autonomous investment spending
R is the interest rate
j is parameter of investment sensitivity to interest rates $j > 0$

Government spending is a function of income and exogenous factors as presented in equation (3).

$$g = g_0 - MPGy_2 \qquad 1 > MPG > 0 \quad (7)$$

Where g is total real government expenditures
MPG is the government's marginal propensity to spend out of real income, $1 > MPG > 0$
g_0 is autonomous government spending, this is set by government policy, $g_0 > 0$

AE is obtained by adding the high income group's consumption (3), the low income group's consumption (5), investment (6) and government spending (7) to produce

$$AE = c + i + g = c_h + c_l + i + g \quad (8)$$

Substituting the right-hand sides of (3), (5), (6), and (7) into (8) yields (9) the expression for aggregate expenditures AE:

$$\begin{aligned} AE = {} & c_{0h} + MPW_h w_h - MPC_h t_{0h} + MPC_h (\alpha y_1 + \rho\Delta y) (1-t_{1h}) \quad (9) \\ & + c_{0l} + MPW_l w_l - MPC_l t_{0l} + MPC_l [(1-\alpha)y_1 + (1-\rho) \Delta y] (1-t_{1l}) \\ & + i_0 - jR + g_0 - MPGy_2 \end{aligned}$$

Grouping like terms together we have:

$$c_0 \equiv c_{0h} + c_{0l} \quad (10.1)$$

$$MPWw \equiv MPW_h w_h + MPW_l w_l \quad (10.2)$$

$$\mu \equiv MPC_h t_{0h} + MPC_l t_{0l} \quad (10.3)$$

$$\begin{aligned} \{\Sigma_1-\Sigma_2\}y_1 + \Sigma_2 y_2 \equiv {} & \{[MPC_h\, \alpha(1-t_{1h}) + MPC_l (1-\alpha) (1-t_{1l})] - [MPC_h\, \rho(1-t_{1h}) + \\ & MPC_l (1-\rho)(1-t_{1l})]\}y_1 + [MPC_h\, \rho(1-t_{1h}) + MPC_l (1-\rho)(1-t_{1l})]\, y_2 \quad (10.4) \end{aligned}$$

Where equation (10.4) utilizes the notation saving definitions Σ_1 and Σ_2.

$\Sigma_1 \equiv MPC_h\ \alpha(1-t_{1h}) + MPC_l\,(1-\alpha)\ (1-t_{1l})$ (10.5)

$\Sigma_2 \equiv MPC_h\ \rho(1-t_{1h}) + MPC_l\ (1-\rho)(1-t_{1l})$ (10.6)

And Appendix 1B provides the derivation of the identity equation (10.4) as well as the demonstration that the relationships in (11) below hold given our standing assumptions: $\rho > \alpha$, $MPC_h < MPC_l$, and $t_{1h} > t_{1l}$.

$\Sigma_1 - \Sigma_2 > 0$ so that $\Sigma_1 > \Sigma_2$ (11)

Substituting (10.1-10.3) and (11) into (9) the expression for aggregate expenditures AE becomes

$AE = c_0 + i_o + g_0 - \mu + MPWw - jR - MPG\ y_2 + (\Sigma_1-\Sigma_2)y_1 + \Sigma_2\ y_2$ (12)

Redefining autonomous aggregate expenditures as

$AE_0 \equiv [c_0 + i_0 + g_0 - \mu + MPWw]$ (13)

we have equation (14) as the revised expression for aggregate expenditure.

$AE = AE_0 - jR + (\Sigma_1-\Sigma_2)y_1 + (\Sigma_2 - MPG)\ y_2$ (14)

Setting AE equal to y_2 and solving for y_2 yields

$y_2 = (AE_0 - jR + (\Sigma_1-\Sigma_2)y_1) / [1-\Sigma_2 + MPG)]$ (15)

Or less compactly

$$y_2 = \frac{AE_0 - jR + (\Sigma_1-\Sigma_2)y_1}{1-[MPC_h\rho(1-t_{1h})+MPC_l(1-\rho)(1-t_{1l})]+MPG} \qquad (16)$$

Substituting the expression for the interest rate R derived in ADC (Please see Appendix 1C) and equation (10.6) for Σ_2 into equation (16) yields equation (17).

$$y_2 = \frac{AE_0 - j\,\{ey_2 - \frac{M_0^S}{P_0}]\,/\,v\} + (\Sigma_1-\Sigma_2)y_1}{1-\Sigma_2+MPG} \qquad (17)$$

Solving (17) for y_2 gives the expression for aggregate demand y^d presented in equation (18).

$$y^d{}_2 = \frac{AE_0 + (\Sigma_1-\Sigma_2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1-\Sigma_2+MPG+(\frac{ej}{v})} \qquad (18)$$

Or less compactly by writing out the denominator fully.

$$y^d{}_2 = \frac{AE_0 + (\Sigma_1-\Sigma_2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1-[MPC_h\rho(1-t_{1h})+MPC_l(1-\rho)(1-t_{1l})]+MPG+(\frac{je}{v})} \qquad (19)$$

While the general form for aggregate demand found in equation (19) for the *"A Rising Tide"* case (which includes ρ as well as α) is identical to the form of ADC's equation (20) for the "*Reverse Robin Hood*" case (which includes α but not ρ), there are key differences in the parameters. For ease of comparison equation (20) of ADC is reproduced here:

$$y^d_1 = \frac{AE_0 + (\frac{j}{v})\frac{M^S_0}{P_0}}{1 - \Sigma + MPG + (\frac{ej}{v})} = \frac{AE_0 + (\frac{j}{v})\frac{M^S_0}{P_0}}{1 - [MPC_h\alpha(1-t_{1h}) + MPC_l(1-\alpha)(1-t_{1l})] + MPG + (\frac{je}{v})} \tag{20}$$

In the denominator for the "*A Rising Tide*" case's equation (19) Σ_2 and ρ replace Σ and α of the "*Reverse Robin Hood*" denominator. And in the numerator for the "*A Rising Tide*" case there is an additional term $(\Sigma_1 - \Sigma_2)y_1$ which contains α in Σ_2.

Analysis of the Effects on y^d_2 of Changes in Exogenous Variables and in ρ

Despite the difference in the particular parameters found in the solutions for y^d of equations (19) of the *"A Rising Tide"* case and (20) of ADC's (2018) the "*Reverse Robin Hood*" case, the work of Appendix 2B reported in the Table 1 shows that the similarity of form of y^d in the "*Rising Tide*" case as in the "*Reverse Robin Hood*" case leads to the same ***qualitative*** findings in the "*Rising Tide*" case as in the "*Reverse Robin Hood*" case for changes in exogenous variables (see Appendix 2B equations (2B-2) for the effects of AE_0 changes, (2B-4) for M^s_0 changes, (2B-11) and (2B-12) for tax rate changes, and (2B-19) for P_0 changes). Essentially and notably including income distribution leaves undisturbed the conventional understanding.

Table 1 presents the case of Rising Tide for parameter, ρ. Please see Appendix 2A for explanations regarding the case where y_1 is sufficiently small.

It is reassuring that the usual qualitative comparative static results are undisturbed by the inclusion of income distribution. More interesting, however, are the *quantitative* effects of the change in income distribution represented by changes in relative income distribution represented by a change in the parameter ρ.

TABLE 1
"*Rising Tide"* -- ρ CASE

PARAMETER X only in numerator of eq [20]	c_0	i_0	g_0	t_{0h}	t_{0l}	w_h	w_l	M^s_0	P_0				Multiplier $\frac{1}{1-\Sigma+MPG+(\frac{ej}{v})}$
Parameter X only in denominator of [20]										t_{1h}	t_{1l}	ρ	
1 $\delta y^d/\delta x$	+	+	+	-	-	+	+	+	-	-	-	-*	
2 $\delta(\delta y^d/\delta x)/\delta\alpha$	-	-	-			-	-	-	+	?	?		
3 $\delta(\text{multiplier})/\delta x$												-	

"+" indicates $\delta y^d/\delta x > 0$ and "-"$\delta y^d/\delta x < 0$

* The sign of $\frac{\partial y^d}{\partial \rho}$ comes with one condition. Specifically, $\frac{\delta y^d_2}{\delta \rho} < 0$ when y_1 is sufficiently small

The last entry of Table 1's Row 1 and all of Row 2's entries report the effects of a change in ρ derived in Appendix 2B equations (2B-20) –(2B-30). These Table 1 results reveal that the "*Rising Tide*" change in income distribution affects the aggregate demand via the same two avenues—direct headwinds and indirect headwinds—as did a "*Reverse Robin Hood*" change in income distribution. But there is one difference.

Focusing on the nexus of the three derivatives $\frac{\delta y^d_2}{\delta \rho}, \frac{\delta[\frac{\delta y^d_2}{P_0}]}{\delta \rho}$, and $\frac{\delta[\frac{\delta y^d_2}{\delta c_0}]}{\delta \rho}$ reveals that income distribution affects aggregate demand and so the Macroeconomy in two ways. First rising income inequality decreases aggregate demand hence providing a headwind on the economy—reported in row 1 of Table 1's $\frac{\delta y^d_2}{\delta \rho} < 0$.

This headwind is reinforced by the steepening of the aggregate demand due to rising income inequality—Row 2's $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta\rho} > 0$. Second, rising income inequality can retard recovery from a recession by leading to smaller increases in aggregate demand when autonomous aggregate expenditures rise or government runs expansionary policy to aid the recovery from a recession—Row2's $\frac{\delta[\frac{\delta y}{\delta c_0}]}{\delta\rho} < 0$ and Row 3's δ(multiplier)/δρ < 0.

Appendix 2B shows that the signs of $\frac{\delta y_2^d}{\delta\rho}$, $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta\rho}$, and $\frac{\delta[\frac{\delta y_2^d}{\delta c_0}]}{\delta\rho}$ are the same as in the "*Reverse Robin Hood*" case (see (2B-27) for the sign of $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta\rho}$ and (2B-30) for the sign of $\frac{\delta[\frac{\delta y_2^d}{\delta c_0}]}{\delta\rho}$). The sign of $\frac{\delta y_2^d}{\delta\rho} < 0$ (see (2B-24) comes with one condition. Specifically, $\frac{\delta y_2^d}{\delta\rho} < 0$ when $y_1 < y^d_2$. That is, relatively y_1 is sufficiently small. See the discussion of this condition in Appendix 2A. Lastly as the results for $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta\rho}$ and $\frac{\delta[\frac{\delta y_2^d}{\delta c_0}]}{\delta\rho}$ indicate, the magnitudes of the qualitative results are again reduced by an increase in inequality (↑ρ). This is shown in Appendix 2B noting that the multiplier term $\frac{1}{1-\Sigma_2+ MPG+\left(\frac{je}{v}\right)}$ found in (2B-31) as well as (2B-2) appears in all the derivatives and (2B-33) shows that the multiplier declines as ρ rises. Thus, notably we find that the qualitative results of the "*Reverse Robin Hood*" case remain in the "*Rising Tide*" case but there are quantitative differences.

Comparison of the effects of the *"Rising Tide"* and the *"Reverse Robin Hood"* cases reveals that the qualitative results of the *"Reverse Robin Hood"* case remain in the "*Rising Tide*" case but there are quantitative differences. *Qualitatively*, the "*Rising Tide*" case's *organic*, relative change of income distribution affects aggregate demand in the same ways as the "*Reverse Robin Hood*" case's *mechanical*, absolute change in income distribution. Specifically, the rising income inequality decreases aggregate demand hence providing a headwind on the economy. This headwind is reinforced by the steepening of the aggregate demand due to rising income inequality. *Quantitatively* the *"Rising Tide"* case's *organic,* relative change of income distribution leads to smaller increases in aggregate demand than the *"Reverse Robinhood"* case's change in income distribution. Specifically, when autonomous aggregate expenditures rise (whether from an increase in investment or consumption) or government runs expansionary policy (whether by raising autonomous government spending or the money supply) the shifts in aggregate demand are reduced. The key to these small quantitative results is the reduction in the multiplier caused by a less equal income distribution.

Conclusion and Discussion

Notably, our finding of the same qualitative results due to a change in relative income distribution in the "*Rising Tide*" case as in the absolute income distribution change of the "*Reverse Robin Hood*" case underscores the significant insight: a growing level of income fails to eradicate the effects of increased income inequality. Growth in and of itself fails to undo the effects of a change in income distribution. Specifically, this failure is revealed by finding that the effects of a change in income distribution do not depend on the "*Reverse Robin Hood*" case's *mechanical*, absolute change in income distribution taking income from one group and giving it to the other. The same qualitative results also occur in the "Rising Tide" organic, relative change of income distribution even though neither group experiences a decrease in income. Notably, while the qualitative results are the same, the "Rising Tide" case indicates that the qualitative results are smaller quantitatively. In sum, the analysis of the "A Rising Tide" thus reveals the robustness of the results found in the "Reverse Robin Hood" case and underscores the significant finding that growth itself fails to eradicate the effects of increased income inequality. Indeed, as a rising tide raises all boats but some more than other, the effects of income inequality appear to be exacerbated.

ENDNOTES

1. Figure 1 replicates the upper chart in Piketty, Saez, and Zucman's Figure V, displaying the share of national income pretax and posttax going to the top 10% of adults from 1917 to 2014 (Piketty et al., 2018). Significantly this figure reveals that the increase of inequality persists even when all welfare transfers are taken into account, i.e. the Posttax case. The degree of increase in income inequality reported by Piketty, Saez, and Zucman presented here is questioned by Auten and Splinter (2019). See the work of Auten and Splinter as well as of Piketty, Saez, and Zucman in the 2019 AEA Papers and Proceeds for the exchange of their ideas.
2. GDP and the income base used by Saez (2016) in both real and nominal terms have increased too. The illustrative example in the chart 1 below clarifies and illustrates the results of this organic change in income distribution. Applying Saez's income distribution proportions to these higher levels of income shows that while the high-income group receives an increased proportion of total income and so more income, the low-income group also receives more income even though its proportion of total income is falling. This change of income distribution in which each group receives more income in absolute terms, but *relatively* the high-income group receives an even higher proportion while the low-income group receives a lower proportion than before is a "*Rising Tide Lifts All Boats Differently*" change. It is different than the "*Reverse Robin Hood*" change in income distribution in which the inequality is in *absolute* terms as the low income group receives not only a smaller proportion of income but also less absolute income (and the high income group a larger proportion of income as well as more absolute income). This "*Rising Tide Lifts All Boats Differently*" case which raises absolute income of both high- and low-income groups while increasing the high-income group's proportion of total income and decreasing the low-income group's proportion is illustrated by the numerical example promised above. For simplicity the illustration assumes a base level of income of \$1,000, a change of income for \$500, and a current level of income of \$1,500.

CHART 1
ILLUSTRATION OF THE ORGANIX CHANGE IN INCOME DISTRIBUTION

	INCOME y and Δy	% GOING TO HIGH INCOME GROUP: α	TOTAL INCOME HIGH INCOME GROUP: αy		% GOING TO LOW INCOME GROUP: (1-α)	TOTAL INCOME LOW INCOME GROUP: (1-α)y
Base Level	\$1000					
		35%	\$350		65%	\$650
		% of Δy going to High Income group: ρ			% of Δy going to Low Income group: 1-ρ	
	Δy \$500	80%	\$400		20%	\$100
Current level	1500					
		50%	\$750		50%	\$750

This example illustrates the "*A Rising Tide Lifts All Boats Differently*" change in income distribution and the assertion that in this case both high and low income groups receive more absolute dollars of income (\$750 > either \$350 or \$650) when the proportion of income going to the high income group rises if the level of income rises sufficiently. The proportions selected for the α's are very close to those reported by Saez's α_{1975} = 32.62% and α_{2015} = 47.81% for 1975 and 2015. They are rounded to ease the illustration. Here it is assumed that base year income is \$1,000 and initial the proportion of income going to the high-income group α_0 is 35% and in the ending year α_1 is 50%. Further it is assumed that current year total income is \$1,500. These assumptions make the *change* in income is \$500. The proportion ρ is deduced from the proportions α_0 and α_1 applied to the assumed initial income level and change in income. The ρ of 80% greater than α_0 is 35% capturing the text's assumption that ρ>α.

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APPENDIX

Appendix 1A

$y_h + y_l = [\alpha y_1 + \rho\ \Delta y] + [(1\text{-}\alpha)\ y_1 + (1\text{-}\ \rho)\ \Delta y] = \{[\alpha y_1 + (1\text{-}\alpha)y_1] + [\rho\ \Delta y + (1\text{-}\ \rho)\ \Delta y]\} =$
$\{y_1 + \Delta y\} = \{y_1 + y_2\text{-}y_1\} = y_2$ (1A-1)

Appendix 1B

To establish the expression $\{\Sigma_1\text{-}\Sigma_2\}y_1 + \Sigma_2 y_2$ begin with the portion of the high income group's consumption that varies with income—$MPC_h\ (\alpha y_1+\rho\Delta y)\ (1\text{-}t_{1h})$—and add it to the low income group's consumption that varies with income—$MPC_l\,[\ (1\text{-}\alpha)y_1 + (1\text{-}\rho)\ \Delta y\ ]\ (1\text{-}t_{1l})$—found in the text's equation (9). Combining and factor terms as in the following four steps results in $\{\Sigma_1\text{-}\Sigma_2\}y_1 + \Sigma_2 y_2$.

Start with $MPC_h\ (\alpha y_1+\rho\Delta y)\ (1\text{-}t_{1h}) + MPC_l\,[\ (1\text{-}\alpha)y_1 + (1\text{-}\rho)\ \Delta y\ ]\ (1\text{-}t_{1l})$ (1B-1)

Which upon combining like terms that multiply y_1 and Δy respectively and factoring becomes

$\{\ MPC_h\ \alpha(1\text{-}t_{1h}) + MPC_l\,(1\text{-}\alpha)\ (1\text{-}t_{1l})\ \}\ y_1\ +\ \{\ MPC_h\ \rho(1\text{-}t_{1h}) + MPC_l\ (1\text{-}\rho)(1\text{-}t_{1l})\ \}\ \Delta y$ (1B-2)

Which in turn simplifies to $\Sigma_1 y_1 + \Sigma_2\Delta y$. (1B-3)

Where Σ_1 and Σ_2 are defined as

$\Sigma_1 \equiv MPC_h\ \alpha(1\text{-}t_{1h}) + MPC_l\,(1\text{-}\alpha)\ (1\text{-}t_{1l})$ (10.4)

$\Sigma_2 \equiv MPC_h\ \rho(1\text{-}t_{1h}) + MPC_l\ (1\text{-}\rho)(1\text{-}t_{1l})$ (10.5)

Recalling $\Delta y \equiv y_2\text{-}y_1$, (1B-3) becomes (1B-4)

$\Sigma_1 y_1 + \Sigma_2\Delta y = \Sigma_1 y_1 + \Sigma_2(y_2\text{-}y_1)$ which upon rearranging terms becomes (1B-5)

$(\Sigma_1\text{-}\Sigma_2)y_1 + \Sigma_2\ y_2$ (1B-6)

Of use later is establishing that $\Sigma_1\text{-}\Sigma_2 > 0$ by noting that Σ_1 is Σ found in ADC 2018 and further noting that by definition of Σ_1 and Σ_2 that

$(\Sigma_1\text{-}\Sigma_2) = [\ MPC_h\ \alpha(1\text{-}t_{1h}) + MPC_l\,(1\text{-}\alpha)\ (1\text{-}t_{1l})\ ] – [\ MPC_h\ \rho(1\text{-}t_{1h}) + MPC_l\ (1\text{-}\rho)(1\text{-}t_{1l})\ ]$ (1B-7)

Upon collecting terms by income group, the right-hand side of (1B-7) becomes

$(\Sigma_1\text{-}\Sigma_2) = [\ MPC_h\ \alpha(1\text{-}t_{1h}) - MPC_h\ \rho(1\text{-}t_{1h})\ ]\ + [\ MPC_l\,(1\text{-}\alpha)\ (1\text{-}t_{1l}) - MPC_l\ (1\text{-}\rho)(1\text{-}t_{1l})\ ]$ (1B-8)

Factoring out MPC_x and $1\text{-}t_{1x}$ (with the subscript x = h, l respectively) the right-hand side of (1B-8) becomes

$(\Sigma_1\text{-}\Sigma_2) = \{\alpha\text{-}\rho\}[MPC_h\ (1\text{-}t_{1h})\ ] + \{(1\text{-}\alpha) - (1\text{-}\rho)\}[MPC_l\ (1\text{-}t_{1l})\ ]$ (1B-9)

Noting that $\{(1\text{-}\alpha) - (1\text{-}\rho)\}$ is $\rho - \alpha$ the right-hand side of (1B-9) becomes

$(\Sigma_1\text{-}\Sigma_2) = (\alpha\text{-}\rho)[\ MPC_h\ (1\text{-}t_{1h})\ ] + [\ MPC_l\,(\rho\text{-}\alpha)(1\text{-}t_{1l})\ ]$ (1B-10)

Factoring out α-ρ the right-hand side of (1B-10) becomes

$$(\Sigma_1-\Sigma_2) = (\alpha-\rho)\ [\ MPC_h\ (1-t_{1h}) - MPC_l\ (1-t_{1l})] \quad (1B\text{-}11)$$

For our standing assumptions of $\rho > \alpha$, $MPC_h < MPC_l$, $t_{1h} > t_{1l}$ we have

$$\Sigma_1-\Sigma_2 \equiv (\alpha-\rho)[\ MPC_h\ (1-t_{1h}) - MPC_l\ (1-t_{1l})\ ] > 0 \quad (1B\text{-}12)$$

And so for these conditions we have the text's inequalities found in (11).

$$\Sigma_1-\Sigma_2 > 0 \text{ or } \Sigma_1 > \Sigma_2 \quad (11)$$

Appendix 1C

For convenience this Appendix 1C reproduces ADC's (2018) derivation of the interest rate that sets the money market in equilibrium. First, we specify money demand and then money supply.

Money demand is a function of real income and the rate of interest as presented in equation (5).

$$m^d = ey_2 - vR \qquad 1 > e > 0\ \ \&\ \ v \geq 0 \quad (i)$$

Where m^d is total real demand for money balances
e is a parameter representing money demand's dependence on real income y_2, $1 > e > 0$
v is parameter of money demand sensitivity to the interest rate, $v \geq 0$

The real Money Supply is represented as the ratio of nominal money supply $M^s{}_0$ to the price level P.

$$m^s = M^s{}_0/P_0 \quad (ii)$$

Where m^s is the total real supply of money
M^s is the nominal money stock
P_0 is the price level

$m^d = m^s$ is the equilibrium condition in the money market. (iii)

Substituting from (i) for money demand and (ii) for money supply into (iii) yields

$$ey_2 - vR = M^s{}_0/P_0 \quad (iv)$$

solving for the interest rate R yields (v) which is the text's equation.

$$R = \frac{ey_2 - \frac{M_0^S}{P_0}}{v} \quad (v)$$

Appendix 2A

For several reasons, the restriction $y_1 < y^d{}_2$ is rather more lax than one might first think. First, recall that Δy is y_2-y_1 which is assumed greater than zero. Second, to explore the restriction $y_1 < y^d{}_2$ even further rewrite (18) in the following way:

$$y^d{}_2 = \frac{AE_0 + (\Sigma_1 - \Sigma_2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1 - \Sigma_2 + MPG + (\frac{ej}{v})} = \theta_2 + \frac{\beta_2}{P}, \quad (2A\text{-}1)$$

where $\theta_2 \equiv \ \frac{AE_0+\,(\Sigma_1-\Sigma_2)y_1}{1-\Sigma_2+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})}$ and $\beta_2 \equiv \ \frac{(\frac{j}{v})M_0^S}{1-\Sigma_2+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})}$

Now recall that y_1 is a fixed amount while y^d_2 varies inversely with the price level. As P falls the condition of y1 < yd2 is ever more likely to be met for as P falls yd2 rises. Specifically, as P falls this increases the $\frac{\beta_2}{P}$ component of yd2 ≡ $\theta_2 + \frac{\beta_2}{P}$ while the θ_2 remains unchanged. Third, if the fixed amount y1 equals θ_2 then y1 < yd2 as yd2 ≡ $\theta_2 + \frac{\beta_2}{P}$. Lastly, consider the implications of either a small or a large y1. For y1 small consider the extreme y1 = 0. Then Δy is y2-0 = y2 and we essentially have the previous "Reverse Robin Hood" case with ρ instead of α. For y1 large consider if y1 = y2. Then Δy is zero causing ρ and (1-ρ) to fall out of the analysis which they only enter by multiplying Δy. With y1 = y2 we return to the previous "*Reverse Robin Hood*" case.

Appendix 2B

Appendix 2B develops in four steps. First, this appendix presents the analysis of the particular case of an increase in c_0 to confirm that the sign of $\frac{\delta y_2^d}{\delta c_0}$ is again positive. With this template provided, the reader can show the signs of the derivatives when there is a change in i_0, g_0, w, t_{0h}, t_{0l}, or $M^s{}_0$ remain the same as in the "*Reverse Robin Hood*" case. (But their magnitudes are reduced by an increase in income inequality, ↑ρ as we show later in the fourth step). Next, this appendix analyzes the effect of changes in the tax rates t_{1h} and t_{1l}. Third, the appendix analyzes the effect of a change in P_0 on y^d_2 to determine the slope of the aggregate demand and as a prelude to demonstrating that an increase in income inequality, ↑ρ, makes the aggregate demand steeper, i.e. less sensitive to changes in the price level. Fourth, the appendix analyzes quantitative the effects of an increase in income inequality represented by an increase in ρ. Even though the signs of the effects of changes in c_0, i_0, g_0, w, t_{0h}, t_{0l}, $M^s{}_0$, t_{1h} and t_{1l} on y_2^d remain the same as in the "*Reverse Robin Hood*" case, the magnitudes of the effects are reduced. The key is the multiplier appears in all these changes and the multiplier is reduced by an increase in ρ.

Shifting y^d: Effect of a change in c_0 on $y^d{}_2$

For ease recall that text's Equation (18).

$$y^d{}_2 = \frac{AE_0+\,(\Sigma_1-\Sigma_2)y_1+(\frac{j}{v})\frac{M_0^S}{P_0}}{1-\Sigma_2+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})} \tag{18}$$

Differentiation of the text's equation (18) with respect to c_0 yields (2B-1).

$$\frac{\delta y_2^d}{\delta c_0} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta AE_0}{\delta c_0}\right\} - \{AE_0+\,(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\cancelto{0}{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta c_0}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-1}$$

As c_0 does not appear in $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ and canceling a $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ from numerator and denominator (2B-1) becomes (2B-2).

$$\frac{\delta y_2^d}{\delta c_0} = \frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)} > 0 \tag{2B-2}$$

Note that (2B-2) contains the multiplier term $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$.

So here as in the "*Reverse Robin Hood*" an increase in c_0 increases y^d. Note that since g_0, i_0, $-t_{0h}MPC_h$, $-t_{0l}MPC_l$ and w all enter the numerator of (18) via AE_0 as does c_0 one sees that (2B-2) establishes the signs of their effects on $y^d{}_2$ are as expected and their effects would also contain the multiplier term $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$.

Effect of a change in $M^s{}_0$ on $y^d{}_2$

Differentiating (18) with respect to $M^s{}_0$ yields (2B-3).

$$\frac{\delta y_2^d}{\delta M_0^S} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta[\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}]}{\delta M_0^S}\right\} - \{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\cancelto{0}{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta M_0^S}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-3}$$

As $M^s{}_0$ appears neither in the numerator's $\{AE_0 + (\Sigma 1-\Sigma 2)y_1\}$ nor in $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ and canceling a $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ from numerator and denominator (2B-3) becomes (2B-4).

$$\frac{\delta y_2^d}{\delta M_0^S} = \frac{\left(\frac{jM_0^S}{v}\right)}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]} \rightarrow \frac{(+)(+)}{(+)} > 0 \tag{2B-4}$$

Note that (2B-4) contains the multiplier term $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$.

Effect of a change in tax rates t_{1h} and t_{1l} on $y^d{}_2$

Differentiating (18) with respect to t_{1h} yields (2B-5).

$$\frac{\delta y_2^d}{\delta t_{1h}} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta\{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}}{\delta t_{1h}}\right\} - \{AE_0+(\Sigma 1-\Sigma 2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta t_{1h}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-5}$$

Since t_{1h} enters (2B-5) only in the Σ_1 and Σ_2 terms (2B-5) becomes (2B-6).

$$\frac{\delta y_2^d}{\delta t_{1h}} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta\{(\Sigma_1-\Sigma_2)y_1\}}{\delta t_{1h}}\right\} - \{AE_0+(\Sigma 1-\Sigma 2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta[-\Sigma_2]}{\delta t_{1h}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-6}$$

To evaluate (2B-6) it is useful to establish how Σ_2 and $(\Sigma_1-\Sigma_2)$ respond to changes in t_{1h}. Recalling (10.5)'s expression for Σ_2 and differentiating it with respect to t_{1h} yields (2B-7).

$$\frac{\delta\Sigma_2}{\delta t_{1h}} = -\ MPC_h\ \rho \tag{2B-7}$$

Recalling (1B-12)'s expression for $(\Sigma_1-\Sigma_2)$ and differentiating it with respect to t_{1h} yields (2B-8).

$$\frac{\delta(\Sigma_1-\Sigma_2)}{\delta t_{1h}} = (\alpha-\rho)(-\ MPC_h) \tag{2B-8}$$

Substituting (2B-7) and (2B-8) into (2B-6) yields (2B-9).

$$\frac{\delta y_2^d}{\delta t_{1h}} = \frac{\{[1-\Sigma_2+MPG+(\frac{je}{v})]\ [(\alpha-\rho)(-MPC_h)y_1\} - \{AE_0 + (\Sigma 1-\Sigma 2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}\}\{(\rho)(MPC_h]\}}{[1-\Sigma_2+MPG+(\frac{je}{v})]^2} \tag{2B-9}$$

(2B-9) becomes (2B-10) by factoring out $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ and noting that y^d_2 is the text's (18).

$$\frac{\delta y_2^d}{\delta t_{1h}} = \frac{[\ \{(\alpha-\rho)(-MPC_h)y_1\} - (\rho)MPC_h y_2^d\ ]\ [1-\Sigma_2+MPG+(\frac{je}{v})]}{[1-\Sigma_2+MPG+(\frac{je}{v})]^2} \tag{2B-10}$$

(2B-10) becomes (2B-11) by canceling a $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ from (2B-10)'s numerator and denominator and collecting terms.

$$\frac{\delta y_2^d}{\delta t_{1h}} = \frac{[\ \{(-\alpha)(MPC_h)y_1\} + \rho(MPC_h)(y_1-y_2^d)\ ]}{[1-\Sigma_2+MPG+(\frac{je}{v})]} \rightarrow \frac{[(-)(+)(+)]+[(+)(+)(y_1-y_2^d)]}{(+)} \tag{2B-11}$$

If $(y_1 - y_2^d) < 0$ then the sign of $\frac{\delta y_2^d}{\delta t_{1h}}$ is negative. Note this condition that $(y_1 - y_2^d) < 0$ is the condition discussed in Appendix 2A and is the condition that will appear again when signing $\frac{\delta y_2^d}{\delta \rho}$, i.e. (2B-24) found below. We note more generally if $\frac{y_1 - y_2^d}{y_1} < \frac{\alpha}{\rho}$ then $\frac{\delta y_2^d}{\delta t_{1h}}$ is negative.

Again, we note that (2B-11) contains the multiplier term $\frac{1}{1-\Sigma_2+MPG+(\frac{je}{v})}$.

Differentiating (18) with respect to t_{1L} yields (2B-12).

$$\frac{\delta y_2^d}{\delta t_{1l}} = \frac{\left\{[1-\Sigma_2+MPG+(\frac{je}{v})]\frac{\delta\{AE_0+(\Sigma_1-\Sigma_2)y_1+(\frac{j}{v})\frac{M_0^S}{P_0}\}}{\delta t_{1l}}\right\} - \{AE_0 + (\Sigma 1-\Sigma 2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}\}\{\frac{\delta[1-\Sigma_2+MPG+(\frac{je}{v})]}{\delta t_{1l}}\}}{[1-\Sigma_2+MPG+(\frac{je}{v})]^2} \tag{2B-12}$$

Since t_{1l} enters (2B-12) only in the Σ_1 and Σ_2 terms (2B-12) becomes (2B-13).

$$\frac{\delta y_2^d}{\delta t_{1l}} = \frac{\{[1-\Sigma_2+MPG+(\frac{je}{v})]\frac{\delta\{(\Sigma_1-\Sigma_2)y_1\}}{\delta t_{1l}}\} - \{AE_0 + (\Sigma 1-\Sigma 2)y_1 + (\frac{j}{v})\frac{M_0^S}{P_0}\}\{\frac{\delta[-\Sigma_2]}{\delta t_{1l}}\}}{[1-\Sigma_2+MPG+(\frac{je}{v})]^2} \tag{2B-13}$$

To evaluate (2B-13) it is useful to establish how Σ_2 and $(\Sigma_1 - \Sigma_2)$ respond to changes in t_{1l}. Recalling (10.5)'s expression for Σ_2 and differentiating it with respect to t_{1l} yields (2B-14).

$$\frac{\delta \Sigma_2}{\delta t_{1l}} = -\ MPC_l\ (1-\rho) \tag{2B-14}$$

Recalling (1B-12)'s expression for $(\Sigma_1 - \Sigma_2)$ and differentiating it with respect to t_{1l} yields

$$\frac{\delta(\Sigma_1-\Sigma_2)}{\delta t_{1l}} = (\alpha-\rho)(MPC_l) \tag{2B-15}$$

Substituting (2B-14) and (2B-15) into (2B-13) yields (2B-16).

$$\frac{\delta y_2^d}{\delta t_{1l}} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\ [(\alpha-\rho)(MPC_l)y_1]\right\}\ -\ \{AE_0+(\Sigma 1-\Sigma 2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{(1-\rho)(MPC_l\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-16}$$

(2B-16) becomes (2B-17) by factoring out $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ in the numerator and noting that y^d_2 is the text's (18) and then canceling a $\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]$ from (2B-17)'s numerator and denominator and collecting terms.

$$\frac{\delta y_2^d}{\delta t_{1l}} = \frac{[(\alpha-\rho)(MPC_l)y_1]-[(1-\rho)MPC_l y_2^d]}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]} \rightarrow \frac{[(-)(+)(+)]-[(+)(+)(y_2^d)]}{(+)} \rightarrow \frac{[(-)(+)(+)]-[(+)(+)(+)]}{(+)} <0 \text{ for } \rho>\alpha. \tag{2B-17}$$

Again we note that (2B-17) contains the multiplier term $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$.

Slope of y^d: Effect on y^d of a change in P_0 and effect of a change in ρ on the slope of y^d

To see the slope of the aggregate demand is negative, differentiate equation (18)'s expression for y^d_2 with respect to P_0. This differentiation yields equation (2B-18).

$$\frac{\delta y_2^d}{\delta P_0} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta[\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}]}{\delta P_0}\right\}\ -\ \{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\overset{0}{\cancel{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta P_0}}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-18}$$

As P_0 only appears in numerator of (18) and upon cancelling $1-\Sigma_2+MPG+\left(\frac{je}{v}\right)$ (2B-18) becomes (2B-19).

$$\frac{\delta y_2^d}{\delta P_0} = \frac{-\left(\frac{j}{v}\right)\frac{M_0^S}{P_0^2}}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)} < 0 \tag{2B-19}$$

Equation (2B-19) shows that the slope of the aggregate demand is negative as equation (2B-19) is the reciprocal of the slope of the aggregate demand.

The effects of increased income inequality (↑ρ) on y^d_2

Having found that the basic results of the "*Reverse Robin Hood*" case still hold, we now we turn to the effects of increasing income inequality represented in this "*A Rising Tide*" case by considering an increase in the parameter ρ.

Recall that this reduction in equality is not the result of taking income from the low income group and giving it to the high income group as in the "*Reverse Robin Hood*" case but rather the result of the high income group receiving a larger portion ρ of the increase in income Δy, i.e. ρ Δy, than they received of the base amount of income α, i.e. αy_1, and by the low income group receiving a smaller portion (1-ρ) of the increase income Δy, i.e. (1-ρ)Δy, than they received of the base amount of income (1-α), i.e. $(1-\alpha)y_1$ when ρ>α. Thus, it is a "*Rising Tide Lifts All Boats Differently*" change in the distribution of income.

The nexus of the three derivatives $\frac{\partial \delta y_2^d}{\partial \rho}$, $\frac{\delta[\frac{\delta y^d}{\delta P}]}{\delta \rho}$, and $\frac{\delta[\frac{\delta y}{\delta c_0}]}{\delta \rho}$ again reveals the two ways—direct and indirect—by which income distribution affects aggregate demand and so the Macroeconomy. We now show that the signs of the three are the same as in the "*Reverse Robin Hood*" case though the sign of $\frac{\partial y^d}{\partial \rho}$ comes with one new condition.

Specifically:

$$\frac{\delta y_2^d}{\partial \rho} < 0 \text{ when } y_1 < y^d{}_2, \tag{i}$$

$$\frac{\delta[\frac{\delta y^d}{\delta P}]}{\delta \rho} > 0, \tag{ii}$$

$$\frac{\delta[\frac{\delta y}{\delta c_0}]}{\delta \rho} < 0. \tag{iii}$$

Direct Effects of a change in income distribution (↑ρ) on $y^d{}_2$: Shift and Slope

Shift: Demonstration that $\frac{\delta y_2^d}{\partial \rho} < 0$ when y_1 is sufficiently small, that is $(y_1 - y_2^d) < 0$.

Differentiating (18) with respect to ρ yields (2B-20).

$$\frac{\delta y_2^d}{\partial \rho} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta\{AE_0+(\Sigma_1-\Sigma_2)y_1+(\frac{j}{v})\frac{M_0^S}{P_0}\}}{\delta\rho}\right\} - \{AE_0+(\Sigma1-\Sigma2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta\rho}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-20}$$

(2B-20) becomes (2B-21) as ρ only enters equation (18) through the denominator's Σ_2:

$$\frac{\delta y_2^d}{\delta \rho} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta\{-y_1\Sigma_2\}}{\delta\rho}\right\} - \{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta[-\Sigma_2]}{\delta\rho}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-21}$$

(2B-21) becomes (2B-22) by factoring out $\frac{\delta[-\Sigma_2]}{\delta\rho}$.

$$\frac{\delta y_2^d}{\delta \rho} = \frac{[\ \left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right][y_1]\right\} - \{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\ ]\ \ [\frac{\delta[-\Sigma_2]}{\delta\rho}]}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-22}$$

(2B-22) becomes (2B-23) by factoring out $\left[1 - \Sigma_2 + MPG + \left(\frac{je}{v}\right)\right]$ in the numerator and noting that $\frac{\{AE_0+(\Sigma_1-\Sigma_2)y_1+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}$ is y_2^d

$$\frac{\delta y_2^d}{\delta \rho} = \frac{[\ y_1 - y_2^d\ ]\ [\frac{\delta[-\Sigma_2]}{\delta\rho}]\ \left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-23}$$

(2B-23) becomes (2B-24) by canceling a $[1 - \Sigma_2 + MPG + \left(\frac{je}{v}\right)]$ from the numerator and the denominator of (2B-23).

$$\frac{\delta y_2^d}{\delta \rho} = \frac{[\ y_1\ -\ y_2^d\ ]\ [\frac{\delta[-\Sigma_2]}{\delta \rho}]}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)} \quad = \frac{[?]\ [(-)(-)]}{[+]} \tag{2B-24}$$

since $\frac{\delta[\Sigma_2]}{\delta \rho} = MPC_{h(1-t_{1h})} - MPC_{l(1-t_{1l})} < 0$, the sign of $\frac{\delta y_2^d}{\delta \rho}$ is negative when $y_1 < y^d_2$.

Also note that (2B-24) contains the multiplier $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$ the implications of which we draw out after showing that $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho} > 0$ and $\frac{\delta[\frac{\delta y_2^d}{\delta c_0}]}{\delta \rho} < 0$.

Slope: Demonstration that $\frac{\delta[\frac{\delta y^d}{\delta P}]}{\delta \rho} > 0$

Now we examine the effect of increased income inequality (↑ρ) on the slope of the aggregate demand: $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho}$. To do this differentiate (2B-19) expression for $\frac{\delta y_2^d}{\delta P_0}$ with respect to ρ which yields (2B-25).

$$\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho} = \frac{-\left\{-\left(\frac{j}{v}\right)\frac{M_0^S}{P_0^2}\right\}\ \{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta \rho}\}}{[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)]^2} \text{ as } \rho \text{ only appears in denominator of (2B-19) via } \Sigma_2 \tag{2B-25}$$

(2B-25) becomes (2B-26) as ρ only appears in the (2B-25)'s Σ_2 term.

$$\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho} = \frac{-\{-(\frac{j}{v})\frac{M_0^S}{P_0^2}\}\ \{\delta(-\Sigma_2)/\delta \rho]}{[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)]^2} \tag{2B-26}$$

(2B-26) becomes (2B-27) by noting $\frac{\delta \Sigma_2}{\delta \rho} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}) < 0$.

$$\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho} = \frac{-\{(\frac{j}{v})\frac{M_0^S}{P_0^2}\}\ \{MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})]}{[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)]^2} = \frac{-\{(+)(+)\}\{-\}}{+} > 0 \tag{2B-27}$$

Since the slope of the aggregate demand is the inverse of $\frac{\delta y_2^d}{\delta P_0}$ an increase in income inequality, ↑ρ, which makes $\frac{\delta y_2^d}{\delta P_0}$ bigger in turn makes $\frac{\delta P_0}{\delta y_2^d}$ smaller. That is makes the aggregate demand steeper, i.e. less sensitive to changes in the price level.

Indirect Effects of a change in income distribution (↑ρ) on $y^d{}_2$

Demonstration that $\frac{\delta[\frac{\delta y_2^d}{\delta c_0}]}{\delta \rho} < 0$.

Differentiating (2B-2) with respect to ρ yields (2B-28).

$$\frac{\delta\ \frac{\delta y_2^d}{\delta c_0}}{\delta \rho} = \frac{\left\{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right][\frac{\delta 1}{\delta \alpha}]\right\}\ -\ (1)\{\cancelto{0}{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta \rho}}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-28}$$

(2B-28) becomes (2B-29) by knowing $\frac{\delta 1}{\delta \rho} = 0$ and noting that ρ only appears in Σ_2.

$$\frac{\delta \frac{\delta y_2^d}{\delta c_0}}{\delta \rho} = \frac{-(1)\{\frac{\delta\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]}{\delta \rho}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} = \frac{-(1)\{\frac{\delta[-\Sigma_2]}{\delta \rho}\}}{\left[1-\Sigma_2+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{2B-29}$$

(2B-29) becomes (2B-30) by noting that $\frac{\delta \Sigma_2}{\delta \rho} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}) < 0$.

$$\frac{\delta \frac{\delta y_2^d}{\delta c_0}}{\delta \rho} = \frac{MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})}{[1-\Sigma_2+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})]^2} \rightarrow \frac{(-)}{(+)} < 0 \tag{2B-30}$$

Again, we note that (2B-30) contains the multiplier term $\frac{1}{1-\Sigma_2+MPG+\left(\frac{je}{v}\right)}$.

The effects of increased inequality ↑ρ on magnitude on $y^d{}_2$ shifts and $y^d{}_2$ slope

Notably the work of Appendix 1B reveals the magnitude of a change in income distribution's effect on aggregate demand is reduced due to income distribution's impact on the multiplier. Inspecting Appendix 2B's relationships (2B-2) $\frac{\delta y_2^d}{\delta c_0}$ (as well as (2B-2)'s related derivatives for $\frac{\delta y_2^d}{\delta i_0}$, $\frac{\delta y_2^d}{\delta g_0}$, $\frac{\delta y_2^d}{\delta w}$, $\frac{\delta y_2^d}{\delta t_{0h}}$, and $\frac{\delta y_2^d}{\delta t_{0l}}$), (2B-3) $\frac{\delta y_2^d}{\delta M_0^s}$, (2B-11) $\frac{\delta y_2^d}{\delta t_{1h}}$, (1B-17) $\frac{\delta y_2^d}{\delta t_{1l}}$, and (2B-19) $\frac{\delta y_2^d}{\delta P_0}$ one sees that each contains the multiplier term

$$\frac{1}{1-\Sigma_2+MPG+(je/v)} \tag{2B-31}$$

Since ρ enters the (2B-31) via the denominator and only through Σ_2, the effect of ρ on Σ_2 determines the effect of income distribution on the multiplier. Now Σ_2 falls as ρ rises as seen by recalling equation (10.5) $\Sigma_2 \equiv MPC_h\rho(1-t_{1h}) + MPC_l(1-\rho)(1-t_{1l})$ and differentiating Σ_2 with respect to with respect to ρ.

$\frac{\delta[\Sigma_2]}{\delta \rho} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})$; which is less than zero because $MPC_h < MPC_l$ by assumption and $t_{1h} > t_{1l}$ in a progressive tax system.
(2B-32)

Hence an increase in ρ reduces Σ_2 which results in the multiplier falling as shown in (2B-33).

$$\frac{\delta Multiplier}{\delta \rho} = \frac{-\{-[MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})]\}}{[1-\Sigma_2+MPG+(je/v)]^2} \rightarrow \frac{-\{-[-]\}}{(+)} < 0. \tag{2B-33}$$

The multiplier falls when income distribution becomes more unequal, i.e. ↑ρ.

So in each of (2B-2) $\frac{\delta y_2^d}{\delta c_0}$ (as well as (2B-2)'s related derivatives for $\frac{\delta y_2^d}{\delta i_0}$, $\frac{\delta y_2^d}{\delta g_0}$, $\frac{\delta y_2^d}{\delta w}$, $\frac{\delta y_2^d}{\delta t_{0h}}$, and $\frac{\delta y_2^d}{\delta t_{0l}}$), (2B-3) $\frac{\delta y_2^d}{\delta M_0^s}$, (2B-11) $\frac{\delta y_2^d}{\delta t_{1h}}$, (2B-17) $\frac{\delta y_2^d}{\delta t_{1l}}$, and (2B-19) $\frac{\delta y_2^d}{\delta P_0}$ the magnitude of income distribution's effect is reduced by increased ρ as is confirmed by (2B-30) $\frac{\delta \frac{\delta y_2^d}{\delta c_0}}{\delta \rho} < 0$ as well as (2B-27) $\frac{\delta[\frac{\delta y_2^d}{P_0}]}{\delta \rho} > 0$ since the slope of aggregate demand is the reciprocal of (2B-27).