Counterparty Default Risk and Monetary Policy in Unsecured Interbank Lending Markets

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Banks bargain over the cost of unsecured loans in a model of interbank lending markets characterized by counterparty default risk. The risk an individual bank defaults depends on both idiosyncratic and aggregate factors and impacts loan rates and participation in the interbank loan market. After a deterioration of the aggregate factor affecting default risk, participation in interbank lending markets declines, and both individual and the average interbank loan rates rise. The model suggests that some policy actions enacted during the late-2000s financial crisis may have reduced participation and increased spreads in unsecured interbank loan markets.

INTRODUCTION

While each component of the financial markets plays a distinct role in the provision of credit and liquidity, interbank lending remains an integral part of modern financial systems both in regard to their role in facilitating the flow of credit throughout the economy and in implementing monetary policy. However, like other segments of the financial system, interbank credit markets are not immune to turbulence. The turmoil is often minor but, on occasion, can be quite severe.

The financial crisis of 2007-2009 was a period of unprecedented turmoil in interbank credit markets and witnessed significant spikes in the cost and price volatility of both unsecured and secured interbank credit. Empirical evidence highlights the effect of counterparty risk in driving these heightened spreads (Williams and Taylor (2009), Afonso, Kovner, and Schoar (2011), Angelini, Nobili, and Picillo (2011), Gorton and Metrick (2012)). In unsecured interbank lending markets, lending banks face counterparty risk due to the fact that interbank loans may not be repaid if the borrowing bank defaults. During the crisis, as markets and bank balance sheets deteriorated and uncertainty and the risk of default rose, interest rates on interbank credit rose in order to compensate lenders for the higher degree of default risk.

The deterioration in interbank markets also prompted an unprecedented response from monetary policy makers. In addition to lowering the policy rate, the Federal Reserve took the unusual action of lowering the discount rate — the cost of borrowing at its emergency lending facility — in August 2007[1]. In October 2008, the Federal Reserve started paying interest on bank reserves in an attempt to provide a floor on interbank rates in the face of severe downward price pressure caused by quantitative easing.[2]

Given the role that counterparty risk played in interbank lending markets and the response from central banks, this paper constructs a model where the effects of counterparty risk and monetary policy actions on interbank lending markets can be analyzed. The results of the model are driven by two key characteristics. First, banks are capable of default on interbank liabilities. The probability that a bank defaults depends on two factors: an aggregate quality component that affects the risk of default at all banks and a second idiosyncratic quality component that is bank-specific. I focus specifically on unsecured interbank loans given its importance as the main policy target for many central banks. Second, banks bilaterally bargain over the interbank loan rate. This setup aims to reflect the over-the-counter (OTC) nature of interbank lending. The resulting match-specific interbank loan rate is modeled as the solution to a Nash bargaining problem and is affected by counterparty default risk and monetary policy tools.

Using this simple framework, individual loan rates and the average loan rate are derived. The average loan rate depends, of course, on individual loan rates but also on the extent of participation in the interbank market. Although all banks may engage in the interbank loan market, not all banks will participate due to the heterogeneity of banks introduced by the idiosyncratic quality component. For some matches, the borrowing bank will find it more attractive to borrow from the central bank's lending facility and the lending bank will find it more attractive to continue to hold excess liquidity as reserves. Given the result of the bargaining problem, an endogenous threshold level of the idiosyncratic bank quality measure emerges. Only borrowing banks with quality of at least the threshold will participate in interbank markets; lower quality borrowing banks below the threshold do not participate.

I then use the model to analyze events and policies that transpired during the financial crisis. A fall in the aggregate quality component — meant to represent the financial distress and perceived increase in counterparty default risk that occurred during the crisis — increases individual loan rates and reduces participation in interbank markets. The effect on the average interbank loan rate is more nuanced and can be decomposed into two counteracting effects. First, the incumbent interest rate effect: when the aggregate quality component falls, default risk for borrowing banks already participating in the interbank market rises. As a result, risk premia for all borrowers rise, putting upward pressure on the average rate. However, there is a second counteracting effect – the borrower threshold effect. A deterioration in aggregate conditions causes the lowest quality borrowing banks to drop out of the interbank market. Since these borrowers paid the highest risk premia, their exit from the interbank market causes the average premium to fall and puts downward pressure on the average interbank loan rate. I show that, under the assumptions of the model, the incumbent interest rate effect outweighs the borrower threshold effect, and the average loan rate rises when aggregate conditions deteriorate.

The effects of monetary policy actions taken during the financial crisis are also studied. A decrease in the cost of borrowing from the central bank's lending facility decreases both individual and the average interbank loan rates but reduces participation in interbank markets. On the other hand, an increase in interest paid on reserves pushes up individual and average loan rates but reduces participation in interbank markets.

This paper is related to a growing literature on counterparty risk in interbank credit markets. Several models (Freixas and Jorge (2008), Heider and Hoerova (2009), Heider, Hoerova, and Holthausen (2010)) use a setting with informational asymmetries to analyze how interbank credit markets function and when they break down. In these models, loan terms are assumed to result from a no-arbitrage condition, abstracting away from the reality of bilateral bargaining that occurs in interbank markets. In contrast, my model focuses on the impact of credit risk on interbank rates when banks bargain over the terms of the loan. I also focus on a full information setting where the probability of default for the lender and borrower is known by both parties. Participation is determined not by rationing, as in the aforementioned models, but by a participation constraint for both the borrowing and lending bank.

Several models do use a Nash bargaining process to model interbank market transactions. Ennis and Weinberg (2009) and Bech and Klee (2011) use a Nash bargaining process to help explain anomalies concerning the fed funds rate relative to, respectively, the discount window rate and the interest rate paid on reserves. Afonso and Lagos (2012) employ a continuous-time search and bargaining model to investigate the determinants of the fed funds rate, and Acharya, Gromb, and Yorulmazer (2012) model interbank transactions as a two-stage bargaining process with the threat of breakdown. However, these models do not focus on the effect of default risk on the interbank loan rate.

This paper also contributes to the literature on the effects of monetary policy in interbank lending markets. Numerous models look at the effects of interest on reserves in interbank markets (Bech and Klee (2011)), the cost of central bank's lending facility (Ennis and Weinberg (2009), Heider, Hoerova, and Holthausen (2010), Bech and Monnet (2016)), or both (Afonso and Lagos (2012), Afonso, Armenter, and

Lester (2018)). However, none of the aforementioned models analyzes such policies in a setting where counterparty default risk is present and loan rates are determined via bargaining.

My model is most closely related to Vollmer and Wiese (2016) who constructed an interbank lending market with counterparty risk, where the interbank loan rate is determined by bilateral bargaining. In this setting, the authors examine the impact of interest on reserves and the central bank's lending facility rate on the likelihood that an interbank transaction occurs. There is heterogeneity in default probabilities between borrowing and lending banks, but no heterogeneity amongst the set of borrowing banks (nor the set of lending banks). In contrast, my model introduces heterogeneity across the set of borrowing banks and the set of lending banks in regard to default risk. This allows me to derive a distribution of loan rates and pin down the extent of participation in the interbank loan market.[3] It also allows me to examine the impact of monetary policies on both participation and the average interbank loan rate. Factoring in the effect of participation on the average loan rate is important because the set of participating banks affects the resulting average rate, and it is the average rate that serves as a benchmark for many other loan rates, most often referenced in regard to financial market conditions, and also serves as many central banks' main policy instrument.

The remainder of the paper is organized as follows: Section I outlines how the cost of and access to interbank credit is determined. Section II derives and analyzes the average interbank interest rate, and Section III concludes.

THE MODEL

Model Overview

The model consists of three periods $t = 0,1,2$, and is populated by a continuum of financial intermediaries (henceforth referred to as banks) of mass 1. The basic timeline of the model is illustrated in table 1.

In $t = 0$ banks are endowed with 1 unit of funds from risk-neutral depositors. A fraction, $1 - \alpha$, $\alpha \in (0,1)$, of these deposits are held in a long-term illiquid asset and the remaining α are held in a short-term liquid asset (henceforth referred to as reserves). The probability that bank i's long-term assets are successful and return R^L is equal to $P\theta_i$. $P \in [0,1]$ is an aggregate quality component that affects all banks' probability of success; $\theta_i \in [0,1]$ is an idiosyncratic, bank-specific quality component affecting bank i's probability of success. If a bank's long-term assets fail, they earn a return of zero and the bank fails and is shut down by regulators. I assume that θ_i is distributed uniformly on the interval $[0,1]$. I also assume that both P and θ_i are realized in period $t = 0$ and are public information.

To satisfy the liquidity needs of depositors, claims $d_1 > 0$ and $d_2 > 0$ are offered by banks at $t = 0$ for withdrawal at dates 1 and 2.[4] At the beginning of $t = 1$, banks are hit with a liquidity shock. While the aggregate demand for liquidity at each date is certain, banks face uncertain liquidity demand at the individual level. While a fraction λ of depositors withdraw their claims at $t = 1$ (and $1 - \lambda$ withdraw at $t = 2$), half of the banks receive a below-average amount of depositor withdrawals $\lambda_l d_1$, where $\lambda_l < \lambda$. These banks (dubbed "lending banks") are left with an excess supply of liquidity that they can either hold as reserves or lend to another bank on the interbank credit market. The other half of the banks (dubbed "borrowing banks") receive an above-average amount of depositor withdrawals $\lambda_h d_1$, where $\lambda_h > \lambda_l$, and $\frac{1}{2}\lambda_h + \frac{1}{2}\lambda_l = \lambda$. Since it is assumed that banks cannot liquidate any of their illiquid assets, banks with high withdrawals must attempt to borrow funds on the interbank credit market or borrow from the central bank's standing lending facility.

TABLE 1
MODEL TIMELINE

$t = 0$	Banks are endowed with 1 unit of deposits. $1 - \alpha$ are held in an illiquid long-term asset. α are held as liquid reserves. The aggregate quality component and idiosyncratic quality components are realized.
$t = 1$	Liquidity shocks hit. Half of the banks have low withdrawals of $\lambda_l d_1$ and have excess liquidity. Half of the banks have high withdrawals of $\lambda_h d_1$ and need extra liquidity. Each bank with excess liquidity is matched with a bank in need of liquidity and negotiates the interbank loan rate. If negotiations break down, the bank in need of liquidity borrows from the central bank's lending facility and the bank with excess liquidity holds their funds as reserves.
$t = 2$	Loans, interbank liabilities, and remaining deposits are repaid, which means fewer defaults.

I interpret the interbank credit market as being analogous to the federal funds market, where banks lend and borrow excess reserves via unsecured loans. Due to the decentralized nature of the fed funds market, the terms of interbank credit transactions are modeled as the result of a Nash bargaining solution. I assume that borrowing and lending banks are randomly matched and may only match with one counterparty. The consequent interbank interest rate is bank-specific in that the terms of the loan depend on the borrowing bank's probability of success.

However, not all banks in need of liquidity will end up borrowing in the interbank market. Given the results of the Nash bargaining solution, only borrowing banks with an idiosyncratic quality value above a certain threshold will find it advantageous to borrow on the interbank market. Borrowing banks that fall below the threshold will find it more attractive to borrow from the central bank's lending facility. At the same time, lending banks matched with these low-quality borrowing banks will find it more attractive to hold their excess liquidity as reserves, given the expected return to lending that results from the Nash bargaining solution. The threshold that determines participation in interbank markets depends on both aggregate conditions and monetary policy variables. Lastly, in $t = 2$, liabilities are repaid, less any defaults, and bank profits are realized. To simplify the model, I assume that if a bank defaults on their interbank liabilities, the resulting losses are small enough that the lending bank will remain solvent (conditional on the success of the lending bank's long-term asset).

In the remainder of this section, I describe in detail how the cost and use of interbank credit are determined in the model. I begin with interest rate determination and then derive the necessary conditions for a borrowing bank to participate in interbank lending markets. I then derive the average interbank lending rate. Comparative statics are conducted throughout. Of special interest is how the cost of and participation in interbank lending markets changes with the aggregate success probability P as well as monetary policy variables. I interpret the events that precipitated the financial crisis as a sharp increase in the probability of default for all banks, which is modeled as a decrease in P.

Interest Rate Determination

After withdrawal shocks are realized, lending bank l and borrowing bank b are randomly matched and they negotiate over the interbank interest rate $R_{l,b}^{IB}$. The resulting interest rate is the solution to the following Nash bargaining problem:

$$\max_{R_{l,b}^{IB}} [E(\pi_l) - E(d_l)]^{\eta} [E(\pi_b) - E(d_b)]^{1-\eta} \tag{1}$$

where $\eta \in (0,1)$ is the lending bank's relative bargaining power, $E(\pi_i)$ is the expected payoff for bank $i = \{l, b\}$ when a loan is made and $E(d_i)$ is the expected disagreement point for bank $i = \{l, b\}$ — what bank i expects to earn if no loan is made.

At date $t = 1$, banks in need of liquidity face deposit withdrawals of $\lambda_h d_1$ but only have $\alpha < \lambda_h d_1$ of reserves. If it cannot raise the appropriate funds by the end of $t = 1$, the bank is shut down by regulators. If the borrowing bank b receives a loan from lending bank l, its expected profits $E(\pi_b)$ at the end of $t = 2$ are as follows:

$$E(\pi_b) = \theta_b P[(1-\alpha)R^L + \alpha R^R - R_{l,b}^{IB} L^{IB} - (1-\lambda_h)d_2] \tag{2}$$

$$\text{s.t.} \quad \lambda_h d_1 \leq \alpha + L^{IB} \tag{3}$$

With probability $\theta_b P$, bank b is successful and earns a return of R^L on its $1-\alpha$ illiquid assets and R^R on its α of reserves. $R^R \geq 1$ represents the interest paid on reserves held at the central bank. When successful, bank b also repays its interbank liabilities and remaining depositor claims in $t = 2$. I assume that borrowing banks always repay their interbank liabilities, given they do not default, regardless of whether or not the lending bank survives. If the lending bank does fail, bank b's payment is used to repay the lending bank's creditors. Lastly, equation 3 imposes a flow of funds constraint: bank b's reserves and funds borrowed on the interbank market must be large enough to meet total depositor withdrawals in period 1.

If bank b does not receive a loan on the interbank credit market, it must borrow from the central bank's lending facility, lest it is shut down. In this scenario, bank b receives the following expected profits $E(d_b)$:

$$E(d_b) = \theta_b P[(1-\alpha)R^L + \alpha R^R - R^{LF} L^{LF} - (1-\lambda_h)d_2] \tag{4}$$

$$\text{s.t.} \quad \lambda_h d_1 \leq \alpha + L^{LF} \tag{5}$$

where $R^{LF} \in [1, R^R)$ is the cost of funds borrowed from the central bank and L^{LF} is the quantity of funds borrowed from the central bank.[5] The intuition behind equation 4 is analogous to equation 2; the only difference when borrowing from another bank versus the central bank is the cost of the loan.

Banks that receive a positive liquidity shock have period 1 depositor withdrawals of $\lambda_l d_1 < \alpha$. If the lender l makes an interbank loan to bank b, its expected profits are as follows:

$$E(\pi_l) = \theta_l P[(1-\alpha)R^L + \alpha R^R + \theta_b P R_{l,b}^{IB} L^{IB} - (1-\lambda_l)d_2] \tag{6}$$

$$\text{s.t.} \quad \lambda_l d_1 + L^{IB} \leq \alpha \tag{7}$$

The lending bank's expected profit from participation in the interbank lending market is interpreted analogously to the borrowing bank's profit with two notable differences. First, repayment of the interbank loan made to bank b is contingent on bank b's success; thus, bank b's probability of success affects lending bank l's expected profits. Second, the flow of funds constraint for the lending bank states that bank l's reserves must be large enough to cover funds offered on the interbank loan market, plus date 1 deposit withdrawals.

If l does not make the loan to b, the expected payout (and l's disagreement point $E(d_l)$) is given by the following:

$$E(d_l) = \theta_l P[(1-\alpha)R^L + (\alpha - \lambda_l d_1)R^R - (1-\lambda_l)d_2] \tag{8}$$

When bank l does not make a loan to bank b, all $\alpha - \lambda_l d_1$ of bank l's reserves that remain after paying date 1 withdrawals continue to be held as reserves.

Before solving the bargaining problem, a few simplifications can be made in regard to the loan quantities L^{IB} and L^{LF}. First, since the cost of loans from the central bank's lending facility and the equilibrium cost of interbank loans are greater than the opportunity cost of using reserves to meet liquidity needs, borrowing banks will use all of their reserves to meet date 1 deposit withdrawals and then borrow the shortfall either from another bank or from the lending facility. This implies that borrowing banks seek to borrow

$$L^{IB} = L^{LF} = \lambda_h d_1 - \alpha \tag{9}$$

Given the flow of funds constraint in equation 7, the most that lending bank l can lend is $L^{IB} = \alpha - \lambda_l d_1$. To ensure that bank b's liquidity needs can be satisfied by bank l, I assume that $2\alpha \geq d_1$. Additionally, since banks are assumed to be risk-neutral, bank l will be willing to satisfy all of bank b's liquidity demand (as long as bank b's idiosyncratic quality component is high enough to satisfy both banks' participation constraints). This implies that the loan amount will be given by equation 9.

Given equations 2, 4, 6, 8, and 9, the Nash bargaining problem solves the following maximization problem:

$$\max_{R_{l,b}^{IB}} [\theta_l P(\theta_b P R_{l,b}^{IB} - R^R)(\lambda_h d_1 - \alpha)]^{\eta} [\theta_b P(R^{LF} - R_{l,b}^{IB})(\lambda_h d_1 - \alpha)]^{1-\eta} \tag{10}$$

Solving the maximization problem 10 yields the following equilibrium interbank interest rate $R_{l,b}^{IB}$:

$$R_{l,b}^{IB} = \eta R^{LF} + \frac{(1-\eta)R^R}{\theta_b P} \tag{11}$$

Result 1. *$R_{l,b}^{IB}$ satisfies the following properties:*

1. *$R_{l,b}^{IB}$ is decreasing in the borrowing bank's idiosyncratic success probability θ_b.*
2. *$R_{l,b}^{IB}$ is decreasing in the aggregate success probability P.*
3. *$R_{l,b}^{IB}$ is increasing in the central bank lending facility rate R^{LF}.*
4. *$R_{l,b}^{IB}$ is increasing in the interest paid on bank reserves R^R.*

Proof: Each part of result 1 can be shown by partially differentiating equation 11 with respect to relevant right-hand side variable.

Intuitively, if the borrowing bank has a lower probability of success due to either a lower value of θ_b or a lower value of P, the lending bank can negotiate a higher interbank interest rate to compensate it for the higher level of counterparty risk.

An increase in the cost of central bank-borrowed funds, R^{LF}, increases the interbank rate that borrowing banks are willing to pay and therefore increases the equilibrium interbank loan rate. If the premium paid on bank reserves, R^R, rises, the opportunity cost of funds rises for the lending bank and they demand a higher interbank loan rate as result.

Determining Access to Credit

The previous section describes how the interbank interest rate is determined for an individual lender-borrower match. However, not all matches will result in an interbank loan transaction. Participation constraints for both the borrowing and lending bank must be satisfied. If not, bargaining breaks down; the borrowing bank must turn to the central bank's lending facility for its liquidity needs and the lending bank holds any excess liquidity as reserves.

For the lender to participate in the interbank loan, the expected return of lending to borrowing bank b must at least be as large as the interest paid on reserves:

$$\theta_b P R_{l,b}^{IB} \geq R^R \tag{12}$$

$$\Rightarrow \theta_b P \eta R^{LF} + (1-\eta) R^R \geq R^R \tag{13}$$

where equation 13 makes use of the interbank loan rate from the previous section (equation 11).

For the borrower to participate in the interbank loan market, the expected cost of the interbank loan must be no larger than the expected cost of borrowing funds from the central bank's lending facility:

$$\Rightarrow \theta_b P \eta R^{LF} + (1-\eta) R^R \geq R^R \tag{14}$$

$$\Rightarrow \eta R^{LF} + \frac{(1-\eta)R^R}{\theta_b P} \geq R^{LF} \tag{15}$$

where equation 13 makes use of equation 11.

Both the borrower's and lender's participation constraints pin down a single threshold value of $\theta_{b,}$ which determines participation in the interbank lending market.

Result 2. *There exists a threshold level of the borrowing bank's idiosyncratic success probability $\widetilde{\theta_b}$ such that for any interbank loan matches where $\theta_b < \widetilde{\theta_b}$, negotiations break down and the transaction does not take place. Interbank lending transactions do occur, at the interest rate in equation* 11, *for matches with $\theta_b \geq \widetilde{\theta_b}$. The threshold is given by the following condition:*

$$\widetilde{\theta_b} = \frac{R^R}{R^{LF}P} \tag{16}$$

Proof: Result 2 can be shown using equations 13 and 15. Since the left-hand side of the lender's participation constraint (equation 13) is increasing in θ_b, the lowest level of θ_b that satisfies the lender's participation constraint occurs when equation 13 holds with equality:

$$\tilde{\theta}_b P \eta R^{LF} + (1-\eta) R^R = R^R \tag{17}$$

Solving equation 17 for $\tilde{\theta}_b$ yields the value of the threshold in equation16.

At the same time, the borrowing bank's participation constraint implies the same cutoff for θ_b. Since the left-hand side of the borrower's participation constraint (equation15) is decreasing in θ_b, the lowest level of θ_b that satisfies the borrower's participation constraint occurs when equation 15 holds with equality:

$$\eta R^{LF} + \frac{(1-\eta)R^R}{\tilde{\theta}_b P} = R^{LF} \tag{18}$$

Solving equation 18 for $\tilde{\theta}_b$ yields the same value of $\tilde{\theta}_b$ implied by the lender's participation constraint. □
The remainder of this section is concerned with how this idiosyncratic cutoff, $\tilde{\theta}_b$, changes with respect to aggregate conditions. These results are formalized in the following result:

Result 3. *The cutoff idiosyncratic success probability component $\tilde{\theta}_b$ satisfies the following properties:*

1. *$\tilde{\theta}_b$ is decreasing in the aggregate success probability component, P.*
2. *$\tilde{\theta}_b$ is decreasing in the central bank's lending facility rate, R^{LF}.*
3. *$\tilde{\theta}_b$ is increasing in the interest paid on bank reserves, R^R.*

Proof: Each component of result 3 can be proved by differentiating equation 16 with respect to the respective variable of interest.

Intuitively, how each of the variables considered in result 3 affects cutoff $\tilde{\theta}_b$ depends on its effect on the lending and borrowing bank's participation constraints. A higher level of P increases the expected return to lending by increasing the likelihood that an interbank loan is repaid and simultaneously decreases the expected cost of borrowing for all levels of θ_b by lowering the risk premia paid on interbank loans. This loosening of both the lender and borrower participation constraints allows $\tilde{\theta}_b$ to fall and increases participation in the interbank market.

When the cost of loans from the central bank's lending facility (R^{LF}) rises, the expected return to lending rises, loosening the lender's participation constraint and permitting a higher level of idiosyncratic risk. For the borrower, an increase in R^{LF} increases the net cost of borrowing at the lending facility. This implies that the borrower is willing to pay a higher risk premium and still borrow on the interbank market, which results in a decrease in $\tilde{\theta}_b$ and increased participation in the interbank market.

Similarly, when the interest paid on reserves (R^R) rises, the opportunity cost of lending on the interbank market rises. Lending banks require a higher expected return, and $\tilde{\theta}_b$ rises as a result. In addition, the cost of borrowing in the interbank market rises, tightening the borrower's lending constraint. To counteract this, the risk premium must fall, which necessitates an increase in $\tilde{\theta}_b$ and reduced participation in the interbank market.

The previous results reflect several stylized facts regarding the interbank lending market before and after the financial crisis of 2008. During the crisis, perceptions of counterparty risk rose.[6] In the model, this can be thought of as a decrease in P, which results in a decrease in participation in the interbank lending market. Afonso, Kovner, and Schoar (2011) document empirical evidence to support this theoretical result. They find that weak banks, but not strong banks, accessed the Fed's discount window after the collapse of Lehman Brothers in September 2008. This is in line with the model's prediction that after a decrease in P, the marginally weakest banks drop out of the interbank lending market in favor of borrowing from the discount window. They also document a slight decrease in overall participation in the fed funds market after the collapse of Lehman, an additional result in line with the model's predictions.

Interestingly, Afonso, Kovner, and Schoar (2011) find a much larger fall in participation after the Federal Reserve began the policy of paying interest on reserves. This decrease in participation, specifically among depository institutions, as well as reduced aggregate trading volume has persisted well after the worst of the financial crisis ended.[7] Several authors (Afonso, Entz, and LaSueur (2013), Craig and Millington (2017), Afonso, Armenter, and Lester (2018) to name a few) attribute the decrease in participation directly to the policy of paying interest on reserves; this hypothesis is supported by the results of the model.

Aggregation Results

The preceding section describes how the cost of and participation in unsecured interbank credit markets are determined at the individual bank level. This section derives and analyzes the average interbank rate for the entire interbank credit market.

The average, or effective, interbank interest rate, R^{IB}_{eff}, is the value-weighted average of all individual interbank rates that occur given the cutoff level for idiosyncratic risk, $\tilde{\theta}_b$. In relation to the federal funds market in the United States, the average interbank interest rate is analogous to the effective federal funds rate and is of key importance to financial markets, monetary policy, and the economy at large. Not only is the effective federal funds rate the primary policy instrument of the Federal Reserve, it serves as a benchmark rate upon which many other interest rates are based and acts as a strong signal of financial market conditions.

Using equations 11, 16, and the assumption that θ is uniformly distributed on the interval $[0,1]$, R^{IB}_{eff} can be expressed as follows:

$$R^{IB}_{eff} = \eta R^{LF} + \frac{(1-\eta)R^{R}}{P(1-\tilde{\theta}_b)}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta_b}d\theta \tag{19}$$

The first term on the right-hand side of equation 19 can be thought of as a "base rate" that is common to all borrowing banks. The second term is the average risk premium paid by all borrowing banks that participate in the interbank market. Equation 19 shows that the effective interbank interest rate can be expressed as the base rate plus the average risk premium.

Of key interest in this paper is how the effective interbank loan rate responds to periods of financial turmoil. In the model, financial distress in the interbank lending market is modeled as an unexpected drop in the aggregate success probability component P. The effect of P on the effective interbank loan rate can be found by differentiating 19 with respect to P, while internalizing the effect of P on the borrower threshold $\tilde{\theta}_b$:

$$\frac{\partial R^{IB}_{eff}}{\partial P} = \frac{(1-\eta)R^{R}}{P(1-\tilde{\theta}_b)}\left[-\frac{1}{P}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta_b}d\theta - \left(\frac{1}{\tilde{\theta}_b} - \frac{1}{(1-\tilde{\theta}_b)}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta_b}d\theta\right)\frac{\partial\tilde{\theta}_b}{\partial P}\right] \tag{20}$$

A fall in P has two effects on the effective interbank interest rate. The first effect of fall in P, which I dub the "incumbent interest rate effect" (IIR), is captured by the first term within the brackets in equation 20. This measures the effect of P on the risk premium paid by borrowing banks who are already participating interbank markets. Per result 1, when P falls, the interbank interest rate rises for all borrowers who access funds. Thus, ceteris paribus, the IIR effect causes R^{IB}_{eff} to rise after a fall in P, and vice versa.

The second effect of P on the effective interbank rate, the borrower threshold effect (henceforth, the BT effect), is the effect on R^{IB}_{eff} when the borrower threshold $\tilde{\theta}_b$ changes in response to a fall in P. It is captured by the term in parentheses (within the brackets) in equation 20 that is multiplied by $\frac{\partial\tilde{\theta}_b}{\partial P}$.

Looking closer at this second effect, the two components that constitute the BT effect move R^{IB}_{eff} in opposite directions. Per result 3, when P falls, the threshold $\tilde{\theta}_b$ rises, implying that $\frac{\partial\tilde{\theta}_b}{\partial P} < 0$. This also implies that the number of borrowing banks falls, and, all else equal, averaging interbank rates over fewer borrowers causes R^{IB}_{eff} to rise after a fall in P. This effect is captured by the second term inside the parentheses in equation 20. However, when $\tilde{\theta}_b$ rises, some banks that were not rationed prior to the fall in P are now rationed out of the market. Since these borrowers were the riskiest borrowers and paid the highest risk premiums, the average interbank rate falls after a fall in P due to these risky borrowers dropping out of the interbank market. This effect is captured by the first term inside the parentheses in equation 20.

Despite these counteracting effects, the net effect of the BT effect is known with certainty and is summarized in the following result.

Result 4. *The net BT effect from a change in P is positive. That is, the BT effect causes* R^{IB}_{eff} *to fall after a fall in the aggregate success component, and vice versa.*

Proof: For the result to be true, the term in parentheses (with the larger brackets) in equation 20 must be positive. This is true if the following condition is satisfied:

$$B(\tilde{\theta}_b) \equiv \frac{1}{\tilde{\theta}_b} + ln(\tilde{\theta}_b) > 1 \tag{21}$$

To show that equation 21 is true for all $\tilde{\theta}_b \in (0,1)$, first observe that

$$\lim_{\tilde{\theta}_b \to 1^-} B(\tilde{\theta}_b) = 1 \tag{22}$$

$$\lim_{\tilde{\theta}_b \to 0^+} B(\tilde{\theta}_b) = \infty \qquad (23)$$

Second, $B(\tilde{\theta}_b)$ is strictly decreasing in $\tilde{\theta}_b$ for $\tilde{\theta}_b \in (0,1)$:

$$\frac{\partial B(\tilde{\theta}_b)}{\partial \tilde{\theta}_b} = \frac{1}{\tilde{\theta}_b}\left(1 - \frac{1}{\tilde{\theta}_b}\right) < 0 \qquad (24)$$

Taken together, equations 22, 23, and 24 show that equation 21 holds for all $\tilde{\theta}_b \in (0,1)$.

Intuitively, the first term in the BT effect, $\frac{1}{\tilde{\theta}_b}$ respresents the risk premium paid by the threshold borrower, and the second term, $\frac{1}{(1-\tilde{\theta}_b)}\int_{\tilde{\theta}_b}^{1} \frac{1}{\theta} d\theta$, represents the average premium paid by all borrowing banks. Since the threshold borrowing bank is the riskiest borrower, they pay a higher premium than the average, which implies that the BT effect must be positive.

Given the two counteracting effects of P on R_{eff}^{IB}, which effect will dominate? The answer is formalized in result 5.

Result 5. *The effective interbank loan rate,* R_{eff}^{IB}*, rises when the aggregate success probability component,* P*, falls.*

Proof: For the result to be true, equation 20 must be negative. Using the definition of $\tilde{\theta}_b$ (equation16) and some simplification, it can be shown that 20 is negative if the following condition holds

$$A(\tilde{\theta}_b) \equiv \tilde{\theta}_b - ln(\tilde{\theta}_b) > 1 \qquad (25)$$

To show that equation 25 holds for all $\tilde{\theta}_b \in (0,1)$, first note that

$$\lim_{\tilde{\theta}_b \to 1^-} A(\tilde{\theta}_b) = 1 \qquad (26)$$

$$\lim_{\tilde{\theta}_b \to 0^+} A(\tilde{\theta}_b) = \infty \qquad (27)$$

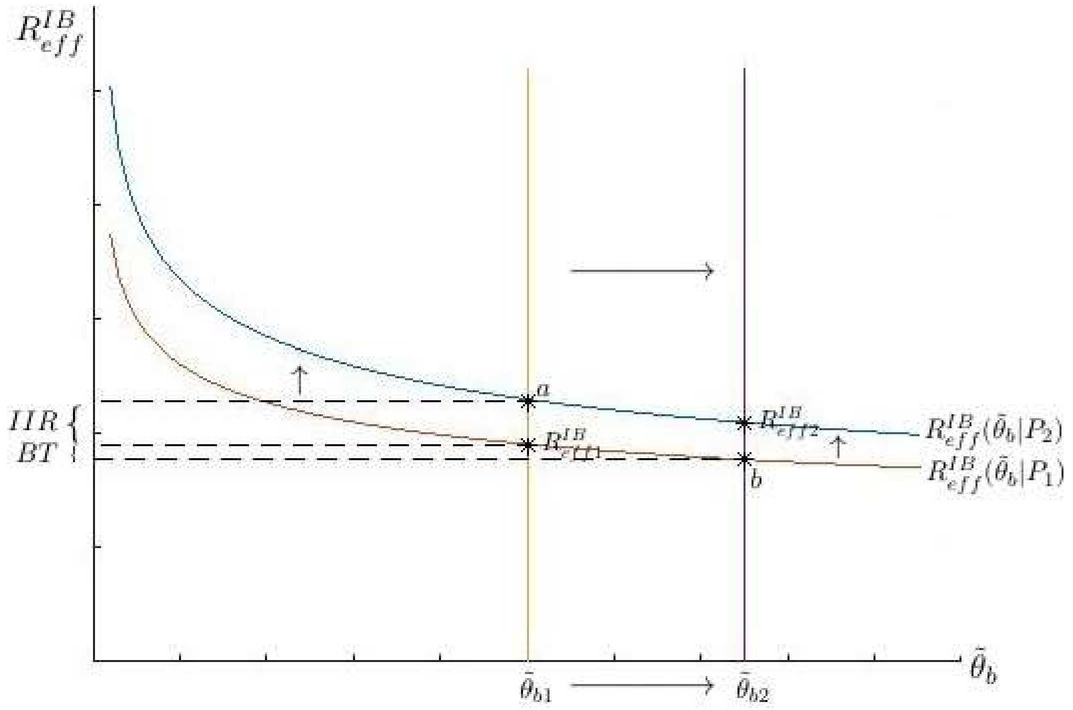
Second, $A(\tilde{\theta}_b)$ is strictly decreasing in $\tilde{\theta}_b$ for $\tilde{\theta}_b \in (0,1)$:

$$\frac{\partial A(\tilde{\theta}_b)}{\partial \tilde{\theta}_b} = 1 - \frac{1}{\tilde{\theta}_b} < 0 \qquad (28)$$

Taken together, equations 26, 27, and 28 show that equation 25 holds for all $\tilde{\theta}_b \in (0,1)$.

Figure 1 illustrates the BT, IIR, and combined effects of a decrease in P. While a decrease in P causes the average interbank rate to rise from R_{eff1}^{IB} to R_{eff2}^{IB}, the BT effect causes the average interbank loan rate to fall after a deterioration in aggregate conditions, the IIR effect causes the average loan rate to rise. The IIR effect is shown as the movement from R_{eff1}^{IB} to point a. When P falls, all borrowers pay higher rates, which pushes up the average rate, given the fixed level of $\tilde{\theta}_b$. In contrast, the BT effect is shown as the movement from R_{eff1}^{IB} to point b. When P falls, the threshold level $\tilde{\theta}_b$ rises. Since this increase in the borrower threshold signifies the exit of the riskiest borrowers that pay the highest rates, the average interbank loan rate falls. Taken together, the IIR effect outweighs the BT effect and the average interbank loan rate rises after a deterioration in aggregate conditions.

FIGURE 1
THE BORROWER THRESHOLD (BT) EFFECT AND INCUMBENT INTEREST RATE (IIR) EFFECT OF A RISE IN AGGREGATE INTERBANK DEFAULT RISK ON THE EFFECTIVE INTERBANK INTEREST RATE



It is also of interest to see how the effective interbank loan rate changes with monetary policy – namely the premium paid on bank reserves, R^R, and the premium charged on funds borrowed from the central bank's lending facility, R^{LF}. This relationship is formalized in the following result.

Result 6. *The effective interbank loan rate is increasing in R^{LF} and increasing in R^R.*

Proof: Differentiating 19 with respect to R^{LF}, while internalizing the dependence of $\tilde{\theta}_b$ on R^{LF}, yields the following condition:

$$\frac{\partial R^{IB}_{eff}}{\partial R^{LF}} = \eta + \frac{(1-\eta)\tilde{\theta}_b}{1-\tilde{\theta}_b}\left(\frac{1}{P\tilde{\theta}_b} - \frac{1}{P(1-\tilde{\theta}_b)}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta}d\theta\right) > 0 \tag{29}$$

which is strictly positive given result 4.

The first term on the right-hand side of equation 29 represents the direct effect of an increase in R^{LF} on R^{IB}_{eff} via its effect on individual loan rates. The second term represents the indirect effect on R^{IB}_{eff} through changes in the threshold level of $\tilde{\theta}_b$. This is equal to the increase in the average risk premium brought about by a rise in R^{LF}. Since an increase in the premium on funds borrowed from the central bank's lending facility causes $\tilde{\theta}_b$ to fall, and thus higher-risk borrowing banks to participate in the interbank loan market, the average risk premium rises and contributes to the increase in R^{IB}_{eff}.

Differentiating 19 with respect to R^R, while internalizing the dependence of $\tilde{\theta}_b$ on R^R, yields the following condition:

$$\frac{\partial R_{eff}^{IB}}{\partial R^{R}} = \frac{(1-\eta)}{1-\tilde{\theta}_b}\left(\frac{1}{P}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta}d\theta - \tilde{\theta}_b\left[\frac{1}{P\tilde{\theta}_b} - \frac{1}{P(1-\tilde{\theta}_b)}\int_{\tilde{\theta}_b}^{1}\frac{1}{\theta}d\theta\right]\right) > 0 \tag{30}$$

Unlike R^{LF}, an increase in R^R has two counteracting effects of R_{eff}^{IB}. The first term in the parentheses on the right-hand side of equation 30 represents the direct effect of an increase in R^R on the effective interbank loan rate via its effect on individual loan rates. The second term (in square brackets) represents the change in R_{eff}^{IB} caused by a change in $\tilde{\theta}_b$ and the resulting average risk premium. Since an increase in R^R causes the threshold $\tilde{\theta}_b$ to rise and the marginally riskiest borrowing banks to drop out of the interbank market, an increase in R^R causes the average risk premium to fall, which exerts downward pressure on R_{eff}^{IB}. However, equation 30 holds if the following condition is met:

$$\tilde{\theta}_b - ln(\tilde{\theta}_b) > 1 \tag{31}$$

which was shown to be true in result 5.

At different points during the financial crisis, the premium paid on the Federal Reserve's discount rate was lowered and the Federal Reserve began to pay interest on reserves. In the model, the policy of paying interest on reserves would be shown as an increase in R^R. This would result in less participation in interbank loan markets and an increase in the effective interbank loan rate. While the policy of paying interest on reserves was designed to provide a floor on the interbank rate in the face of large liquidity injections by the Fed's other liquidity facilities, the model suggests that it raises the average cost of borrowing in the interbank loan market.

The Federal Reserve also lowered the discount rate during the financial crisis. In the model, this would be shown as a decrease in R^{LF}, which would decrease participation in the interbank loan market and decrease the effective interbank rate. While there is evidence that low-quality borrowers were more likely to access the discount window,[8] this does not fit with the rising spreads witnessed.

However, it is possible that the premium on borrowing from the discount window includes costs other than the discount rate. Namely, borrowing from the discount window may carry an extra cost associated with the "stigma" of seeking an emergency loan from the Fed.[9] If the perceived cost of this stigma rose during the crisis, when sending a signal of poor financial health may be particularly harmful, it is possible that the perceived R^{LF} rose during the crisis despite the Federal Reserve's policy actions. Additionally, it could be that the upward pressure on the interbank loan rate caused by deteriorating aggregate conditions during the crisis outweighed the downward pressure introduced by the cut in the discount rate.

CONCLUSION

This paper presents a theoretical framework to analyze the effects of counterparty risk and monetary policy on the cost of and participation in unsecured interbank credit markets. A bank–specific interbank loan rate that results from a Nash bargaining problem is derived. This individual loan rate is increasing in the borrower's probability of default and rises after a decrease in the aggregate component of success probabilities.

Given the individual interbank loan rate, an endogenous threshold for the bank-specific success component emerges, which determines participation in the interbank market. After a deterioration in aggregate conditions, this threshold tightens, fewer banks access interbank credit, and both individual and the average interbank loan rates rise. In this respect, my model is capable of capturing the key effects the financial crisis had on interbank markets both from a market-wide perspective and that of individual borrowing banks.

Central bank policies also affect access and loan rates: lowering the cost of borrowing from the central bank's lending facility and increasing the interest paid on bank reserves reduces participation. However, these policy actions have opposite effects on the average interbank loan rate. Lowering the lending facility rate lowers the average interbank rate while raising the return on reserves increases the average interbank rate.

While I am encouraged by the results my framework is capable of producing, there is still much room for further exploration and improvement moving forward. I would like to extend the model to include a role for conventional monetary policy. This would require introducing a mechanism that permits the quantity of reserves to affect loan rates, which potentially could be achieved by introducing search and matching frictions akin to Bech and Monnet (2016) and Afonso, Armenter, and Lester (2018) while, of course, preserving the role of counterparty risk currently present in this model.

An additional extension is to include more heterogeneity amongst the interbank market participants. Specifically, I would like to allow for different types of institutions: depository institutions (e.g., banks) that are eligible to be paid interest on reserves and non-depository institutions (e.g., government-sponsored enterprises) that can trade in the interbank market but are not eligible to be paid interest on reserves. Since a lender’s outside option is of first-order importance in determining interbank loan rates and participation, such an extension may allow me to better capture key characteristics of the interbank lending market.

ACKNOWLEDGEMENT

The author would like to thank Carl Walsh, Ken Kletzer, Johanna Francis, and participants at the University of California, Santa Cruz Macro and Monetary Economic Workshop for their helpful comments and suggestions. The author declares that she has no relevant or material financial interests that relate to the research described in this paper.

ENDNOTES

1. See FRB Press Release https://www.federalreserve.gov/newsevents/press/monetary/20070817a.htm
2. See FRB Press Release https://www.federalreserve.gov/monetarypolicy/20081006a.htm
3. Bech and Monnet (2016) also derive a distribution of loan rates in an interbank market with bilateral bargaining over loan terms. However, in their model, the underlying heterogeneity causing rate dispersion stems from differences in reserves, not differences in default risk.
4. I assume that the values of d_1 and d_2 are chosen by regulators to ensure that banks with successful long-term assets are solvent.
5. For simplicity, I assume that loans from the central bank’s lending facility require no collateral.
6. See Williams and Taylor (2009), Afonso, Kovner, and Schoar (2011), Angelini, Nobili, and Picillo (2011), and Gorton and Metrick (2012).
7. See Afonso, Entz, and LaSueur (2013)
8. See Afonso, Kovner, and Schoar (2011).
9. See Ennis and Weinberg (2009).

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