

The Emergence of Renminbi as an International Invoicing Currency and The Spillover

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China has aggressively promoted Renminbi(RMB), or Chinese Yuan, to be the next international currency since 2005. Has China's effort been paid off as RMB gradually emerged as an invoicing currency? What are the spillover effects of the RMB internationalization on the current vehicle currency country, US, and the rest of the world? It's hard to estimate the externality of RMB internationalization because of the endogeneity of the regime change. We adopt a dynamic trading post model developed by (Rey, 2001) and (Devereux & Shi, 2013), that exchange rates, prices, consumption, and trade flows are endogenously determined. Hence we can solve the endogeneity and estimate the quantity of externalities by simulating the international goods and foreign exchange market. We find positive(negative) externality on import and export prices when RMB becomes an invoicing(reserve) currency in international trades. The externality on the real side of the economy, such as, export, import and real exchange rate, is significant only if RMB rises as an international reserve currency. The spillover effect is also sensitive to the relative size of China, which means the timing of RMB internationalization matters.

INTRODUCTION

China, the second-largest economy and exporter in the world, has aggressively been pursuing the internationalization of its currency, yuan or renminbi (RMB) in recent decades. If Renminbi arises as an international currency to compete with US dollar, do we expect a spillover effect on the US and its trading partners? And by how much?

The goal of this paper is to calibrate a dynamic general equilibrium model of the international currency and estimate the externalities of the emergence of RMB as an international currency. China started to allow cross border trading in RMB since 2004. China's central bank, People's Bank of China (PBOC), appointed Bank of China (Hong Kong Branch) as the RMB Clearing Bank in Hong Kong in 2003, which marks the opening of the Hong Kong offshore RMB market. PBOC introduced the Cross-Border Trade RMB Settlement Pilot Project in 2008 to promote RMB as an international currency. In the second half of 2010, the pilot project had extended to 20 provinces and to trading partners from all countries. Since 2011, PBOC has further pushed the use of RMB in international trades and foreign direct investment ("At the Centre of RMB Internationalization- A brief guide to offshore RMB". <https://www.db.com/en/media/At-the-centre-of-Renminbi-internationalisation--A-brief-guide-to-offshore-RMB.pdf>). Since 2012, all companies in mainland China with import and export qualifications are permitted to make and receive payments in RMB for goods, services and other current account transactions with their counterparts all over the world. shows that the use of RMB in China's cross-border Table 1 trade settlement has significantly increased. In December 2015, the RMB denominated settlement

accounted for 22.6% of China's total import and export, according to PBOC. Meanwhile, the RMB-denominated deposits and financial instruments has been growing steadily. By the end of October 2015, the RMB became the fifth most used payment currency according to the Society for Worldwide Interbank Financial Telecommunication (SWIFT).

TABLE 1
RMB SETTLEMENT UNDER CURRENT ACCOUNT

Year	Goods	Service and Others	Total
2009	1.95	0.61	2.56
2010	303.4	46.7	350.1
2011	1381.07	207.86	1588.93
2012	2603.98	275.75	2879.73
2013	4136.84	499.94	4636.78
2014	5894.65	656.37	6551.02
2015	6391.14	843.22	7234.36
2015	6391.14	843.22	7234.36

Data: PBOC

Unit: Billion Yuan

The BIS Triennial Central Bank Survey on Foreign exchange turnover in April 2016 ranks Renminbi to the 20th place in 2007, 17th in 2010, 9th in 2013 and 8th in 2016. Comparing with other major currencies, such as Canadian dollar, Swiss Franc and Swedish Krona, we believe the rise of Chinese Yuan will lead to more profound economic changes to other economies. Among the top 10 currencies on the BIS Foreign exchange turnover list, China is the second largest exporter and third largest importer. The WDI import and export index shows that the total trade volume of China is in the third place, next to European Union and the US, by 2015. The size of the economy has always been an important parameter in the international currency literature. (Matsuyama, Kiyotaki, & Matsui, 1993) and (Randall, 2001) propose a search based model to explain the pricing mechanism of the foreign currency. The size of a foreign economy is positively related to the expected turnover of the foreign currency, hence the potential of being an international currency. In the New Keynesian study of international currency, such as (Goldberg & Tille, 2008) and (Devereux, Engel, & Storgaard, 2004), the relative size of the exporter country matters that smaller countries are less likely to issue dominant invoicing currency. In a different strand of international currency literature, foreign exchange trades take place through decentralized meetings at trading posts, where agents can buy or sell one currency for another. The vehicle currency emerges if and only if the relative size of the economy is large enough. Hence, the current and future size of the Chinese economy predicts the importance of the Chinese currency in future.

Endogeneity is the major challenge when estimating the spillover effect on RMB emergence. Let P be prices, such as, import and export prices, exchange rates. Y be Macroeconomic fundamentals, such as GDP, PPP and inflation. If the US dollar is the sole dominant currency, the exchange rate regime is indicated by a binary variable, $Z=0$. If Chinese Yuan emerges as a competing international currency, the exchange rate regime is $Z=1$. The spillover effect of RMB internationalization is $E[P_i|Y_i, Z = 1] - E[P_i|Y_i, Z = 0]$ where the subscript i refers to countries other than China. However, both P_i and Y_i are correlated with the regime dummy Z . We can only observe $E[P_{1i}|Y_{1i}, Z = 1] - E[P_{0i}|Y_{0i}, Z = 0]$, instead of $E[P_{1i} - P_{0i}|Y_{1i}, Z = 1]$, or $E[P_{1i} - P_{0i}|Y_{0i}, Z = 0]$. If we can identify $P_{1i} = F(Y_i, Z)$ and $P_{0i} = F(Y_i, Z)$ in a general equilibrium, the endogeneity problem would be solved. Hence we adopt the dynamic general equilibrium model using trading post framework, proposed in (Devereux & Shi, 2013), to study the transition of China from the use of USD to the use of RMB in its cross-border trade.

The trading post framework can be traced back to (Krugman, 1980). It develops a static partial equilibrium model in a three-country world with transaction costs on the currency markets. (Krugman, 1980) proposes systematic study of international financial market with transaction costs. It considers two

types of exchange structures. If two currencies are traded directly, it's defined as the direct exchange structure. If one currency is traded for the third currency only to acquire the target currency of interest in future, it's called the indirect exchange structure. And the third currency is called the vehicle currency. If transaction costs are a decreasing function of the volume of transactions, the structure of exchange can be related to the structure of payments in economically sensible ways. Hence the currency of a country that is important in world payments can serve as an international medium of exchange. (Rey, 2001) extends the static model in (Krugman, 1980) to a dynamic general equilibrium model and imposes cash-in-advance constraints on goods markets. Trade in goods market determines the money demand, and in turn the currency exchange affects the economic allocation and therefore the relative price. Pushing further, (Devereux & Shi, 2013) extends the dynamic general equilibrium in (Rey, 2001) to $N \geq 3$ countries. It compares two types of equilibria: The Symmetric Trading Equilibrium (STE) and Vehicle Currency Equilibrium (VCE). In the STE, agents in each country can use domestic currency to buy foreign currencies directly and there are $N(N-1)/2$ active bilateral foreign exchange trading posts. In the VCE, there is one central currency and $N-1$ peripheral currencies. There are only $N-1$ active trading posts, which means the trade of any two peripheral currencies has to follow the indirect exchange structure. (Devereux & Shi, 2013) also studies four types of deviations from the VCE: bilateral deviations, an introduction of a single currency union, a switch to an alternative vehicle currency undertaken by all peripheral countries, and coexistence of vehicle currencies.

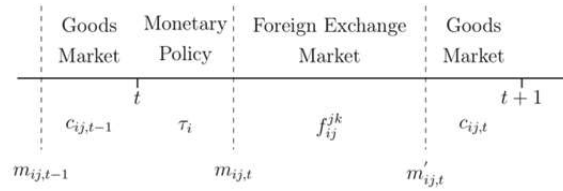
In our paper, we use the one vehicle currency equilibrium to describe the foreign exchange market before the emergence of RMB, where US dollar is the central currency. Then we focus on the bilateral deviations of RMB and other peripheral countries to evaluate the emergence of RMB. Each deviating pair would add one extra active trading post to the equilibrium. As the use of RMB having increased in the settlement of cross border trades, more countries join the bilateral deviation with China. Last, we run an experiment on two vehicle currency equilibrium, where US dollar is served as the global vehicle currency and Chinese Yuan is served as local vehicle currency. The calibration and numerical experiment shows that the bilateral deviation is more likely to cause appreciation of RMB and depreciation of US dollar, while the deviation to the two vehicle currency equilibrium would depreciate RMB and lead to a larger scale of US dollar appreciation. Besides, the emergence of RMB as invoicing currency would raise both import and export price of US if the output ratio between China and US is large enough. If RMB were a local vehicle currency shoulder to shoulder with the US dollar, US would expect a decline in current account/trade deficit, or even a current account/trade surplus. Meanwhile, China is more likely to benefit from the strong Yuan acting as a secondary international currency. In summary, the externality of RMB internationalization is significant and sensitive to the relative size of China's economy. (Goldberg & Tille, 2008) (Head & Shi, 2002) (Krugman, 1980) (Matsuyama, Kiyotaki, & Matsui, 1993) (Prasad, 2016) (Randall, 2001) (Rey, 2001) (Trejos, 1996)

The rest of the paper is organized as follows. The second section briefly introduces the model and three equilibrium regimes. The third section explains the calibration strategy and presents the estimation results and numerical experiments. The last section concludes.

MODEL

Environment

Similar to (Devereux & Shi, 2013), we consider an open economy with $N \geq 3$ countries. Let $n_i, \forall i = 1, 2, \dots, N$, be the endowment of country i . Normalize the total endowment to one that, $\sum_{i=1}^N n_i = 1$. Country i produces a continuum n_i types of goods $z, z \in Z_i$. Time is discrete, indicated by $t = 0, 1, 2, \dots$. Every period, money transfer, foreign exchange trade, and consumption happen in order as follows.



In the first sub-period, domestic households in country i receive money transfer, τ_i , from the monetary authority of country i . Then households visit currency trading posts to adjust their money holdings of domestic currency and foreign currencies. Assume that they can visit multiple trading posts simultaneously, but can only visit each trading post once. Households visit goods market in the last sub period. They can visit both domestic and foreign goods market. The present value of discounted utility of country i household is,

$$U^i = \sum_{t=1}^{\infty} \beta^t \left[\sum_{j=1}^{j=N} n_j u(c_{ijt}) \right], \quad (1)$$

where β is the discount factor, and $c_{ijt}(z)$ is country i household's consumption of country j 's type z product. Symmetry implies that the utility of country i household consuming country j product is $n_j u(c_{ijt})$.

From here, we will suppress the time subscript t , using $+1$ for time $t+1$ and -1 for time $t-1$. Assume country i household holds m_{ij} units of currency j at the beginning of the current period, and m'_{ij} after they visit trading posts. Assume the money growth rate of currency j is γ_j which is implemented by a lump-sum money transfer τ_i of currency j to domestic households. We normalize the stock of currency $j=1, 2, \dots, N$ to one every period. Then $\tau_j = (1 - \frac{1}{\gamma_j})/n_j$.

CIA constraint. Cash-In-Advance(CIA) applies to every goods market, that Country i household needs to bring enough currency j to the goods market of country j for c_{ij} ,

$$m'_{ij} \geq p_j c_{ij} n_j, \forall j \quad (2)$$

where p_j is the price index of country j . The inter-temporal budget constraints of country i households are

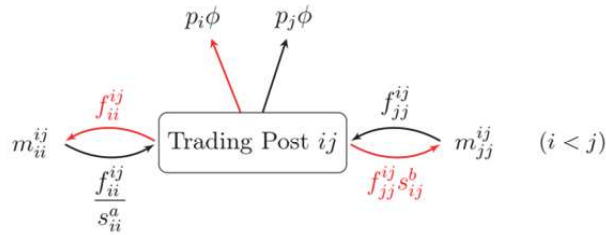
$$m_{ii} = \frac{1}{\gamma_i} [m'_{ii(-1)} - p_{i(-1)} c_{ii(-1)} n_i + w_{i(-1)} + \pi_{i(-1)}] + \tau_i \quad (3)$$

$$m_{ij} = \frac{1}{\gamma_j} [m'_{ij(-1)} - p_{j(-1)} c_{ij(-1)} n_j], \forall j \neq i \quad (4)$$

where the subscript -1 refers to one period before the current period.

Trading Post

Managers of trading post jk ($j < k$) sells currency k in exchange for j at the "ask" price s_{jk}^a , and buys currency k with the "bid" price, s_{jk}^b units of currency j . Following (Devereux & Shi, 2013), we impose fix costs to maintain the trading post jk , that ϕ_t units of goods k and ϕ_t units of goods j has to be paid every period. The transaction cost coefficient ϕ_t is determined by the institutional foundation of the global financial market, and hence calibrated by year. Besides, assume country i household brings f_{ij}^{jk} units of currency j to the trading post jk , in exchange for f_{ij}^{jk}/s_{jk}^a of currency k if $j < k$; $f_{ij}^{jk} s_{jk}^b$ if $j > k$. Then the exchange rates s_{jk}^a and s_{jk}^b are determined by market clear conditions of currency j and k . For example, the flows of currency i and j of trading post ij are showed by the red and black arrow in the following chart.



There are $N(N-1)/2$ possible trading posts; but not all of them are necessarily active. A trading post is less likely to be active if the trade volume of the bilateral foreign currency transaction is too low to cover the fix cost. A trading post may be desolated if traders have other options. For example, all peripheral countries adopt a single vehicle currency for goods trade, then only $N-1$ trading posts between vehicle and peripheral currencies are needed. Moreover, capital market immobility can shut down a trading post, such as China's foreign exchange control last century.

The trading post model has multiple equilibria. In this paper, we consider three types of equilibria: one vehicle currency equilibrium, an interim equilibrium with one vehicle currency and one deviating currency, and two vehicle currency equilibrium. One can find out how to solve the equilibrium in (Devereux & Shi, 2013). To be concise, we will briefly describe the equilibrium to introduce the calibration and numerical experiment.

One Vehicle Currency Equilibrium (OVCE)

If there exist one vehicle currency of the N country economy, all $N-1$ peripheral countries engage in indirect currency trade in order to purchase the foreign currency. Only $N-1$ trading posts are active in this equilibrium. Let currency 1 be the vehicle currency. Peripheral country $i=2, 3, \dots, N$ buys vehicle currency 1 from the trading post $1i$ and then carry the vehicle currency to the next period to buy foreign currency j from the trading post $1j$. For peripheral country i , the cost of obtaining one unit of currency j through currency 1 is $\gamma_1 s_{1j(+1)}^a / s_{1i}^b$, which should be smaller than the cost of direct currency trade s_{ij}^a (if $i < j$), or $1/s_{ij}^b$ (if $i > j$).

The money balances right after the currency trades are determined by equation (5), (6), and (7),

$$m'_{ii} = m_{ii} - f_{ii}^{1i}, \quad (5)$$

$$m'_{i1} = m_{i1} - \sum_{j \neq \{1, i\}} f_{i1}^{1j} + s_{1i}^b f_{ii}^{1i}, \quad (6)$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{1j}^a} f_{i1}^{1j}, j \notin \{i, 1\}, \quad (7)$$

$$m_{i1} \geq \sum_{j \neq i} f_{i1}^{1j}. \quad (8)$$

Equation (8) is the vehicle currency constraint that the total amount of vehicle currency brought to trading posts $1j$ ($j \neq 1, i$) should not exceed country i 's balance of vehicle currency at the beginning of the period.

Let m_i be the money supply of country i , then the money market clear implies that

$$m_i \triangleq \sum_{j=1}^{j=N} n_j m_{ji} = n_i Y_i p_i,$$

Equilibrium

In the equilibrium, households choose $c_{ij}, m'_{ij}, f_{ij}, \forall j = 1, \dots, N$, to maximize the present value of the expected discounted utility $E[U^i]$ subject to budget constraints (3), (4), CIA constraints, (2), and money balance constraints (5), (6), (7), and (8)

Assume $\gamma_j > 1, \forall j$, that it's always costly to carry any currency over time. In the equilibrium, every country only carries necessary amount of currency for goods trade. Then the vehicle currency constraint

(8) is always binding, $m_{i1} = \sum_{j \neq i} f_{i1}^{1j} > 0$. And CIA constraints of peripheral countries consuming peripheral goods ($j > 1$), (2), are always binding, that $m'_{ijt} = p_{jt} c_{ijt} n_j, \forall i > 1, j > 1$. No country carries foreign currency issued by peripheral countries over period, that $m_{ij} = 0 \forall j \notin \{1, i\}$. As a result, $m_{iit} = \frac{1}{n_i}$, for all peripheral countries $i > 1$. Substitute $m_{ij} = 0$ to (6) and the CIA constraint, we can get $f_{i1}^{1j} = s_{1j}^a p_j c_{ij} n_j$.

The bid and ask price of currency i in terms of the vehicle currency are determined by the market clear conditions of trading post $1i$.

$$s_{1i}^a [n_i f_{ii}^{1i} - \phi_i p_i] = n_1 f_{11}^{1i} + \sum_{j \notin \{i, 1\}} n_j f_{j1}^{1i},$$

$$s_{1i}^b n_i f_{ii}^{1i} = n_1 f_{11}^{1i} + \sum_{j \notin \{i, 1\}} n_j f_{j1}^{1i} - \phi_1 p_1.$$

Interim Equilibrium

Now we consider country 2 and a group of peripheral countries, I_d deviate from the above OVCE. In the interim equilibrium, country 2 visits trading post $2j$ to purchase foreign currency j directly if $j \in I_d$. The peripheral deviating country buy currency 2 directly from trading post $2j$, but acquires peripheral foreign currency $j > 2$ indirectly by using vehicle currency 1. In the interim equilibrium, there are $2N-3$ active trading posts, $N-1$ trading posts $1i$ ($i > 1$) and $N-2$ trading post $2j$ ($j > 2$). To ensure the existence of this specific interim equilibrium, the following inequalities should be satisfied,

$$\frac{\gamma_1 s_{12}^a}{s_{1i}^b} \geq \frac{1}{s_{2i}^b} \text{ (or } s_{2i}^a), \forall i > 2,$$

$$\frac{\gamma_1 s_{1j}^a}{s_{1i}^b} \leq \frac{\gamma_2 s_{2j}^a}{s_{2i}^b} \left(\text{or } \frac{1}{s_{ji}^b}, \text{ or } s_{ij}^a \right), \forall i > 2, \forall j > 2,$$

$$\gamma_1 s_{1j}^a / s_{1i}^b \geq s_{1j}^a, \forall j > 1.$$

Peripheral country prefers currency 1 than 2 as the vehicle currency in the trade of currency $j \neq 2$; while prefers to trade for currency 2 directly than indirectly. Country 2 prefers to trade all foreign currencies directly than indirectly. For peripheral country $i > 2$, the new money balances right after visiting trading posts are

$$m'_{ii} = m_{ii} - f_{ii}^{1i} - f_{ii}^{2i}, \tag{9}$$

$$m'_{i1} = m_{i1} - \sum_{j \neq \{1, 2, i\}} f_{i1}^{1j} + s_{1i}^b f_{ii}^{1i}, \tag{10}$$

$$m'_{i2} = m_{i2} + s_{2i}^b f_{ii}^{2i}, \tag{11}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{1j}^a} f_{i1}^{1j}, j \notin \{1, 2, i\}. \tag{12}$$

The total amount of vehicle currency carried to trading posts $1j$ by country i should subject the following vehicle CIA constraint,

$$m_{i1} \geq \sum_{j \neq i} f_{i1}^{1j}. \tag{13}$$

For the deviating country 2, the adjusted money balances are determined as follows,

$$m'_{22} = m_{22} - \sum_{j > 2} f_{22}^{2j} - f_{22}^{12},$$

$$m'_{2i} = m_{2i} + \frac{1}{s_{2i}^a} f_{22}^{2i}, \forall i > 2,$$

$$m'_{21} = m_{21} + s_{12}^b f_{22}^{12}.$$

Similar to the OVCE equilibrium, CIA constraints of peripheral countries consuming peripheral goods $j > 1$ are binding. And vehicle CIA constraints always binding. The bid and ask exchange rates of currency i are decided by the foreign exchange market clear conditions. The market clear conditions and solutions are in the Appendix.

Two Vehicle Currency Equilibrium

Consider currency 1 as the global vehicle currency, and currency 2 as the local vehicle currency. We divide N countries to two groups: I_1 and I_2 . Countries in I_1 trade all foreign currencies indirectly using global vehicle currency 1. Countries in I_2 trade foreign currencies issued by I_2 country with local vehicle currency 2, but trade other foreign currencies using global vehicle currency 1. For two vehicle currencies coexisting, the cost of indirect currency purchases through currency 1 and 2 should be equivalent,

$$\gamma_1 s_{1j}^a / s_{1i}^b = \gamma_2 s_{2j}^a / s_{2i}^b$$

The money balance, CIA constraints and vehicle CIA constraints equations are set in a similar manner with previous sections. The constraints, optimality conditions and solutions are in the Appendix.

SIMULATION OF THE EXTERNALITY IN NUMERICAL EXPERIMENTS

Calibration

We include the US, China and 29 peripheral countries who have signed Bilateral Swap Agreement (BSA) with China.

TABLE 2
LIST OF RMB BILATERAL SWAP AGREEMENTS

Country	Time	Currency	Country	Time	Currency
South Korea	2008	KRW	Hungary	2013	HUF
Argentina	2009	ARS	Canada	2014	CAD
Hong Kong	2009	HKD	Nepal	2014	NPR
Indonesia	2009	IDR	Qatar	2014	QAR
Malaysia	2009	MYR	Switzerland	2014	CHF
Singapore	2010	SGD	Chile	2015	CLP
New Zealand	2011	NZD	South Africa	2015	ZAR
Pakistan	2011	PKR	Denmark	2016	DKK
Russia	2011	RUB	Egypt	2016	EGP
Thailand	2011	THB	Mexico	2016	MXN
Australia	2012	AUD	Norway	2016	NOK
Turkey	2012	TRY	Poland	2016	PLN
Ukraine	2012	UAH	Saudi Arabia	2016	SAR
UAE	2012	AED	Sweden	2016	SEK
Brazil	2013	BRL			

Data source: PBOC/Monetary Policy Department. Time column shows the year when a RMB BSA is first signed.

US dollar is the single vehicle currency in our calibration. Hence, we exclude other major international currencies, such as, Euro, Pound Sterling and Japanese Yen. Table 2 shows the list of the 29 chosen peripheral countries. The 'Time' column shows the year when the peripheral country first signed RMB BSA with China. In the calibration, we use this information to identify the timing of bilateral deviation. China's central bank, PBOC, started to allow foreign currencies exchange since late 2004. From 2005 to 2008, there is no record on the cross border RMB settlement. We consider the economy is in one vehicle currency equilibrium, OVCE. During this period, US dollar was the only dominating invoicing and reserve currency. Since 2009, the emergence of BSA has increased the cross border RMB settlement. According to the 2016 RMB Internationalization Report published by the PBOC, the RMB

settlement under the Current Account was 2.56 billion yuan in 2009, 350.1 billion yuan 2010, 1588.93 billion yuan in 2011. From 2011 to 2015, RMB gradually emerges as an important invoicing currency that the cross board RMB settlement has reached 7234.36 billion yuan by the end of 2015, which counts more than 15\% of the cross border trade. RMB ranked second next to the US dollar. Hence we consider the economy has entered the interim equilibrium regime since 2009. The deviating peripheral countries are identified by the RMB BSA.

We adopt $\beta = 0.99$ from the literature. We extract macroeconomic indicators from the World Development Indicators (WDI)(<http://documents.worldbank.org/curated/en/2011/01/16251192/total-factor-productivity-across-developing-world>). The endowment of country i , $n_i, \forall i = 1, \dots, N$, is estimated by its real GDP as a share of the total GDP (WDI/NY.GDP.MKTP.KD). The money growth rate $\gamma_i, \forall i = 1, \dots, N$, is estimated by the PPI inflation rate. We consider three candidates: 1) Broad Money Growth Rate, WDI/FM.LBL.BMNY.ZG; 2) Inflation Rate, WDI/NY.GDP.DEFL.KD.ZG; 3) PPI inflation rate, Federal Reserve Economic Data. The graph presented in this paper uses the PPI inflation rate. According to (Chinn, 2005), PPI is a better measure of the tradable goods price. Then we calibrate the currency transaction cost parameter ϕ_t to target the weighted average of FX bid-ask price spreads, where the daily bid and ask exchange rates are extracted from Bloomberg.

**TABLE 3
CALIBRATED PARAMETERS**

Calibrated Parameters	Model Moments	Data Moments
n_i	n_i	Share of RGDP
m_i	p_i/p_1	Purchasing Power Parity (PPP)
γ_i	$(\gamma_i - 1) \times 100$	Inflation (Based on GDP Deflator)
ϕ_t	$\frac{s^a - s^b}{s^b}$	Bid-ask spread of the exchange rate

Table 3 shows the model and data moments chosen for the calibration. Other specifications of the calibration are summarized in Table 4.

**TABLE 4
CALIBRATION SPECIFICATIONS**

Endogenous Variables	
Prices	p_i
Exchange Rates	s_{ij}^b, s_{ij}^a
Consumptions	c_{ij}
Exogenous Variables	
Number of countries	N=31
Deviating Countries	D
Endowment	n_i

The Emergence of Renminbi and Model Fit

We first calibrate period 2005-2015 to the one vehicle currency equilibrium (OVCE) regime. Then we assume that the rising RMB change the FX equilibrium from OVCE to the Interim Equilibrium in 2009, calibrating 2009 - 2015 to the interim equilibrium regime. In each year of the second period,

deviating peripheral countries are those with concurrent RMB BSA according to Table 2. For example, Singapore joined the RMB BSA in 2010. We include Singapore to the deviating group in 2011-2015. We calibrate the macroeconomic moments, discount factor, endowments and money growth rates, to the same data moments in both scenarios.

Table 5 shows the R-square of the interim equilibrium and the OVCE. Both Interim Equilibrium and OVCE can explain more than 70% of the export price volatility of the US and China. Interim Equilibrium Regime performs better than OVCE regime on the import price of China and REER (real effective exchange rate) of the US; while OVCE regime fits better in terms of export prices and China's REER. We extract the 2005-2015 US and China REER from the Federal Reserve Economic. Overall, the interim equilibrium captures the price volatilities well, except the REER of US. This is because the BIS REER is based on a currency basket with all major currencies, including Euro, GBP and JPY; whereas the model REER puts a heavy weight on USD-CNY exchange rate instead.

TABLE 5
R SQUARE OF INTERIM EQUILIBRIUM

	R^2	REER	Export Price	Import Price
Interim	US	5.1%	75.1%	62.8%
	China	19.5%	72.1%	77.1%
OVCE	US	4.5%	97.6%	81.8%
	China	29.1%	88.5%	33.1%

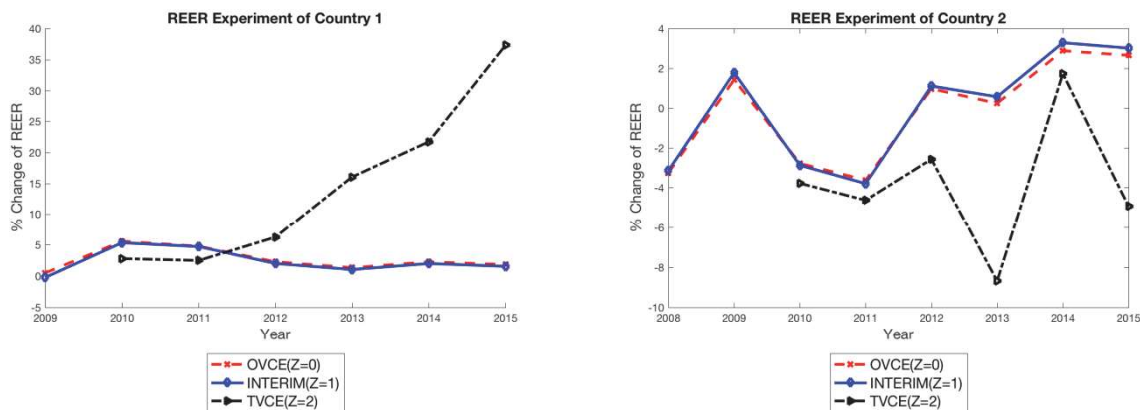
Externality of The Rising RMB

Let Z be the regime indicator. We run OVCE ($Z=0$) and TVCE ($Z=2$) experiments using parameters calibrated to the interim equilibrium regime ($Z=1$) as explained above. The changes of estimated prices, exchange rates and consumptions are thereby results of the regime change only.

REER

Figure 1 shows the changes of REER in the TVCE experiment. If the RMB were local vehicle currency instead of deviating peripheral currency during 2009-2015, the US dollar would appreciate up to 40% and the Chinese Yuan would become more volatile and depreciate.

FIGURE 1
REAL EFFECTIVE EXCHANGE RATE



We then estimate three types of spillover effects $\Delta_1 = E \left[\frac{P_{1i} - P_{0i}}{P_{0i}} \middle| Y_{1i} \right]$, $\Delta_2 = E \left[\frac{P_{2i} - P_{0i}}{P_{0i}} \middle| Y_{1i} \right]$ and $\Delta_3 = E \left[\frac{P_{2i} - P_{1i}}{P_{1i}} \middle| Y_{1i} \right]$. When RMB emerges as the invoicing currency, the spillover effect from $Z=0$ to $Z=1$ is estimated by Δ_1 . If RMB rises to the regional vehicle currency, the spillover effect from $Z=0$ to $Z=2$ is Δ_2 and from $Z=1$ to $Z=2$ is Δ_3 .

Import and Export

Table 6 shows the above spillover effects in terms of import and export. Column "US (ROW)" shows the externalities of US export to the ROW and import from the ROW. Column "US (China)" refers to US export to China and import from China. Column "China" refers to Chinese export to the ROW and import from the ROW. Column "Deviating Countries" and "Non-Deviating Countries" is the group average of the export to the ROW and import from the ROW. China and deviating peripheral countries did not benefit from the bilateral deviation according to the negative Δ_1 on export, but would be able to improve export and total trade according to Δ_3 . The deviating countries share similar externalities on import and export with the US. The bilateral deviation from OVCE to the interim does not cost US economy much, -0.5% on export and 3.1% on import. If RMB became the regional vehicle currency, US would enjoy an increased trade balance with 35.7% increase in export and about 12.3% decrease in import, where 90% of the export growth attributes to China and 70% of the import decline also attributes to China.

TABLE 6
EXTERNALITIES ON EXPORT AND IMPORT

Externality (%)		US (ROW)	US (China)	China	Deviating Countries	Non-Deviating Countries
Export	Δ_1	-0.5	0.5	0	-4.1	-10.8
	Δ_2	35	32.6	13.3	30.7	-31.8
	Δ_3	35.7	31.9	13.3	36.4	-23.6
Import	Δ_1	3.1	1.8	-23.1	-1.3	-3
	Δ_2	-9.8	-7.2	7.9	-12.3	60.3
	Δ_3	-12.3	-8.7	42	-11.2	65.6

Non-deviating countries suffer 10.8% loss on export and 3% loss on import when the regime changed from OVCE to the interim equilibrium. If the currency regime further changed to TVCE, they would increase import by 65.6% and decrease export by 23.6%. Non-deviating countries are more likely to shift from exporting countries to importing countries if RMB rose as international currency.

Table 7 shows that the import and export price of both US and China would experience mild inflation moving to bilateral deviation. If deviation to TVCE happened, the export price of China would jump up while the import price would decline; US would expect a much lower price level where export price decreases more than import price. The substitution effect implies that China would import more and export less, and US would export more and import less. When RMB became international currency in TVCE, although the US dollar appreciates significantly, the changed purchasing power parity will reverse the trade balance between China and US.

TABLE 7
EXTERNALITIES ON IMPORT AND EXPORT PRICE

Externality (%)	Import Price		Export Price	
	US	China	US	China
Δ_1	3.3	6.2	3.8	4.6
Δ_2	-15.1	-18.8	-28.7	150
Δ_3	-17.6	-23.9	-32	135.9

Besides, wealth effect also affects the welfare outcome. If the increased price inflated the income, current account, the home country would prefer to consume more.

Current Account

Table 8 shows the spillover effects in terms of the current account as a percentage of the GDP. China would receive higher net income from abroad when deviating to the interim stage. As RMB became the regional vehicle currency, the current account of China would go into deficit. US, on the other hand, would be able to reduce the current account deficit and eventually receive a surplus as RMB emerging as an international currency. Deviating countries would increase the CA by 20% if RMB became the international currency, meanwhile non-deviating countries would experience 66.8% decrease.

TABLE 8
EXTERNALITIES ON CURRENT ACCOUNT

Externality (% of GDP)			Deviating Countries	Non-Deviating Countries
	US	China		
Δ_1	-1.4	5.9	-4.6	-15.5
Δ_2	3.9	0.1	20.4	-66.8
Δ_3	5.3	-5.8	25.1	-51.3

DISCUSSION

First, we discuss several assumptions made in this paper. We assume that all international trades are denominated in local currency (LCP) and subject to Cash-In-Advance constraints. The LCP assumption is not essential in the trading post mechanism. Take Korean Won as example. If the Korean seller set prices in Won, Russian buyer would visit trading post Won-Ruble and buys Won. If the trade is settled in Ruble, the Korean seller would sell the Ruble in exchange of Won immediately. The demand of Won in terms of Ruble depends on the export from Korea to Russian no matter the invoice currency is Won or Ruble.

The CIA constraint is critical to explain the demand of foreign currencies. If credit is available, would it kill the exchange rate risk? In this paper, if peripheral country is able to issue domestic currency denominated debt, it doesn't have to carry vehicle currency over time, which is similar to the interim equilibrium as a deviating peripheral country. If a peripheral country can only issue vehicle currency denominated debt, it's equivalent to buy vehicle currency and carry it over time (OVCE/TVCE).

The search-based model is another candidate to model the foreign exchange market, for example, (Matsuyama, Kiyotaki, & Matsui, 1993) and (Trejos, 1996). Moreover, (Head & Shi, 2002) introduces a hybrid model combining the location setting of trading posts assumption and search-based pricing mechanism. The results of these models are more intuitive to explain the emergence of vehicle currency, because of the rich underline micro-foundation. However, the non-linear optimality conditions in

equilibria are hard to estimate, especially when dealing with multiple countries. Besides, the search-based model doesn't significantly improve the estimation of externality caused by the regime change.

From OVCE to Interim, RMB emerges as a more popular invoicing currency. But China does not benefit much from this deviation according to our numerical experiment. Why has China been so eager to build a strong Yuan? Currency internationalization provides a number of advantages to the country of issuance. It reduces exchange rate risks, allows the issuance of domestic-currency denominated debt on the global capital market, and thereby improves risk management of cross-border transactions. The ultimate goal of RMB internationalization will push the exchange rate equilibrium to a TVCE, which would greatly increase the total trade of China. (Prasad, 2016) ; Also suggests that RMB internationalization is a strategic move to help the capital market reform.

Last, the estimation result is sensitive to the relative outputs. The output of China increases faster relative to US, which reduces the share of US output more in the model since we exclude other major economies. As a result, the estimated export price of US grows faster than data. Figure 6 and Figure 7 in the Appendix show that the externality on export price is negatively correlated with the size of country 1 and positively correlated with the size of country 2. The bilateral deviation of RMB to the Interim regime slightly increases both countries export price if country 1 has a smaller share of output and country 2 has a larger share. Figure 8 and Figure 9 show that the import price indices of both countries decrease if the exchange rate regime shift from OVCE/Interim to TVCE. Figure 13 and Figure 15 show that country 2 would benefit more in TVCE if the relative size of country 2 to country 1 is larger.

CONCLUSION

The RMB has risen as an invoicing currency, but there is still a long way to go for RMB to become an international currency. The numerical experiment shows that the deviation from OVCE to the Interim would not lead to significant spillover effect; while the further deviation to the TVCE would cause remarkable externalities on both US and other peripheral countries. As RMB becomes a reserve currency, the net export and current account increases in US and deviating peripheral countries, decreases in China and non-deviating peripheral countries. The spillover into US would be larger if more peripheral countries took RMB as a regional vehicle currency in TVCE.

The timing of the RMB internationalization is critical to China. If the relative output of China is large, there is welfare gain of the RMB internationalization; otherwise, it's costly to provide an international currency.

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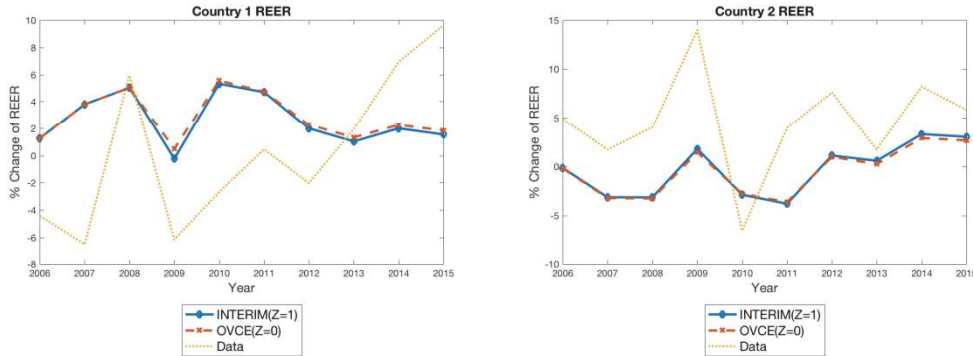
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APPENDIX

GRAPHS

Model Fit

REER

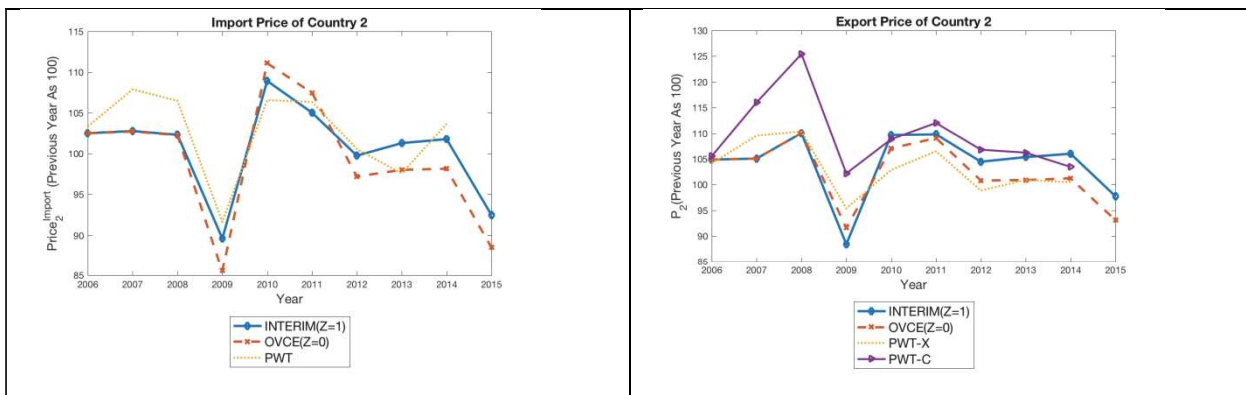


Above graphs show the percentage growth rate of Country 1 and Country 2 real effective exchange rate (REER).

Import Export Prices

In the following graphs, 'OVCE' refers to the first equilibrium regime; 'Interim' refers to the regime change specification, which shows the emergence of RMB. Also let OVCE be regime $Z=0$, Interim be regime $Z=1$ and TVCE be regime $Z=2$.

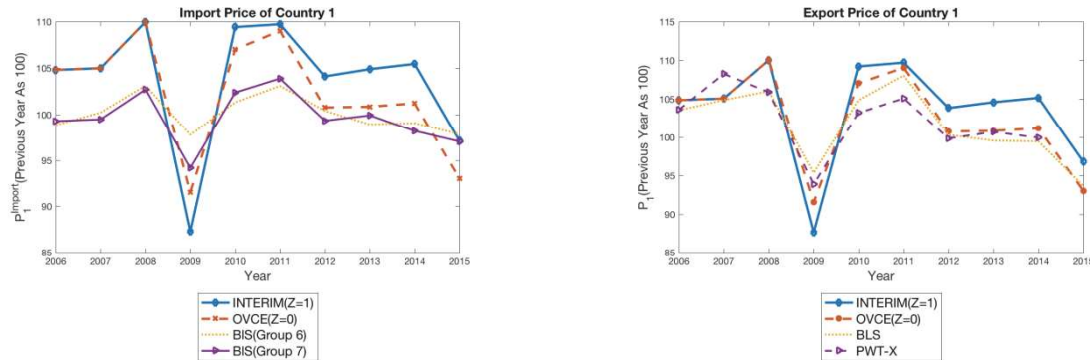
FIGURE 2
IMPORT/EXPORT PRICE OF COUNTRY 2



The left panel of Figure 2 compares the two model predicted import price of Country 2 and data moment cited from the Penn World Table 2016 (abbreviated as PWT). The gap between the two model moments widens as more peripheral country deviates in the Interim Equilibrium. There is 1 deviating peripheral country in 2009, 5 in 2010, 6 in 2011, 10 in 2012, 14 in 2013, 16 in 2014, and 20 in 2015. The 'Interim' regime fits the data better both in levels and volatilities. The 'Interim' moment replicates the mild recovery of import price from 2012 to 2014, where the OVCE model predicts a sharp decline of the import price from 2010 to 2015. The right panel of Figure 2 compares the model predicted Country 2 price, p_2 and the export/consumer price indices cited from PWT. The model doesn't distinguish between

the consumer price and export price of the tradable goods. Hence the model predicted p_2 is somewhere in-between.

FIGURE 3
IMPORT PRICE OF COUNTRY 1



The left panel of Figure 3 illustrates Country 1 import price predicted by OVCE and Interim Equilibrium, and the BLS import price index based on two groups of countries¹. Group 6 includes China, Japan, Australia, Brunei, Indonesia, Macao, Malaysia, New Zealand, Papua New Guinea, Philippines, and the Asian Newly Industrialized Countries. Group 7 refers to Asian Newly Industrialized Countries - Hong Kong, Singapore, South Korea, and Taiwan. Both Interim and OVCE are able to simulate the trough in 2009, while only Interim is able to predict the import price recovery in 2015. The right panel of Figure 3 shows Country 1 export prices from 2005 to 2015. The dotted yellow line shows the US export price index, estimated by Bureau of Labor Statistics¹. The dashed arrow line shows the export price index estimated by PWT. Both models predict the deflation from 2008 to 2009 and the recovery afterwards.

TVCE Experiment and Externalities

Import and Export Prices

FIGURE 4
EXPORT PRICE EXPERIMENT

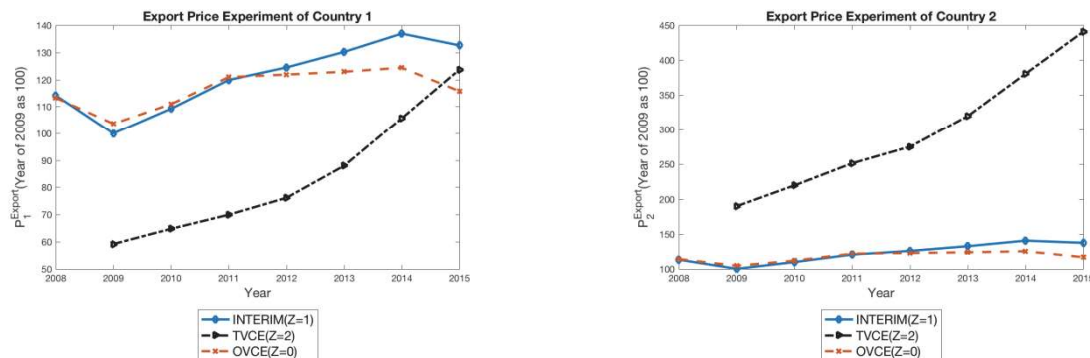


FIGURE 5
IMPORT PRICE EXPERIMENT

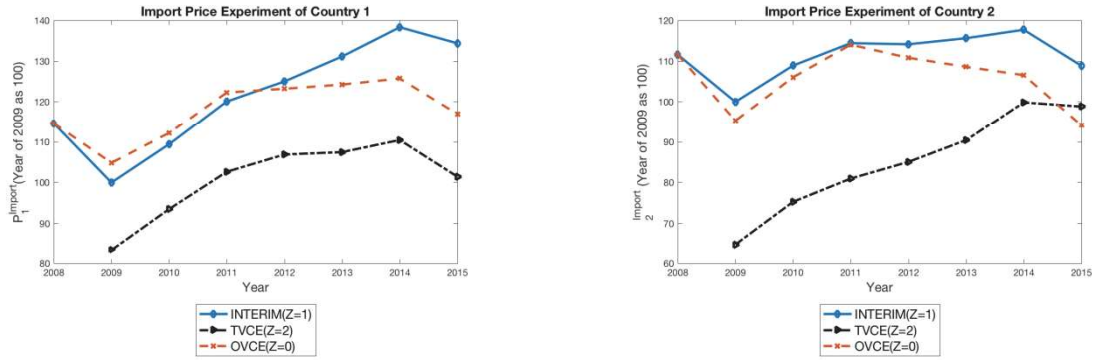


FIGURE 6
EXPORT PRICE OF COUNTRY 1 AND SHARE OF GDP

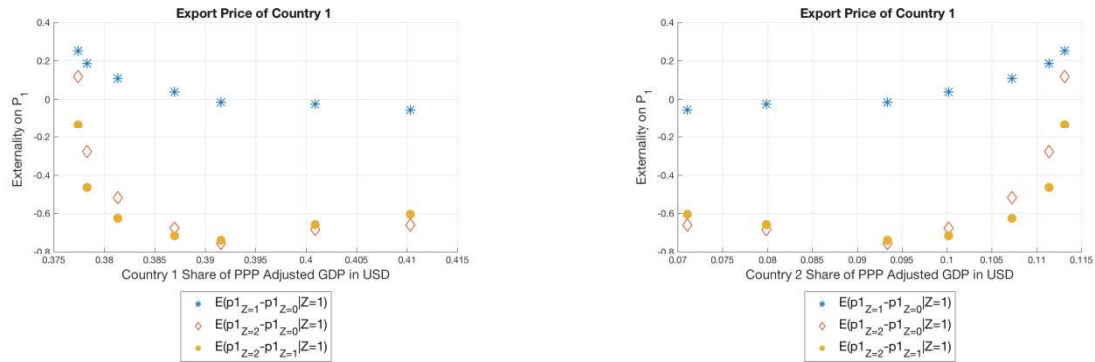


FIGURE 7
EXPORT PRICE OF COUNTRY 2 AND SHARE OF GDP

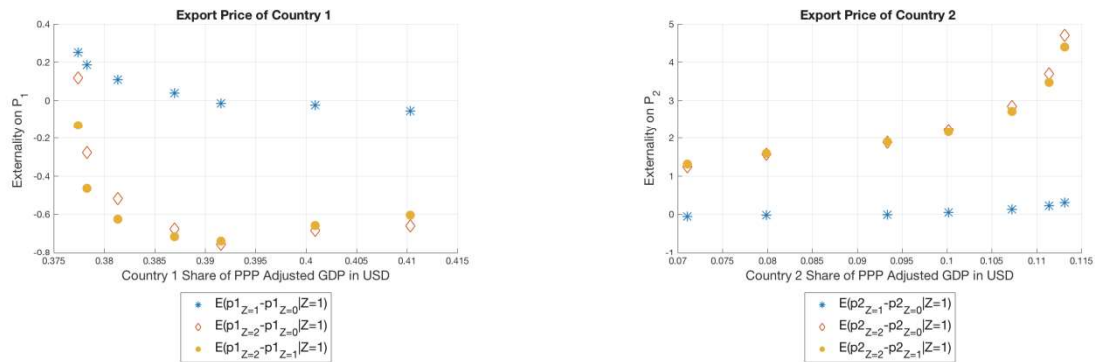


FIGURE 8
IMPORT PRICE OF COUNTRY 1 AND SHARE OF GDP

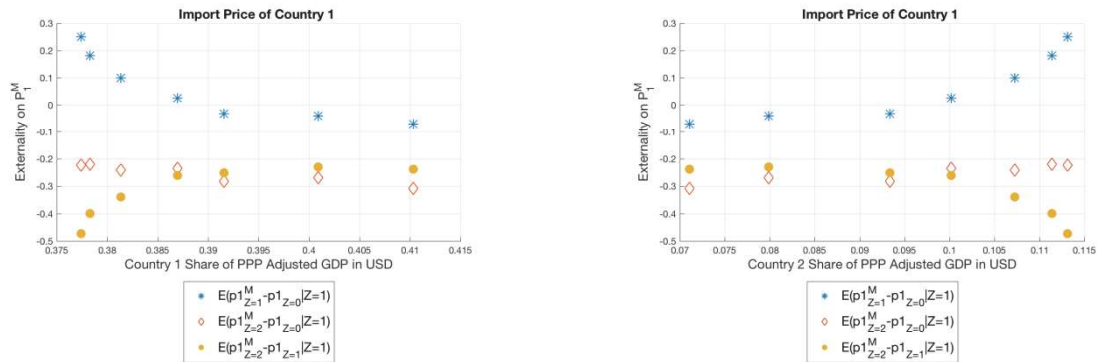
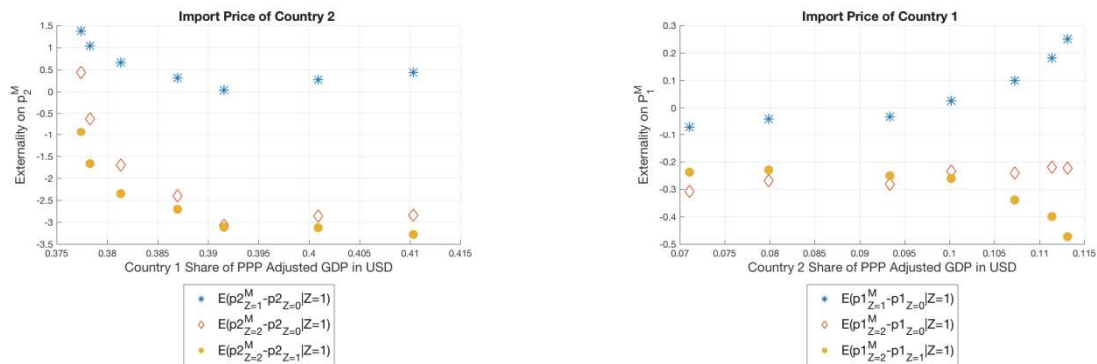


FIGURE 9
IMPORT PRICE OF COUNTRY 2 AND SHARE OF GDP



Current Account and Welfare

The CA deficit of country 1 would increase when the RMB deviate from the OVCE with other peripheral countries. If RMB rises as a local vehicle currency in TVCE, there is positive externality on country 1 CA.

The CA of country 2 increases when the exchange regime shifting from OVCE to Interim, decreases with approximately the same amount when the exchange regime changing from Interim to TVCE. As the two effects canceled out, the CA of country 2 in TVCE remains the same with the one in OVCE. But the welfare of country 2 in TVCE has improved comparing with OVCE, since the import increases more than the export according to Figure 13 and Figure 15.

FIGURE 10
EXTERNALITY ON CURRENT ACCOUNT OF COUNTRY 1

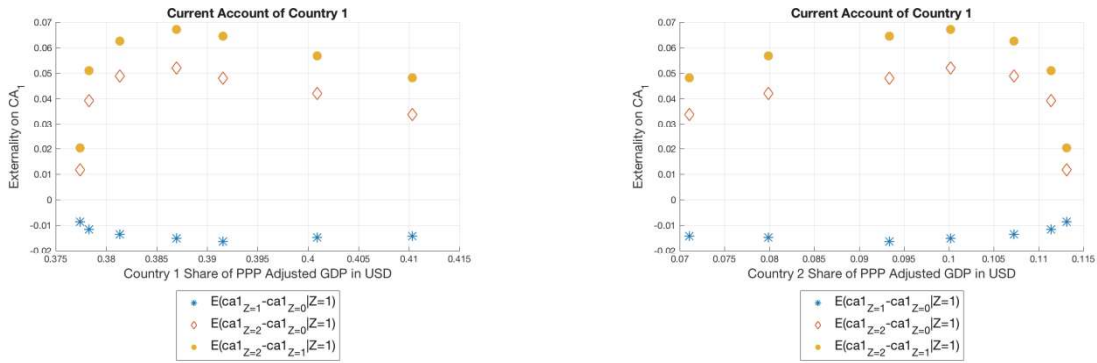


FIGURE 11
EXTERNALITY ON CURRENT ACCOUNT OF COUNTRY 2

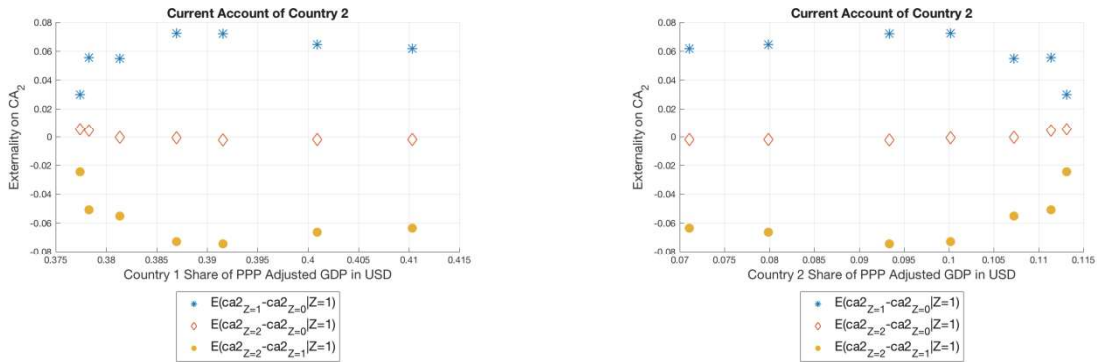


Figure 12 and Figure 14 show that country 1 would expect a decrease in import and an increase in export when the exchange regime shifts from OVCE/Interim to TVCE. When USD is the only vehicle currency, peripheral countries buy USD with their endowment to facilitate the foreign exchange trades. If RMB rises as a local vehicle currency, the demand of USD as a vehicle currency decreases. As a result, the trade deficit and current account deficit of country 1 decreases.

FIGURE 12
EXTERNALITY ON IMPORT OF COUNTRY 1

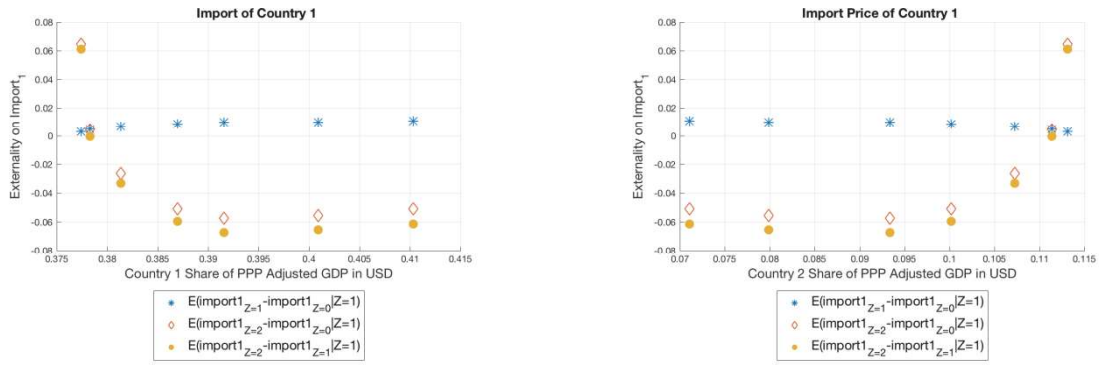


FIGURE 13
EXTERNALITY ON IMPORT OF COUNTRY 2

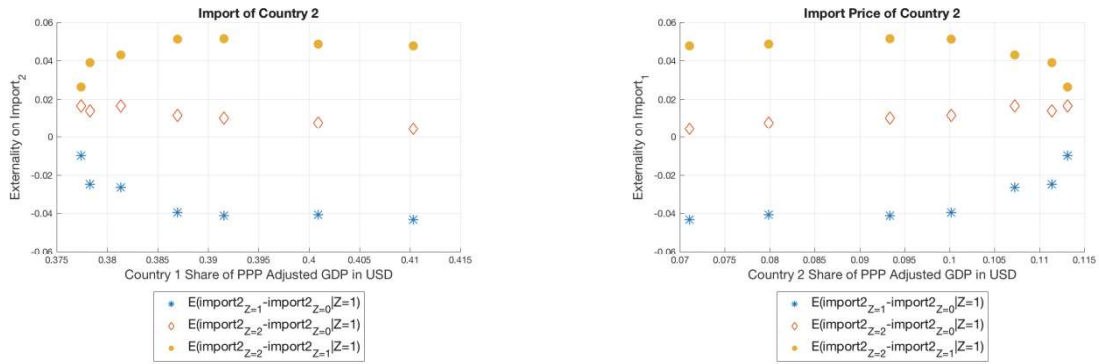


FIGURE 14
EXTERNALITY ON EXPORT OF COUNTRY 1

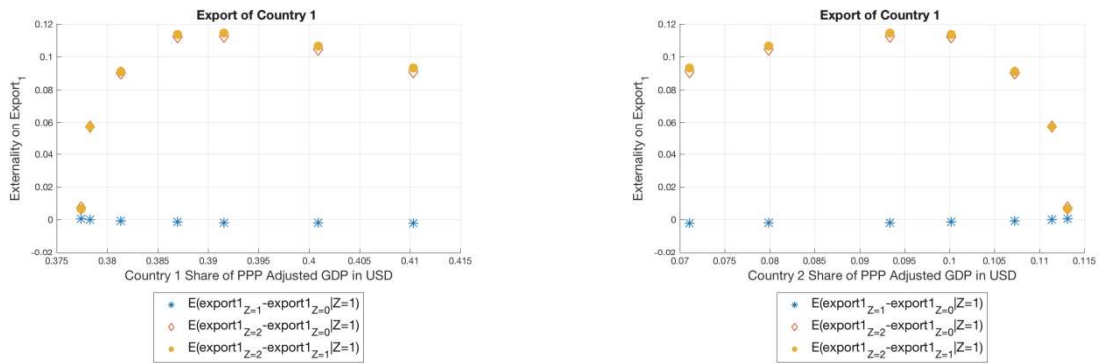
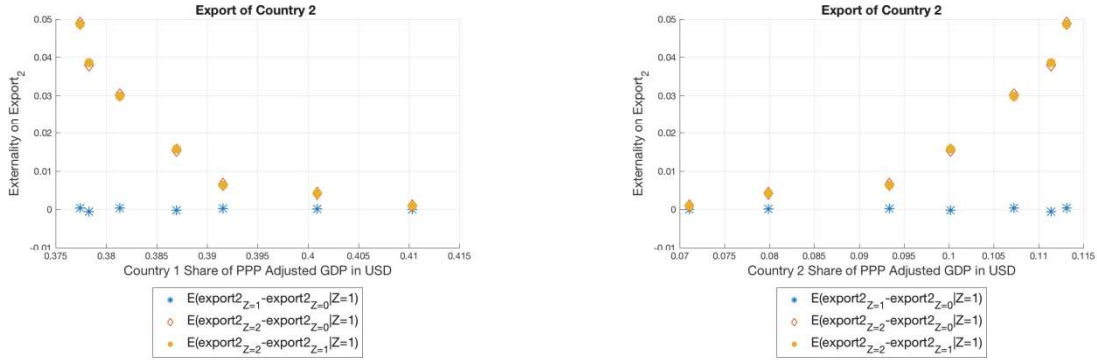


FIGURE 15
EXTERNALITY ON EXPORT OF COUNTRY 2



DESCRIPTION OF EQUILIBRIA

One vehicle currency equilibrium (OVCE)

Consumption

$$c_{ii} = \frac{Y_i}{\delta_i}, \forall i > 1; c_{ij} = \beta \frac{n_j}{n_i \delta_i \gamma_1} \frac{s_{1i}^b(-1)}{s_{1j}^a} Y_j, \forall i > 1, j > 1, j \neq i; c_{i1} = \frac{s_{1i}^b}{n_i \delta_i}, \forall i > 1$$

$$c_{11} = \frac{m_{11}}{p_1}; c_{1i} = \frac{m_{11}}{s_{1i}^a p_i}, \forall i > 1$$

where $\delta_i = n_1 + n_i + \beta(1 - n_1 - n_i)$.

Money Balance

$$m'_{ij} = p_j c_{ij} n_j, \forall j = 2, \dots, N; m'_{i1} = s_{1i}^b \left(\frac{1}{n_i} - \frac{1}{\delta_i} \right) m_{ij} = 0, \forall j \neq i, j \neq 1; m_{ii} = \frac{1}{n_i}, \forall i > 1$$

$$m_{i1} = \frac{\beta(1-n_1-n_i) s_{1i,t-1}^b}{n_i \delta_i \gamma_1}; m_{11} = \frac{1}{n_1} \left(1 - \frac{\beta}{\gamma_1} \sum_{i \neq 1} \frac{(1-n_1-n_i) s_{1it}^b}{\delta_i} \right); m_{1j} = 0, \forall j > 1$$

Currency carried to trading posts

$$f_{ii}^{1i} = \frac{1}{n_i} - \frac{1}{\delta_i}; f_{i1}^{1j} = \frac{\beta n_j s_{1i,t-1}^b}{n_i \delta_i \gamma_1}; f_{11}^{1i} = s_{1i}^a n_i p_i c_{1i} = n_i m_{11}$$

Price

$$p_i = \frac{1}{n_i Y_i}; p_1 = \frac{1}{n_1 Y_1} [1 - \gamma_{1,t+1} (1 - n_1 m_{11,t+1})]$$

Trading post li

$$s_{1i}^a = \frac{n_1 f_{11}^{1i} + \sum_{j \notin \{i,1\}} n_j f_{j1}^{1i}}{n_i f_{ii}^{1i} - \phi_i p_i}; s_{1i}^b = \frac{n_1 f_{11}^{1i} + \sum_{j \notin \{i,1\}} n_j f_{j1}^{1i} - \phi_i p_i}{n_i f_{ii}^{1i}}$$

Interim equilibrium solution

Consumption

$\forall i > 2,$

$$c_{i1} = \frac{1}{p_1} \frac{s_{1i}^b}{n_i \xi_i}; c_{i2} = \frac{1}{p_2} \frac{s_{2i}^b}{n_i \xi_i}; c_{iit} = \frac{1}{n_i p_i \xi_i} = \frac{Y_i}{\xi_i}; c_{ijt} = \frac{\beta n_j Y_j s_{1i,t-1}^b}{n_i \xi_i \gamma_1 s_{1j}^a}, \forall j > 2$$

where $\xi_i = n_1 + n_2 + n_i + \beta(1 - n_1 - n_2 - n_i)$.

$$c_{11} = \frac{m_{11}}{p_1}; c_{1j} = \frac{m_{11}}{s_{1j}^a p_j}, \forall j > 1; c_{22} = \frac{m_{22}}{p_2} = Y_2; c_{2j} = \frac{m_{22}}{s_{2j}^a p_j} = \frac{1}{s_{2j}^a p_j n_2}, \forall j > 2; c_{21} = \frac{s_{12}^b m_{22}}{p_1} = \frac{s_{12}^b}{p_1 n_2}$$

Money Balance

$$m_{ijt} = 0 \forall i > 2, j > 1, j \neq i; m_{iit} = \frac{1}{n_i}, \forall i > 1; m_{i1t} = (1 - n_1 - n_2 - n_i) \frac{\beta s_{1i,t-1}^b}{\gamma_1 n_i \xi_i}, \forall i > 2$$

$$m_{11t} = \frac{1}{n_1} \left(1 - \frac{\beta}{\gamma_1} \sum_{i>2} \frac{(1-n_1-n_2-n_i)s_{1i,t-1}^b}{n_i \xi_i} \right); m_{1it} = 0, \forall i > 1; m_{2i} = 0, \forall i \neq 2$$

$$m'_{ij} = p_{jt} c_{ijt} n_j, \forall i > 2, j > 1; m'_{i1} = \frac{\xi_i - n_2 - n_i}{n_i \xi_i} s_{1i}^b, \forall i > 2; m'_{1i} = p_{it} c_{1it} n_i; m'_{2i} = p_{it} c_{2it} n_i$$

Currency carried to trading posts

$$f_{i1}^{1j} = \frac{\beta n_j}{\gamma_{1t}} s_{1i,t-1}^b c_{ii,t-1} p_{ii,t-1} = \frac{\beta n_j s_{1i,t-1}^b}{\gamma_{1t} n_i \xi_i}, \forall i > 2, j > 2, j \neq i;$$

$$f_{ii}^{2i} = n_2 p_i c_{ii} = \frac{n_2}{n_i \xi_i}, \forall i > 2; f_{ii}^{1i} = \frac{1}{n_i} - \frac{n_2 + n_i}{n_i \xi_i}, \forall i > 2;$$

$$f_{11}^{1i} = s_{1i}^a n_i p_i c_{1i} = n_i m_{11}, \forall i > 1; f_{22}^{2i} = s_{2i}^a n_i p_i c_{2i} = \frac{n_i}{n_2}, \forall i > 2; f_{22}^{12} = \frac{1}{s_{12}^b} n_1 p_1 c_{21} = \frac{n_1}{n_2}$$

Price

$$p_i = \frac{1}{n_i \gamma_i}, \forall i > 1; p_1 = \frac{1}{n_1 \gamma_1} [1 - \gamma_{1,t+1} (1 - n_1 m_{11,t+1})]$$

Trading Post 1i, i > 2

Country $i \in D, i \neq 2$

$$s_{1i}^a = \frac{n_1 n_i m_{11} + \sum_{j \in D, j \neq 2, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \xi_j} + \sum_{j \in D, j \neq 1, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \delta_j}}{\frac{M_i}{\xi_i} [n_1 + \beta(1 - n_1 - n_2 - n_i)] - \phi_i p_i};$$

$$s_{1i}^b = \frac{n_1 n_i m_{11} + \sum_{j \in D, j \neq 2, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \xi_j} + \sum_{j \in D, j \neq 1, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \delta_j} - \phi_1 p_1}{\frac{M_i}{\xi_i} [n_1 + \beta(1 - n_1 - n_2 - n_i)]}.$$

Country $i \notin D, i \neq 1$

$$s_{1i}^a = \frac{n_1 n_i m_{11} + \sum_{j \in D, j \neq 2, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \xi_j} + \sum_{j \in D, j \neq 1, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \delta_j} + M_2 \frac{\beta n_i s_{12}^b}{\gamma_1 \delta_D}}{\frac{M_i}{\delta_i} [n_1 + \beta(1 - n_1 - n_i)] - \phi_i p_i};$$

$$s_{1i}^b = \frac{n_1 n_i m_{11} + \sum_{j \in D, j \neq 2, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \xi_j} + \sum_{j \in D, j \neq 1, i} M_{j,-1} \frac{\beta n_i s_{1j}^b}{\gamma_1 \delta_j} + M_2 \frac{\beta n_i s_{12}^b}{\gamma_1 \delta_D} - \phi_1 p_1}{\frac{M_i}{\delta_i} [n_1 + \beta(1 - n_1 - n_i)]}.$$

Trading Post 2i, i > 2

$$s_{2i}^a = \frac{M_2 n_i / \delta_D}{M_i n_2 / \xi_i - \phi_i p_i}; s_{2i}^b = \frac{M_2 n_i / \delta_D - \phi_2 p_2}{M_i n_2 / \xi_i}.$$

Trading Post 12

$$s_{12}^a = \frac{n_1 n_2 m_{11} + \frac{\beta}{\gamma_1} \sum_{j \in \text{end}} n_2 \frac{M_j s_{1j}^b}{\delta_j}}{M_2 [n_1 + \beta(1 - n_1 - n_D)] / \delta_D - \phi_2 p_2}; s_{12}^b = \frac{n_1 n_2 m_{11} + \frac{\beta}{\gamma_1} \sum_{j \in \text{end}} n_2 \frac{M_j s_{1j}^b}{\delta_j} - \phi_1 p_1}{M_2 [n_1 + \beta(1 - n_1 - n_D)] / \delta_D}.$$

Two Vehicle Currency Equilibrium

Consumption

$\forall i \in I_1,$

$$c_{ii} = \frac{Y_i}{\delta_i}, i \in I_1; c_{ij} = \frac{\beta s_{1i(-1)}^b n_j Y_j M_i}{\gamma_1 s_{1j}^a n_i \delta_i M_j}, i \in I_1.$$

$\forall i \in I_2,$

$$c_{ii} = \frac{Y_i}{\xi_i} M_i; c_{ij} = \frac{s_{1i(-1)}^b \beta n_j Y_j M_i}{s_{1j}^a \gamma_1 n_i \xi_i M_j}, j \in I_1; c_{ij} = \frac{s_{2i(-1)}^b \beta n_j Y_j M_i}{s_{2j}^a \gamma_2 n_i \xi_i M_j}, j \in I_2.$$

where $\xi_i = n_1 + n_2 + n_i + \sum_{j>2, j \neq i} \beta n_j$

Country 2, $i = 2,$

$$c_{22} = \frac{m_{22}}{p_2}; c_{21} = \frac{m_{22}}{p_1} s_{12}^b; c_{2j} = \frac{\beta m_{22} n_j Y_j s_{12(-1)}^b}{\gamma_1 M_j s_{1j}^a}, if j \in I_1; c_{2j} = \frac{m_{22} n_j Y_j}{M_j s_{2j}^a}, if j \in I_2.$$

Country $i = 1, c_{11} = \frac{m_{11}}{p_1}.$

Money Balance

Country $\forall i \in I_1,$

$$m_{ij} = 0, \forall j > 1, j \neq i; m_{ii} = \frac{M_i}{n_i}; m_{i1} = \frac{\beta \sum_{j \neq 1, i} n_j}{\gamma_1 \delta_i n_i} s_{1i(-1)}^b M_{i,-1};$$

$$m'_{ij} = \frac{\beta s_{1i(-1)}^b n_j}{\gamma_1 s_{1j}^a n_i \delta_i} M_i, j > 1; m'_{i1} = \left(\frac{1}{n_i} - \frac{1}{\delta_i}\right) s_{1i}^b M_i.$$

Country $\forall i \in I_2,$

$$m_{ij} = 0, \forall j > 2, j \neq i; m_{ii} = \frac{M_i}{n_i}; m_{i1} = \frac{\beta}{\gamma_1} s_{1i(-1)}^b \frac{\sum_{j \in I_1} n_j}{n_i \xi_i} M_i; m_{i2} = \frac{\beta}{\gamma_2} s_{2i(-1)}^b \frac{\sum_{j \in I_2, j \neq i} n_j}{n_i \xi_i} M_i.$$

Country $i = 2,$

$$m_{2j} = 0, \forall j > 2; m_{22} = \frac{1}{n_2} \left(M_2 - \frac{\beta}{\gamma_2} \sum_{i \in I_2} s_{2i(-1)}^b \frac{\sum_{j \in I_2, j \neq i} n_j}{\xi_i} \right); m_{21} = m_{22} \frac{\beta}{\gamma_1} s_{12(-1)}^b \sum_{j \in I_1} n_j;$$

$$m'_{21} = s_{12}^b n_1 m_{22}; m'_{2j} = \begin{cases} \left(\frac{s_{12(-1)}^b}{s_{1j}^a}\right) m_{22} n_j \frac{\beta}{\gamma_1}, & \forall j \in I_1 \\ \frac{1}{s_{2j}^a} m_{22} n_j, & \forall j \in I_2 \end{cases}; m'_{22} = n_2 m_{22}.$$

Country $i = 1,$

$$m_{1j} = 0, \forall j > 1; m'_{11} = n_1 m_{11}; m'_{1j} = \frac{1}{s_{1j}^a} n_j m_{11};$$

$$m_{11} = \frac{1}{n_1} \left(1 - \frac{\beta}{\gamma_1} [s_{12(-1)}^b N_1 (M_2 - \frac{\beta}{\gamma_2} \sum_{i \in I_2} s_{2i(-1)}^b \frac{N_2 - n_i}{\xi_i}) + \sum_{i \in I_1} s_{1i(-1)}^b \frac{1 - n_1 - n_i}{\delta_i} M_i + \sum_{i \in I_2} s_{1i(-1)}^b \frac{N_1}{\xi_i} M_i] \right).$$

Currency carried to trading posts

Country $\forall i \in I_1,$

$$f_{i1}^{1j} = \frac{\beta n_j}{\gamma_1 n_i \delta_i} s_{1i(-1)}^b M_i; f_{ii}^{1i} = \left(\frac{1}{n_i} - \frac{1}{\delta_i}\right) M_i.$$

Country $\forall i \in I_2,$

$$f_{i1}^{1j} = \frac{\beta s_{1i(-1)}^b n_j}{\gamma_1 n_i \xi_i} M_i, j \in I_1; f_{i2}^{2j} = \frac{\beta s_{2i(-1)}^b n_j}{\gamma_2 n_i \xi_i} M_i, j \in I_2, j \neq i;$$

$$f_{ii}^{1i} = \frac{M_i}{n_i \xi_i} \left(\beta \sum_{j \in I_1} n_j + n_1 \right); f_{ii}^{2i} = \frac{M_i}{n_i \xi_i} \left(\beta \sum_{j \in I_2, j \neq i} n_j + n_2 \right).$$

Country $i = 1, f_{11}^{1j} = n_j m_{11}$.

Country $i = 2,$

$$f_{22}^{21} = m_{22} \left(n_1 + \sum_{j \in I_1} n_j \right); f_{21}^{1j} = \begin{cases} m_{22} \frac{\beta}{\gamma_1} s_{12(-)}^b n_j, \forall j \in I_1, \\ 0, \forall j \in I_2 \end{cases}; f_{22}^{2j} = \begin{cases} 0, \forall j \in I_1 \\ n_j m_{22}, \forall j \in I_2 \end{cases}.$$

Prices

$$p_i = \frac{M_i}{n_i}, \forall i \in I_1; p_i = \frac{M_i}{n_i}, \forall i \in I_2.$$

$$p_2 = \frac{1}{n_2} [M_2 - \gamma_{2(+)} (M_2 - n_2 m_{22(+)})]; p_1 = \frac{1}{n_1} [1 - \gamma_{1(+)} (1 - n_1 m_{11(+)})].$$

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Country $i \in I_2,$

$$s_{1i}^a = \frac{n_1 n_i m_{11} + \sum_{j \in I_1} \frac{\beta n_i}{\gamma_1 \delta_j} s_{1j(-)}^b M_j}{\frac{M_i}{\xi_i} (\beta \sum_{j \in I_1} n_j + n_1) - \phi_i p_i}; s_{1i}^b = \frac{n_1 n_i m_{11} + \sum_{j \in I_1} \frac{\beta n_i}{\gamma_1 \delta_j} s_{1j(-)}^b M_j - \phi_1 p_1}{\frac{M_i}{\xi_i} (\beta \sum_{j \in I_1} n_j + n_1)}.$$

Country $i \in I_1,$

$$s_{1i}^b = \frac{n_1 n_i m_{11} + \frac{\beta}{\gamma_i} \left(\sum_{\substack{j \in I_1 \\ j \neq i}} \frac{s_{1j(-)}^b}{\delta_j} n_i M_j + \sum_{j \in I_2} \frac{s_{1j(-)}^b}{\xi_j} n_i M_j \right) - \phi_1 p_1}{M_i \left(1 - \frac{n_i}{\delta_i} \right)};$$

$$s_{1i}^a = \frac{n_1 n_i m_{11} + \frac{\beta}{\gamma_i} \left(\sum_{\substack{j \in I_1 \\ j \neq i}} \frac{s_{1j(-)}^b}{\delta_j} n_i M_j + \sum_{j \in I_2} \frac{s_{1j(-)}^b}{\xi_j} n_i M_j \right)}{M_i \left(1 - \frac{n_i}{\delta_i} \right) - \phi_i p_i}.$$

Trading Post 2i, $i \in I_2,$

$$s_{2i}^b = \frac{n_2 n_i m_{22} + \sum_{\substack{j \in I_2 \\ j \neq i}} \frac{\beta n_i}{\gamma_2 \xi_j} s_{2j(-)}^b M_j - \phi_2 p_2}{\frac{M_i}{\xi_i} (\beta \sum_{\substack{j \in I_2 \\ j \neq i}} n_j + n_2)}; s_{2i}^a = \frac{n_2 n_i m_{22} + \sum_{\substack{j \in I_2 \\ j \neq i}} \frac{\beta n_i}{\gamma_2 \xi_j} s_{2j(-)}^b M_j}{\frac{M_i}{\xi_i} (\beta \sum_{\substack{j \in I_2 \\ j \neq i}} n_j + n_2) - p_i \phi_i}.$$

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$$s_{12}^b = \frac{n_1 m_{11} + \sum_{j \in I_1} \frac{\beta s_{1j(-)}^b}{\gamma_1 \delta_j} M_j - \frac{\phi_1 p_1}{n_2}}{m_{22} (n_1 + \sum_{j \in I_1} n_j)}; s_{12}^a = \frac{n_1 m_{11} + \sum_{j \in I_1} \frac{\beta s_{1j(-)}^b}{\gamma_1 \delta_j} M_j}{m_{22} (n_1 + \sum_{j \in I_1} n_j) - \frac{\phi_2 p_2}{n_2}}.$$