Estimation and Prediction of Gasoline Price in Mexico from Evans Price Adjustment Model

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This work adds to the mathematical proposition, which is aligned to the vision of the educational model at Tecnologico de Monterrey. It is a part of a set of situations that have been prepared to cover the mathematical training of students, these have been tested successfully at Campus State of Mexico. All these challenges have the same philosophy: propose situations of real context whose answers could be interesting by any sector of society. The controversy among economic specialists inspired the present challenge; at the beginning of 2018, they were deciding the possibility that Magna gasoline could reach an average price of 20 pesos in Mexico. This situation was proposed to the students of the second semester of mathematics at the beginning of 2018, in order to analyze the problem under the model of Evans price adjustment. They had to generate a prediction based on the question of whether gasoline Magna would reach 20 pesos at the end of this year.

Keywords: competencies, prediction, price, challenge

INTRODUCTION

A substantial element of the Tecnologico de Monterrey's educational model is that which refers to RETO. A challenge is an activity, task or situation that involves the student in a stimulating and challenging way. In reality and in practice, the context of this pedagogical approach seems reasonable when a CHALLENGE is located in the second or third quarter of some professional career, even more so if it is linked to some teaching partner (company or government, for example, that accompanies the process of execution of the CHALLENGE). However, for the first quarter in particular, regarding the mathematical education of the basic core where we work with first time students with the corresponding cognitive and personal immaturity, the answer does not always seem so natural.

In the academic field, it is a consensus that every university student should develop good mathematical competence; however, how can this be done and how can it be done in the face of the approach of what a CHALLENGE entails? This work reports a possible response and constitutes one more educational experience that is part of a wide network of situations of a real nature. The goal of this type of activity is to contribute in the mathematical area within the educational model already mentioned. However, against the background of a real situation generated in Mexico from 2017 with the so-called "fuel price liberation", this work is an example of a didactic situation that resorts to the application of a mathematical model, called Evans' price adjustment model, which establishes that "the price of a good changes in time proportionally to its scarcity".

THEORETICAL FRAMEWORK

This work was proposed to students of the second semester of the engineering degrees, and is based on an Evans mathematical model of adjustment (or prediction) of prices that allows estimating-forecasting, in the short term, the price of a good if these are known in advance: price, demand and supply. The problem-situation proposed here, which was worked on by the students in the January-May 2018 semester, fits well into the context of the new TEC 21 Educational Model. At the same time, in the context of educational mathematics it belongs to the so-called Brousseau's Didactic Situation Theory (Brousseau, 2000).

The challenge was posed in the context of a real situation that is being experienced in Mexico from January 2017, namely, the federal government took the decision to release the fuel prices: Magna, Premium, Diesel and LPG gas leaving the prices to be regulated by supply, demand, international prices and the exchange rate. Under these new circumstances, the Ministry of Finance, formerly the sole regulator of these fuels prices, would gradually be left out of these determinations except for the fact that it would soften, through certain incentives in certain regions of the country (such as on the northern border with the United States), the abrupt fluctuations that the prices of these fuels might have. This price release is already being fully experienced in 2018 with the implications that this provision has had, particularly on national inflation. In fact, inflation rate in Mexico in 2017 reached its highest point in the last 17 years, reaching a rate of 6.77%, which, according to specialists, is largely explained by the increase in fuel prices (Migueles, 2018).

To obtain a forecast on gasoline prices (Magna was the most representative case study) throughout 2018, a mathematical model based on two pillars was proposed to the students: Evans' price adjustment model, which translates into a first-order differential equation, and the adjustment by linear regression of supply and demand using the information generated by the PEMEX Paraestatal, until two years ago, the only distributor of all types of fuels in Mexico. This CHALLENGE (like others of the same type) is governed by the following principles:

a) They require interaction between students and a real context.
b) They require teamwork so that communication between students develops the language of the discipline as part of the learning process.
c) Validation of the processes and the solution through oral presentation of the ideas discussed, if possible before a group of experts. Otherwise, comparing with specialists' opinions on the subject that understand the challenge.
d) If relevant, the incorporation of social aspects such as ethics and/or citizenship, or some socio-economic problem such as the current one.
e) Finally, in the group of challenges proposed for professional training, one more element has been added: the use of technology.

It can be said that posing and solving a problem like this makes sense in the context of a competency-based education in which the overall planning of an entire institution is aware of the administration of a complex process that transcends the traditional professorship. The educational intentions behind this kind of situation are linked, among other things, to enabling students to learn through concrete experiences, linked to their immediate reality and updated in educational matters. Along the same lines, students are provided with the skills that they require in their professional training and that industry or different organizations demand of them when they get into professional jobs.

This work took 5 weeks between the assignment and the delivered solution. Students had to specify Evans' mathematical model from his theoretical description (Draper and Klingman, 1967). To obtain information from PEMEX (Petróleos Mexicanos, 2018) on the supply and demand of the chosen fuel and from this information, by linear regression, to determine two expressions for supply and demand in terms of price throughout 2017, the year in which the transition to the new economic model of fuel prices in Mexico was initiated. It can be said that this model could have been solved directly from the data found using numerical methods or, a non-linear regression adjustment could have been more appropriately applied (which would have improved the forecast); however, the students who worked on the challenge

did not have enough knowledge and/or skills to address a solution that incorporated more than what is stated here.

METHODOLOGY

As previously mentioned, given the context of fuel price release and the notable increase in its price from January 2017, specialists speculated that the price of Magna gasoline would reach 20 pesos by the end of 2018. Evans' price adjustment model is based on the principle that the rate of change in the price of a good is proportional to the scarcity, defined as the difference between demand and supply. Therefore, this principle is translated into the following differential equation:

$$\frac{dp}{dt} = k(D(p) - O(p))$$

Where p is the good's price, in this case, the liter of gasoline Magna, t is the time, $D(p)$ and $O(p)$ is the demand and offer respectively, both dependent, as is known, on the price p of the good; k is the proportionality constant. Solving the differential equation is not difficult at first, because by means of variable separation, it can be written in the form:

$$\frac{dp}{D(p) - O(p)} = kdt$$

If two expressions are known for the demand and supply (depending on the price of the good), what would follow is to carry out an integration process and calculate the adjustment parameters (k, and the constant of integration).The point is that these aforementioned expressions for $D(p)$; O(p), should be obtained from reliable real data, preferably from a government source. The source the teams used was the PEMEX yearbook (Indicadores Petroleros, 2018). Getting information was not as simple as it seemed at first glance. The following table contains information on price per liter and demand, in barrels per day and in liters per day.

TABLE 1
DEMAND VERSUS PRICE. MAGNA GASOLINE (INDICADORES PETROLEROS, 2018)

Month	Price per liter (in pesos)	Sales of Magna gasoline in liters	Sale of Magna gasoline in barrels
January	15.99	95646.75968	601,6
February	15.99	101036.4292	635,5
March	15.87	104565.9472	657,7
April	16.50	101131.8215	636,1
May	16.42	106457.8961	669,6
June	16.22	107634.4021	677,0
July	16.25	104152.5802	655,1
August	16.43	108063.6678	679,7
September	16.63	105631.1621	664,4
October	16.71	104025.3904	654,3
November	16.76	107904.6805	678,7

Supply information is not published directly. First, it must be obtained as the sum between domestic production and imports. In addition, PEMEX offers information on supply considering both gasolines: Magna and Premium. It was necessary to seek information from the Energy Regulatory Commission

(Gasoline and Diesel Prices, 2017) to determine what percentage corresponded to Magna gasoline within the total supply of gasoline. Magna gasoline represents 81.5% of the total

TABLE 2
SUPPLY VERSUS PRICE. MAGNA GASOLINE (GASOLINE AND DIESEL PRICES, 2017)

Month	Price per liter (in pesos)	Total gasoline supply (thousands of barrels per day)	Supply of Magna gasoline (in thousands of barrels per day)	Supply of Magna gasoline (in liters) 81.5% of total supply
January	15.99	861,3	701,9595	111602.6456
February	15.99	838	682,97	108583.5563
March	15.87	839,3	684,0295	108752.0033
April	16.50	767,3	625,3495	99422.62856
May	16.42	837,5	682,5625	108518.769
June	16.22	813,1	662,6765	105357.1475
July	16.25	834,5	680,1175	108130.045
August	16.43	843,4	687,371	109283.2594
September	16.63	788,9	642,9535	102221.441
October	16.71	774,6	631,299	100368.5235
November	16.76	823,1	670,8265	106652.894

Once the above information was available, it was easy to draw up two charts that gave a clearer idea of the type of function that would adjust the data for both demand and supply. These charts are shown in Figures 1 and 2 below.

FIGURE 1
DEMAND VERSUS PRICE

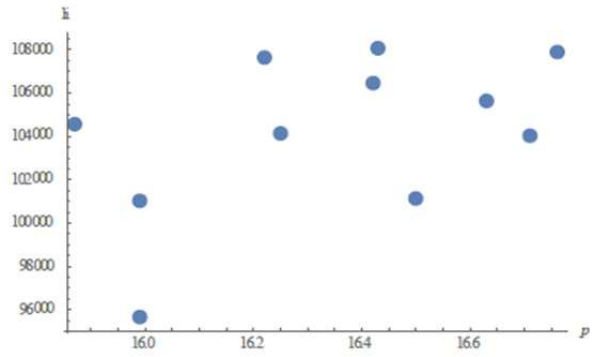
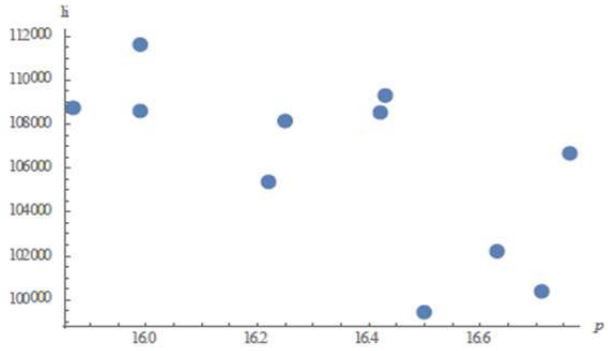


FIGURE 2
SUPPLY VERSUS PRICE



As can be seen, the data variability is considerable. This is why a linear type adjustment seems to be a bad fit by far. However, as indicated above, given the level of mathematical competence of the participants in this challenge, a linear-type adjustment was proposed. The charts of these (linear) adjustments, placed on top of the previous dotted charts, emphasize what has been indicated.

FIGURE 3
ADJUSTMENT. DEMAND VERSUS PRICE

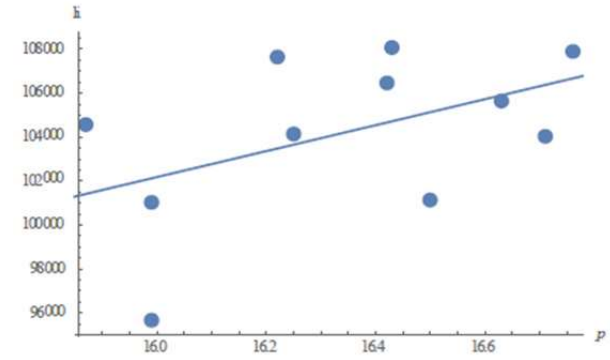
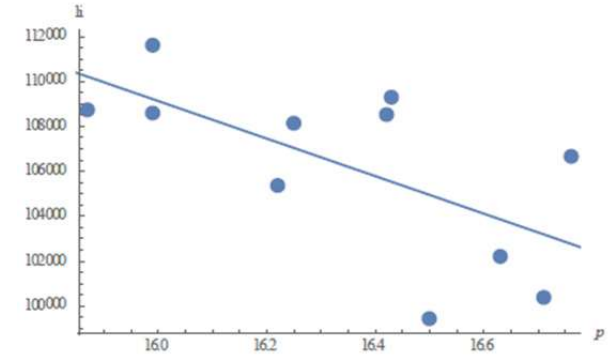


FIGURE 4
ADJUSTMENT. SUPPLY VERSUS PRICE



This CHALLENGE was a good reason to use technology. Although they were motivated and gave a brief workshop on the Mathematics software, most preferred to use Excel. Anyway, and under the limitations that have already been pointed out, it was found that the linear adjustment formulas for the demand and supply functions are, respectively:

$$D(p) = 7739.925 + 5902.606\,p$$

$$O(p) = 242920.34 - 8361.967\,p$$

Consequently, the scarcity is:

$$D(p) - O(p) = -235180.415 + 14264.573\,p$$

The challenge was assigned to be worked on for five weeks. It took about two weeks for the students to find and understand the kind of information they needed. Later, their work focused on the solution and interpretation of results. At the same time, the theoretical knowledges that the students required to solve the CHALLENGE were developed. The solution they proposed entails the following ideas:

By placing the scarcity in the differential equation of separable variables, it is determined:

$$\frac{dp}{-235180.415 + 14264.573\,p} = kdt$$

Therefore, by integrating, it is obtained:

$$0.00007010374290618697 \ln(-235180.415 + 14264.5736\,p) = kt + C$$

In order to find the constants k and C, it was considered (arbitrarily) that $t = 0$ corresponded to the month of December2017 (month in which no information was reported in the PEMEX yearbook). So, for example, $t = 1$ would mean January 2018. The last two historical data were taken, corresponding to October and November 2017. For the variable t, this would mean: $t = -2$ and $t = -1$, respectively. For these times, the price of gasoline was: 16.71 and 16.76, respectively. With these values, it was obtained:

$$k = 0.000014183566157794226, C = 0.0005937417625861285$$

Substituting these values and clearing the price function, we found that:

$$p(t) = 16.487 + 0.334187e^{0.211629t}$$

RESULTS

The conclusions will indicate those associated with the didactics used, their pros and cons. In this section it will be highlighted the results on the average price of Magna gasoline that were forecasted (unfortunately, the reality has exceeded the forecast). A comparison between forecast and reality is provided in the following table:

TABLE 3
FORECAST VERSUS RECORDED COST FOR MAGNA GASOLINE, 2018

Month	Forecast (pesos/liters)	Reality (pesos/liters)
January	16.90	16.90
February	17.00	17.48
March	17.12	17.54
April	17.27	18.26
May	17.45	18.40
June	17.68	18.50
July	17.96	¿?
August	18.30	¿?
September	18.73	¿?
October	19.26	¿?
November	20.72	¿?

Even though the forecast has remained below the real prices, one conclusion is immediately apparent: Magna gasoline will in fact reach 20 pesos, approximately in the last quarter of the year (if the trend continues as it is and politics do not intervene in this economic matter).

CONCLUSIONS

Some virtues of this kind of didactics can be pointed out roughly. Perhaps the most outstanding thing lies in the combination of knowledge and thinking skills to solve the proposed situation, as well as the use of technology, motivation and the significance level achieved by an application of mathematics such as this. It was complicated to develop specific aspects of the course while providing the basic elements of the software. Quite a few students were allowed to learn for themselves, but the less well-informed had difficulty in combining theory, software and application. From the analysis of this situation and the results obtained in this work, perhaps one of the most important conclusions is that the students managed to understand the value of this tool that we refer to as differential equation, a natural instrument for research in many fields of science and engineering. The results obtained in the forecasting were compared with the actual values of the gasoline price. The positive evaluation that was made regarding this challenge by the students comes from the comparison versus reality, a comparison that generated throughout the rest of the semester a very rewarding experience for the students.

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