On the Survival of Conservatism Traders in an Asset Market with Strategic Interaction

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This paper examines a static model of an asset market with rational traders, conservatism traders and noise traders. Both rational and conservatism traders receive an informational signal about the asset payoff before any trade takes place. Conservatism traders are slow to update their conditional mean of the asset payoff relative to rational traders. To maximize their own expected profits, all rational and conservatism traders strategically submit their market orders to the market maker. This paper proves analytically that conservatism traders cannot survive. The implication of the results suggests that the anomaly of asset price underreaction to new information is a short-lived phenomenon.

INTRODUCTION

Conservatism bias is a type of behavioural bias well documented in the psychologists' experiments (see Edwards (1968)). Traders with conservatism bias are slow in updating their beliefs (relative to rational Beyesian traders) when forming their posterior beliefs. Conservatism bias has been viewed by some behavioral models as a cause of asset price underreaction to new information (see Douks and McKnight (2005), Jegadeesh and Titman (2001) and Barberis, Shleifer and Vishny (1998)). Now, the question is whether traders with conservatism bias would survive in the market competition in the long run? Can the phenomenon of asset price underreaction (caused by conservatism bias) be long lived?

This paper attempts to examine the long-run survival of traders with conservatism bias in an asset market allowing for strategic interaction among traders. Specifically, this paper, in the spirit of Kyle (1985), builds a one-period model of an asset market. The asset payoff is unknown to all traders in the beginning of the period but traders receive an informational signal about the asset payoff before any trade takes place. There are rational traders, conservatism traders and noise traders in the market. Conservatism traders are slow to update their conditional mean of the asset payoff relative to rational traders. Rational and conservatism traders are both risk neutral. Noise traders trade for their liquidity needs. Hence, their demand for the asset is assumed to be random. There is one market maker who supplies the liquidity to the market. The cost of doing so is assumed to be zero. To maximize their own expected profits, all rational and conservatism traders strategically submit their market orders for the asset to the market maker. After observing the aggregate market orders of all traders, the market maker sets the asset price equal to the expected asset payoff conditional on the observed aggregate market orders for the asset. The market maker does not observe the informational signal about the asset payoff.

In the equilibrium, the market order for the asset coming from each of rational and conservatism traders is generated from maximizing his expected profit given all others' equilibrium market orders for the asset and given the equilibrium pricing rule of the market maker. Given the equilibrium market orders of all rational and conservatism traders, asset price must equal the expected asset payoff conditional on the observed aggregate demand for the asset.

This paper proves that conservatism traders lose money every period and they cannot possible survive in the long run in any evolutionary dynamic. The implication of this result suggests that the anomaly of asset price underreaction to new information caused by conservatism bias cannot be long lived.

The remainder of this paper consists of three sections. The next section describes the model. Section 3 presents the results. Section 4 concludes the paper.

THE MODEL

Consider an asset market with one asset and one market maker. The payoff of the asset is unknown to all traders. But traders have prior information about the asset payoff. In other words, traders know that the payoff of the asset (denoted as θ) is normally distributed with the mean θ (where θ>0) and variance of σ_θ^2. In addition, traders receive an informational signal about the asset payoff in the beginning of the period before any trade occurs. The informational signal about the asset payoff (denoted as S) is modeled as: $S = \theta + \varepsilon$, where ε is normally distributed with the mean of zero and variance of σ_ε^2; furthermore, ε is independent of θ.

There are three types of traders in the market: rational traders, conservatism traders and noise traders. Since θ and ε are independent and normally distributed, rational traders, after receiving the informational signal, update their belief about the asset payoff according to[1]

$$E(\theta|(S,r)) = \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}(S-\bar{\theta}), \tag{1}$$

where the parameter r indicates a rational trader.

Conservatism traders are those who display conservatism bias. Conservatism bias is a type of behavioural bias identified in the psychologists' experiments (see Edwards (1968)). Conservatism traders are slow in updating their beliefs about the asset payoff relative to rational traders. Conservatism traders' conditional mean of the asset payoff is modeled as the summation of their prior knowledge plus the partial adjustment towards rational traders' conditional mean of the asset payoff. Specifically,

$$E(\theta|(S,c)) = \bar{\theta} + m(E_r(\theta|S) - \bar{\theta}) = \bar{\theta} + \frac{m\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}(S-\bar{\theta}), \tag{2}$$

where the parameter c indicates a conservatism trader and $m \in (0,1)$. The parameter m reflects the trader's conservatism bias. The lower the parameter m, the greater is the traders' conservatism bias. In addition, if the realization of the informational signal is larger than the expected asset payoff, then conditional mean of the asset payoff for conservatism traders is smaller than that for rational traders; on the other hand, if the realization of informational signal is smaller than the expected asset payoff, then the conditional mean of the asset payoff for conservatism traders is larger than that for rational traders.

Noise traders trade based on their liquidity needs and their demand for the asset is assumed to be a normally distributed random variable (denoted as x) with the mean of zero and variance of σ_x^2. It is assumed that the random variables θ, ε and x are mutually independent.

There is a total of N rational and conservatism traders in the market. Among the N traders, the proportion of traders being conservatism traders is denoted as f, (where $f \in [0,1]$).

The market maker provides the liquidity to the market and the cost of supplying the liquidity to the market is assumed to be zero. After receiving the informational signal in the beginning of the period, all traders submit their market orders for the asset to the market maker. The market maker can observe the aggregate demand for the asset of all traders, but he cannot observe individual market orders for the asset. Hence, after observing the aggregate market orders for the asset (denoted as D) of all traders, the market maker sets the price for the asset (denoted as P) to equal to the conditional mean of the asset payoff. That is,

$$P = E(\theta|D). \tag{3}$$

Given the pricing rule described in equation (3), taking into account of the impact on the asset price of their market orders for the asset, rational and conservatism traders strategically set their demand for the asset to maximize their own expected profits conditional on observing the informational signal about the asset payoff. Specifically, trader i ($i \in \{1,2,\ldots N\}$) of either type of traders ($j = r, or\ c$) sets his demand for the asset (denoted as X_{ij}) according to

$$\max_{X_{ij}}\left((E(\theta|(S,j)) - E\left(P\middle|(S,X_{ij})\right))X_{ij}\right). \tag{4}$$

Denote N_r as the number of rational traders in the market and N_c as the number of conservatism traders in the market. Hence, $N_r + N_c = N$. The equilibrium in the market is characterized by X_{ir}, for $i = 1,2,\ldots N_r$, and X_{ic}, for $i = 1,2,\ldots N_c$, where X_{ir} and X_{ic} solve the optimization problem (4) and the asset price P is determined from equation (3). Note that rational and conservatism traders are risk neutral in this model.

THE RESULTS

This section presents the solution to the above optimization problem and proves that conservatism traders cannot survive in the long run.

Specifically, as shown in the appendix, the equilibrium strategies for conservatism and rational traders and the equilibrium asset price are as follows:

$$X_r = \frac{\eta(Nf(1-m)+1)(S-\bar{\theta})}{\lambda(N+1)}, \tag{5}$$

$$X_c = \frac{\eta[N(m-1)(1-f)+m)](S-\bar{\theta})}{\lambda(N+1)}, \tag{6}$$

and

$$P = \bar{\theta} + \lambda x + \frac{\eta N(1-f+fm)(S-\bar{\theta})}{N+1}, \tag{7}$$

respectively, where the parameter λ is characterized by:

$$\lambda^2 = \frac{\eta N\sigma_\theta^2(fm-f+1)(Nf(1-m)+1)}{\sigma_x^2(N+1)^2}. \tag{8}$$

Equation (8) indicates that there is one positive and one negative solution for the parameter λ. The positive solution is used as the value of the parameter λ to ensure that the equilibrium price increases in the total demand for the asset and to ensure that the second-order condition of the optimization problem (4) holds.

Furthermore, the expected profit of trader i ($i = r, c$) (denoted as π_i) is computed as $\pi_i = E((\theta - P)X_i)$. The difference in the expected profits of conservatism and rational traders is

$$\pi_c - \pi_r = \frac{\sigma^2}{\lambda}\frac{\eta}{N+1}(m-1)(Nf - Nfm + 1). \tag{9}$$

Note from equation (9) that due to the conservatism bias (i.e., $m < 1$), conservatism traders make less expected profit than rational traders.

To examine whether conservatism traders would survive in the long run, the one period model described above is embedded in an evolutionary process that is characterized as follows. Assume that traders are able to observe others' profits and imitate others' strategies if they are proven to be more profitable than their own, then the fraction of the population being rational traders is modeled by

$$f_{t+1} = f_t + g(\pi_c(f_t) - \pi_r(f_t); f_t), \tag{10}$$

where f_t is the proportion of traders being conservatism traders in time period t; and $g(\cdot)$, a mapping from $(-\infty, +\infty) \times (0,1) \rightarrow (0,1)$, is a continuous function with the following properties[2]:

$(i) g(\cdot) < 0 \; if \; \pi_c(f_t) - \pi_r(f_t) < 0 \; and \; f_t < 0,$

$(ii) \; g(\cdot) \; = 0 \; if \; \lim_{f_t \rightarrow 0^+}(\pi_c(f_t) - \pi_r(f_t)) \leq 0.$

In other words, the fraction of the population being conservatism traders will decrease in next time period if in the current period rational traders make more expected profit than do conservatism traders. As this fraction gets closer to zero, $g(\cdot)$ will move closer to zero. This ensures the steady state can be eventually locked at the corner point.

Since rational traders indeed make more expected profit every time period than conservatism traders do in the market (due to equation (9)), according to the dynamics described in equation (10), the fraction of a population being conservatism traders decreases each time period until it reaches zero at the corner point and it will be locked at the corner point in the limit. In other words, in the long run, conservatism traders in this asset market will not survive. This is stated in the proposition below:

Proposition 1 Conservatism traders will lose money to rational traders and conservatism traders cannot survive in the long run. The phenomenon of asset price underreaction to new information (caused by conservatism bias) is short-lived and will disappear in the long run.

This proposition suggests that the phenomenon of asset price underreaction to new information (caused by conservatism bias) is short-lived and will disappear in the long run. This is because the expected profit of conservatism traders is smaller than that of rational traders. Hence, the conservatism traders will not survive in the long run.

CONCLUSION

This paper examines the survival of conservatism bias in a one-asset market with one market maker. The payoff of the asset is unknown to all market participants. But rational and conservatism traders receive an informational signal about the asset payoff before any trade takes place. Traders are risk neutral and they act strategically to maximize their expected profits. In this model, the market maker supplies the liquidity to the market. The cost of doing so is assumed to be zero. Rational and conservatism traders submit their market orders to the market maker. After observing the aggregate demand of all traders (including noise traders' demand for the asset), the market maker sets the asset price equal to the expected asset payoff conditional on the observed aggregate demand for the asset of all traders. This paper is able to obtain a unique equilibrium where conservatism traders make less expected profit than rational traders. Consequently, conservatism traders cannot survive in the long run. The implication of this result suggests the phenomenon of asset price under reaction to new information (caused by conservatism bias) is short-lived and will disappear in the long run.

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1. See Hoel (1962).
2. This class of dynamics is very general, and it is consistent with the replicator dynamics and many other types of selection dynamics used in the evolutionary game theory (see Taylor and Jonker (1978), Weibull (1995), Luo (1999)). This dynamic has been applied in securities market (see Hirshleifer and Luo (2001), and Fischer and Verrecchia (1999)).

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APPENDIX

Derivation of equations (5), (6) and (7): Suppose that the equilibrium strategies for both rational and conservatism traders are linear functions of the informational signal and the asset price is a linear function of the total demand for the asset of all traders, then the following will prove the existence of such an equilibrium. Specifically, the market order for the asset of trader i of type j, for $j = r, c$, is assumed to be

$$X_{ij} = a_{ij} + b_{ij}S, \tag{11}$$

for $i = 1,2,..,N_r$ if $j = r$ and for $i = 1,2,..,N_c$ if $j = c$; and the asset price is assumed to follow the linear pricing rule:

$$P = \mu + \lambda D, \tag{12}$$

where $D = \sum_{i=1}^{N_r} X_{ir} + \sum_{i=1}^{N_c} X_{ic} + x$; and all the coefficients μ, λ, a_{ij}, and b_{ij} (for $i = 1,2,..,N_r$ when $j = r$ and for $i = 1,2,..,N_c$ when $j = c$) are to be determined in the following:
Substituting equation (11) and (12) into the optimization problem (4), and using equations (1) and (2), one can solve the first-order condition for optimization problem (4) as the following: for $\eta = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$,

$$\bar{\theta} + \eta(S - \bar{\theta}) - \mu - \lambda(2X_{ir} + \sum_{\substack{n=1 \\ n\neq i}}^{N_r}(a_{nr} + b_{nr}S) + \sum_{n=1}^{N_c}(a_{nc} + b_{nc}S)) = 0, \tag{13}$$

and

$$\bar{\theta} + m\eta(S - \bar{\theta}) - \mu - \lambda(2X_{ic} + \sum_{n=1}^{N_r}(a_{nr} + b_{nr}S) + \sum_{\substack{n=1 \\ n\neq i}}^{N_c}(a_{nc} + b_{nc}S)) = 0. \tag{14}$$

Substituting equation (11) into equations (13) and (14) yields:

$$a_{ij} = \frac{\bar{\theta}-\mu-R_j\eta\bar{\theta}}{\lambda} - A, \tag{15}$$

and

$$b_{ij} = \frac{R_j\eta}{\lambda} - B, \tag{16}$$

where $A = \sum_{n=1}^{N_r} a_{nr} + \sum_{n=1}^{N_c} a_{nc}$, and $B = \sum_{n=1}^{N_r} b_{nr} + \sum_{n=1}^{N_c} b_{nc}$; and $R_j = 1$ if $j = r$; and $R_j = m$, if $j = c$.
Note from equations (15) and (16) that for $i' \neq i, a_{ij} = a_{i'j}$ and $b_{ij} = b_{i'j}$ for $j \in \{r, c\}$. Hence, let $a_{ir} = a_r$, $b_{ir} = b_r$ for $R_j = 1$; and $a_{ic} = a_c$, $b_{ic} = b_c$ for $R_j = m$. Using equations (15) and (16), then the following four equations are true:

$$a_r = \frac{\bar{\theta}-\mu+\eta\bar{\theta}(Nf(m-1)-1)}{\lambda(N+1)}, \tag{17}$$

$$a_c = \frac{\bar{\theta}-\mu+\eta\bar{\theta}(N(1-f)(1-m)-m)}{\lambda(N+1)}, \tag{18}$$

$$b_r = \frac{\eta(1+Nf(1-m))}{\lambda(N+1)}, \tag{19}$$

and

$$b_c = \frac{\eta(m - N(1-f)(1-m))}{\lambda(N+1)}. \tag{20}$$

In addition, equation (3) implies that

$$P = E(\theta | A + BS + x = D) = \bar{\theta} + \frac{B\sigma_\theta^2(D - A - B\bar{\theta})}{B^2\sigma_S^2 + \sigma_x^2}. \tag{21}$$

Also, equations (12) and (21) along with the definitions of A and B imply that

$$\mu = \theta, \tag{22}$$

and

$$\lambda^2 = \frac{\eta N \sigma_\theta^2(fm - f + 1)(Nf(1-m) + 1)}{\sigma_x^2(N+1)^2}. \tag{23}$$

Note from equation (23) that there is one positive and one negative solution for the parameter λ. The positive solution is used as the value of the parameter λ. This ensures the equilibrium price to increase in the total demand for the asset of all traders. In addition, the positive λ also ensures that the second-order condition of optimization problem (4) holds.

The coefficients described in equations (17) through (20), (22) and (23) uniquely solve the optimization problem (4) and equation (12). Therefore, the equilibrium strategies in equations (5) and (6); and equilibrium asset price in equation (7) are obtained from equations (17) through (11), (12), (20) and (22).