Distribution Matters: The Reverse Robin-Hood Macroeconomic Effects

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This paper examines income distribution's impact on aggregate demand. Income distribution is introduced by replacing consumption as a function of aggregate income with consumption as the sum of two consumptions: a high income group's and low income group's. Notably the high income group's MPC is assumed to be less than the lower income group's. Analysis reveals two key results. As the high income group's proportion of income rises: (1) aggregate demand falls and (2) autonomous spending changes cause smaller aggregate demand shifts. A reduced multiplier is the key. Empirical work supports the assumption that the high income group's MPC is less than the low income group's.

INTRODUCTION

Macroeconomics misses income distribution matters due to a blind spot. This blind spot is rooted in the assumption that consumption depends on aggregate income. This paper removes the blind spot by incorporating income distribution in the consumption function. Consumption is formed as the sum of the consumption of two income groups—one with high income and the other with low income with the marginal propensity to consume (MPC) of the high income group less than the MPC of the low income group. Analysis of this innovated macro model reveals that distribution matters. In particular we find that as a greater proportion of income goes to the high income group aggregate demand falls, the aggregate demand curve becomes more price level inelastic, and there is a reduction in the magnitude of aggregate demand shifts in response to exogenous changes in consumption, investment, government spending, and money supply. The key is that the expenditure multiplier is reduced as a greater proportion of income goes to the high income group.

Our empirical work lends support for the key assumption that the MPC of the high income group is less than the MPC of the low income group. To estimate MPCs, we utilize decile data for annual consumption and income for the period 1984 to 2011. One glimpse of our confirming findings is that we find the average of the MPCs of the bottom nine deciles is 0.651 while the MPC of the top decile 0.514.

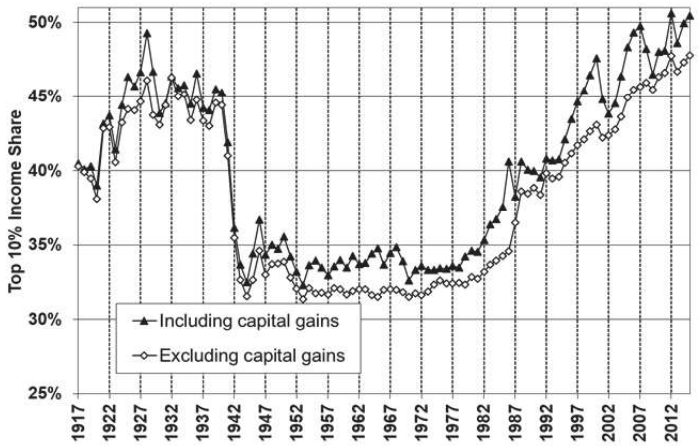
The remainder of this paper sets out in five sections. Section One provides perspective on past work on income distribution's effect on the Macroeconomy. Section Two sets up a conventional macro model with consumption a function of aggregate income in order to establish a baseline. Section Three sets up and analyzes a revised model which is identical to Section Two's with one exception. The exception is the innovation that aggregate consumption is the sum of the consumption of two income groups—one with high income and the other with low income—whose MPCs differ. The MPC of the high income group is less than the MPC of the low income group. Section Four presents our data set and the results of regression work. Section Five summarizes.

SECTION ONE: PAST TREATMENT OF INCOME DISTRIBUTION'S EFFECT

Early Keynesians thought income distribution affected the Macroeconomy. Stressing the marginal propensity to consume (MPC) and relying on their intuition that the MPC of higher income group is lower than that of the lower income group, it seemed clear to them that redistributing income from the higher income to the lower income group would raise consumption. Since the work in the 1940s and 1950s, by Bronfenbrenner, Yamane, and Lee (1955), Conrad (1955), and Lubell (1947), economists have paid scant attention to the effect of the MPC-income-distribution connection. Perhaps this reduced interest is due to the work on the Life Cycle Hypothesis (LC) of Ando and Modigliani (1963) and Friedman (1957) on the Permanent Income Hypothesis (PIH) which reduced the role of the MPC in macroeconomic thinking. Surprised that the Keynesian view had not been investigated directly, Blinder (1975) investigated it both theoretically and empirically. His theoretical work using a permanent income hypothesis framework finds that the MPC linchpin of the Keynesian intuition could but does not necessarily decline as one's income rises. His empirical work on the effect on total consumption of the MPCs of high and low income groups was, to quote him, "doomed to failure" (p. 459) due to considerable collinearity, the relatively stable income distribution during the time frame of his data set 1947-72, and paucity of data. He concludes that while the empirical results reported in his table 2 give "no indication that MPCs decline in higher income brackets...Precisely what they (MPCs) do is not illuminated very well by table 2". Blinder's inconclusive empirical results and the "it depends" nature of the theoretical results concerning the MPC seems to have contributed to the scant attention macroeconomists have given to the study of the effect of income distribution. Indeed Stiglitz (2013, p. 298-99) laments "the distribution of income is seldom mentioned in macroeconomics, and that's exactly the point."

Despite the influence of LC-PIH and the key work of Blinder, the early Keynesian intuition persists. Husby's (1971) little noted empirical work suggests the MPC of high income families is lower than that of low income families. Stiglitz (2013, p. 106-07) states "Moving money from the bottom to the top lowers consumption" and "in some sense, the entire shortfall in aggregate demand…today can be blamed on the extremes of inequality." The work of Saez et al (2015) on income distribution underscores the significance of Husby's empirical work and Stiglitz's view. Figure 1 (from Piketty and Saez, 2003) illustrates the top decile of the population's income share and reveals the dramatic change in income distribution from the relative stability in the years 1947-72 examined by Blinder to the dramatic surge since 1982. Combined the work of Husby, Stiglitz, Piketty, and Saez warrant renewed investigation of the macroeconomic effects of the income distribution.

FIGURE 1
THE TOP DECILE INCOME SHARE (1917-2015)



We pursue two leads in our investigation of income distribution matters. First, we develop the implications of the Keynesian intuition extending them beyond income distribution's impact on consumption to an explicit investigation of income distribution's effect on aggregate demand and the Macroeconomy. Second, we add to the empirical investigation of income distribution effects in two key ways. Like Blinder, our data is annual. But unlike his 1947-1972 time span in which income distribution was fairly stable, ours is for the years 1983-2011 in which there is much more variability in income distribution. Additionally, unlike Blinder who lacked consumption data at even the quintile level and relied on total consumption, we utilize decile specific consumption data that has become available after his work was done.

SECTION TWO: BASELINE MACRO MODEL

This section builds a baseline macro model with consumption as a function of income level. This baseline model will be used to judge the effects of including income distribution as a determinant of consumption in Section Three's model. Constructing this baseline model also establishes the notation and provides the explicit derivation of aggregate demand also used in the Section Three.

First the behavioral relations in the commodity market equations (1)-(4) and money market (5)-(7) are set out and then used to derive aggregate demand. Consumption is a function of current income and wealth as presented in equation (1).

$$c = c_0 + MPWw + MPC(y - t) \tag{1}$$

Where

1. c is total real consumption
2. c_0 is real survival consumption independent of wealth and income, $c_0 > 0$
3. MPW is the marginal propensity to consume out of wealth, $1 > MPW > 0$
4. w is real wealth
5. y is real income
6. MPC is the marginal propensity to consume out of income, $1 > MPC > 0$
7. t is total tax = $t_0 + yt_1$ where t_0 is lump sum tax and t_1 is the tax rate, $1 > t_1 > 0$

Investment is a function of the real rate of interest and exogenous factors as presented in equation (2).

$$i = i_0 - jR,\ i_0 > 0\ \&\ j > 0 \tag{2}$$

Where
1. i is total real investment
2. i_0 is autonomous investment spending
3. R is the real interest rate
4. j is parameter of investment sensitivity to interest rates $j > 0$

Government spending is a function of income and exogenous factors as presented in equation (3)

$$g = g_0 - MPGy,\ 1 > MPG > 0 \tag{3}$$

Where
1. g is total real government expenditures
2. MPG is the government's marginal propensity to spend out of real income, $1 > MPG > 0$
3. g_0 is autonomous government spending, this is set by government policy, $g_0 > 0$

Aggregate Expenditure, AE, given in equation (4) is found by adding equations (1) - (3) for c + i + g, substituting from the right-hand sides of (1) - (3), grouping like terms together, and defining AE_0 as (4.1):

$$AE_0 \equiv (c_0 + i_0 + g_0 - MPC\ t_0 + MPWw) \tag{4.1}$$

$$AE = AE_0 - jR + y\ (MPC\ (1 - t_1) - MPG) \tag{4}$$

Turning to the money market, we specify money demand and then money supply. Money demand is a function of real income and the rate of interest as presented in equation (5).

$$m^d = ey - vR,\ 1 > e > 0\ \&\ v \geq 0 \tag{5}$$

Where
1. m^d is total real demand for money balances
2. e is a parameter representing money demand's dependence on real income y, $1 > e > 0$
3. v is parameter of money demand sensitivity to the interest rate, $v \geq 0$

The real Money Supply is the ratio of nominal money supply M^s_0 to the price level P.

$$m^s = M^s_0/P_0 \tag{6}$$

Where
1. m^s is the total real supply of money
2. M^s_0 is the nominal money stock

3. P_0 is the price level

We derive expressions for y from the commodity equilibrium equation (7) and for R from the money market equilibrium equation (9).

$$AE = y \tag{7}$$

Substituting (4) for left-hand side of (7) and solving for y yields

$$y = \frac{AE_0 - jR}{1 - \text{MPC}(1-t_1) + MPG} \tag{8}$$

$$m^d = m^s \tag{9}$$

Substituting (5) for money demand and (6) for money supply into (9) and solving for R yields

$$R = \frac{ey - \frac{M_0^S}{P_0}}{v} \tag{10}$$

Substituting the right-hand side of equation (10) for R into equation (8) and solving for y yields equation (11) for aggregate demand y^d which expresses aggregate demand as a function of the parameters and exogenous variables

$$y^d = \frac{AE_0 + \left(\frac{j}{v}\right)\frac{M_0^S}{P_0}}{1 - \text{MPC}(1-t_1) + MPG + \left(\frac{je}{v}\right)} \tag{11}$$

Table 1 summarizes the results of Appendix A's analysis of the effects on aggregate demand due to changes in the exogenous variables. All the derivatives have the expected signs. For example the "+" in the c_0 column says that an increase in autonomous consumption c_0 increases (i.e. rightward shift) aggregate demand. An increase in the price level P_0 causes a decrease in y^d. The slope of aggregate demand is negative.

TABLE 1
STANDARD RESULTS

PARAMETER X	C_0	I_0	G_0	T_0	W	M^S_0	T_1	P_0	MULTIPLIER
$\delta y^d/\delta x$	+	+	+	-	+	+	-	-	$\frac{1}{1 - \text{MPC}(1-t_1) + MPG + \left(\frac{je}{v}\right)}$

Combining these aggregate demand results with an aggregate supply of the form $y^s = \overline{y} + \pi(P_0 - P^e)$ found in Mankiw (2016, p 410), equilibrium price level and output level both increase (decrease) in response to an increase (a decrease) in aggregate demand when aggregate supply is positively sloped ($\pi \neq 0$). For a vertical aggregate supply ($\pi = 0$) output is constant while the price level rises when aggregate demand rises. For the extreme Keynesian horizontal AS ($\pi = \infty$), output rises while price level is constant when aggregate demand increases.

SECTION THREE: REVISED MACRO MODEL - CONSUMPTION DEPENDS ON INCOME DISTRIBUTION)

This section brings income distribution into the model and analysis. While retaining all the relationships of Section Two, the consumption relationship is revised by making two assumptions. First, we assume there are two distinct income groups: one with high income and one with low income. Second, we assume that the Marginal Propensity to Consume (MPC) of the high income group MPC_h is less than the MPC_l of the low income group, $0 < MPC_h < MPC_l < 1$. This revision of the consumption relation (1) introduces a new parameter for income distribution α while retaining all the well-known exogenous variables and parameters determining aggregate demand, y^d. This parameter α allows examination of income distribution's effect on y^d while continuing to permit analysis of the effects of the well-known exogenous variables on y^d.

Household Consumption

We denote the consumption of the high income group by c_h and the consumption of the low income group c_l with total consumption as their sum:

$$c = c_h + c_l. \tag{12}$$

Letting α be $1 \geq \alpha \geq 0$, we identify αy as the proportion of total current income y going to the high income group and $(1-\alpha)y$ going to the low income group. Incorporating these assumptions we obtain c_h as equation (13) for the high income group and c_l as equation (14) for the low income group.

High Income Group

$$c_h = c_{0h} + MPW_h w_h - MPC_h t_{0h} + MPC_h (\alpha y(1-t_{1h})),\ c_{0h} > 0\ \&\ 1 > MPW_h\ \&\ MPC_h > 0,\ 1 > t_{1h} > 0 \tag{13}$$

Low Income Group

$$c_l = c_{0l} + MPW_l w_l - MPC_l t_{0l} + MPC_l ((1-\alpha) y (1-t_{1l})),\ c_{0l} > 0\ \&\ 1 > MPW_l\ \&\ MPC_l > 0,\ 1 > t_{1l} > 0 \tag{14}$$

Note for taxes we assume a progressive tax income system so that $0 < t_{1l} < t_{1h} < 1$; t_x is total tax, t_{0x} is lump sum tax, and t_{1x} is the tax rate with $1 > t_{1x} > 0$ where x is h or l respectively so that $t_h = t_{0h} + \alpha y t_{1h}$ and $t_l = t_{0l} + (1-\alpha)yt_{1l}$.

AE which is $c + i + g \equiv c_h + c_l + i + g$ becomes equation (16) by substituting the right-hand sides of (13), (14), (2), and (3) and grouping like terms given in identities (15.1)-(15.4):

$$c_0 \equiv c_{0h} + c_{0l} \tag{15.1}$$

$$MPWw \equiv MPW_h w_h + MPW_l w_l \tag{15.2}$$

$$\mu \equiv MPC_h t_{0h} + MPC_l t_{0l} \tag{15.3}$$

$$\Sigma \equiv MPC_h \alpha(1-t_{1h}) + MPC_l(1-\alpha)(1-t_{1l}) \tag{15.4}$$

$$AE = c_0 + i_o + g_0 - \mu + MPWw - jR + y\,\Sigma - y\,MPG \tag{16}$$

Redefining autonomous aggregate expenditures AE_0 as (17)

$$AE_0 \equiv (c_0 + i_0 + g_0 - \mu + MPWw) \tag{17}$$

the expression for AE given in equation (16) becomes equation (18)

$$AE = AE_0 - jR + y\,\Sigma - y\ MPG \tag{18}$$

Setting equation (18)'s AE equal to y, solving for y, and substituting the right-hand side of equation (10) for R yields equation (19)

$$y \ = \frac{AE_0 + MPWw - j\,\{ey - \frac{M_0^S}{P_0})\ /\ v\}}{1 - \Sigma + MPG} \tag{19}$$

Solving (19) for y gives the expression for aggregate demand y^d_1 presented in equation (20). The subscript 1 in y^d_1 is used *solely* to distinguish aggregate demand of this Section Three from that of the previous Section Two.

$$y^d{}_1 = \frac{AE_0 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1 - \Sigma + MPG + (\frac{ej}{v})} = \frac{AE_0 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1 - [MPC_h\alpha(1 - t_{1h}) + MPC_l(1 - \alpha)(1 - t_{1l})] + MPG + (\frac{je}{v})} \tag{20}$$

Comparing the Section Two's equation (11) for y^d and equation (20)'s revised y^d_1 reveals that though they have the same general form there is one significant difference. The income distribution parameter α appears and it appears only in the denominator. Specifically equation (20)'s denominator term $\mathrm{MPC_h}\alpha(1 - \mathrm{t_{1h}}) + \mathrm{MPC_l}(1 - \alpha)(1 - \mathrm{t_{1l}})$ replaces equation (11)'s denominator term$\mathrm{MPC}(1 - t_1)$.

As the forms for y^d and y^d_1 are the same and as both terms $\mathrm{MPC_h}\alpha(1 - \mathrm{t_{1h}}) + \mathrm{MPC_l}(1 - \alpha)(1 - \mathrm{t_{1l}})$ and $\mathrm{MPC}(1 - t_1)$ are greater than zero, the signs of the qualitative comparative static results presented in Table 2 Row 1 are the same as those reported in Table 1. Significantly this means that incorporating income distribution into the model leaves intact the standard qualitative results. (See Appendix B.)

Of most interest are the effects of the change in income distribution represented by changes in the parameter α. Recall that an increase in α means that the high income group receives a larger share of total income while the low income group a smaller. So for an increase in α income inequality increases.

In particular Table 2 reports three key effects of a change in the income distribution demonstrated in Appendix B. First, Row 1 states that $\frac{\delta y_1^d}{\partial \alpha} < 0$ (see equation (B11), Appendix B). This says that as the high income group receives a larger proportion of total income (i.e. α increases), aggregate demand decreases (y^d_1 shifts to the left). Second, $\frac{\delta(\frac{\delta y_1^d}{\delta P_0})}{\delta \alpha} > 0$ (see equation (B14). This means that $\frac{\delta y_1^d}{\delta P_0}$ rises as α rises and simultaneously its reciprocal, the slope of the aggregate demand $\frac{\delta P_0}{\delta y_1^d}$, falls. Since $\frac{\delta P_0}{\delta y_1^d}$ is negative, a falling $\frac{\delta P_0}{\delta y_1^d}$ means a steepening aggregate demand, y^d_1, making aggregate demand more price-level inelastic. Third $\frac{\delta \frac{\delta y_1^d}{\delta c_0}}{\delta \alpha} < 0$ (see (B16)). This says that as α rises the effect of a change in autonomous spending such as autonomous consumption is reduced, i.e. an increase in c_0 causes a smaller rightward shift in y^d.

TABLE 2
COMPARATIVE STATIC RESULTS

PARAMETER X ONLY IN NUMERATOR OF EQ (20)		C_0	I_0	G_0	T_{0H}	T_{0L}	W_H	W_L	M^S_0	P_0				MULTIPLIER $\frac{1}{1-\Sigma+MPG+(\frac{EJ}{V})}$
Parameter X only in denominator of (20)											t_{1h}	t_{1l}	α	
1	$\delta y^d/\delta x$	+	+	+	-	-	+	+	+	-	-	-	-	
2	$\delta(\delta y^d/\delta x)/\delta\alpha$	-	-	-			-	-	-	+				
3	δ(multiplier)/δx												-	

The nexus of these three results reveals that income distribution matters. It matters in two significant but different ways. First, consider the *direct effect* of a change in α presented by $\frac{\delta y_1^d}{\partial\alpha} < 0$. This derivative says even if autonomous spending, tax rates, and the money supply (exogenous variables) are unchanged, rising income inequality by itself decreases aggregate demand shifting it to the left. This falling aggregate demand due to a change in income distribution provides a headwind on the economy. Furthermore, this headwind is reinforced. The steepening aggregate demand due to increased α revealed by $\frac{\delta(\frac{\delta P_0}{\delta y_1^d})}{\delta\alpha} < 0$ indicates that a shift in the aggregate demand to the left results in an even greater decrease in income than if income distribution had been constant.

Distinct from this *direct effect* of income redistribution on aggregate demand, there is a second, *indirect effect* of income distribution. This second effect is activated by a change in autonomous spending as indicated by $\frac{\delta(\frac{\delta y_1^d}{\delta c_0})}{\delta\alpha} < 0$. This derivative indicates that an increase in α reduces the *magnitude* of the effect of c_0 on y^d even while the *direction* of the effect (sign of the derivatives of $\delta y^d/\delta c_0 > 0$) remains as expected. Appendix B reveals a common thread driving the change in magnitude. It is the multiplier $\frac{1}{1-\Sigma+MPG+(\frac{ej}{v})}$ which falls when α increases.

Equation (21) shows that a change in y^d_1 (see (B2)) due to a change in c_0 results in the multiplier expression (similarly for a change in i_0 see (B2.2)).

$$\frac{\delta y_1^d}{\delta c_0} = \frac{1}{1-\Sigma+MPG+(\frac{ej}{v})} \equiv \text{multiplier}. \tag{21}$$

Differentiating (21) with respect to with respect to α yields equation (22) (see (B19)).

$$\frac{\delta Multiplier}{\delta\alpha} = \frac{MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})}{(1-\Sigma+MPG+(\frac{ej}{v}))^2} \rightarrow \frac{(-)}{(+)} < 0. \tag{22}$$

With its denominator positive, the sign of equation (22) is determined by its numerator. With $MPC_h < MPC_l$ by assumption and with a progressive income tax's $t_{1h} > t_{1l}$, the numerator of equation (22) is negative. When equation (22) is negative, an increase in α leads to a smaller multiplier. Hence an increase in the high income group's share of total income reduces the shift in aggregate demand due to a change in c_0.

The significance of this second avenue for income distribution's effect is emphasized most notably in the context of recovery from a recession, in particular the 2008-09 great recession. Rising income inequality (recall Figure 1) reduces the multiplier thus adding another headwind, slowing the recovery from recession. Specifically even as autonomous spending recovers, the reduction in the multiplier reduces the increase in aggregate demand arising due to the recovery in autonomous spending. Thus in addition to all the other factors impeding the recovery, the high income group's rising share of income documented by Saez (2015) adds to the list of headwinds hindering the recovery and contributes to the explanation of the record length of time for the recovery from the great recession 2008-09.

Interestingly, the falling multiplier also reduces the ability of policy to stimulate the economy. The effect on aggregate demand of an increase in money stock and hence the economy is found by differentiating equation (20)'s expression for y^d_1 with respect to M^s_0 giving equation (23), (see Appendix (B3).

$$\frac{\delta y_1^d}{\delta M_0^s} = \left(\frac{1}{1-\Sigma+MPG+\left(\frac{\mathrm{ej}}{\mathrm{v}}\right)}\right)\left(\frac{j}{vP_0}\right) \tag{23}$$

(23)'s first bracketed term is once again the multiplier which is reduced by the increase in α as the interpretation of equation (22) established. In this respect the extensive quantitative easing conducted by the Fed would not have as strong an effect as it would if income distribution had remained unchanged.

Lastly the falling multiplier also suggests that a given increase in government spending would have less ability to increase aggregate demand. Noticing that g_0 enters y^d_1 as does c_0, equation (21) by extension becomes equation (24) (see (B2.3)).

$$\frac{\delta y_1^d}{\delta g_0} = \frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)} > 0 \tag{24}$$

The effect of an increase in income inequality reducing the multiplier reduces the potency of a government spending fiscal policy. Other things equal the dosage of a government spending fiscal policy needed to achieve a desired shift in aggregate demand would need to be larger than if income distribution had remained unchanged.

SECTION 4: EMPIRICAL RESULTS

Motivation

Our empirical work supports the key innovation in Section Three's model that the high income group's MPC is less than the low income group's MPC. In the early 1970's Blinder (1975) was unable to find reliable estimates of the MPC in large part due to the lack of quintile specific consumption data. His empirical work was also hindered by a paucity of annual data (24 years) relative to the number of estimated coefficients (8), considerable collinearity, and relatively stable income distribution in his data's time period. Husby (1971) likewise was hampered by a lack of data on consumption and disposable income for individual households (panel data). Yet from his data he was able to estimate a nonlinear consumption function suggesting the marginal propensity to consume of high income families is lower than that of low income families.

The Data

Table 3 presents the results of separate regressions on each of the ten income deciles as well as two additional regressions on groupings aggregating deciles (e.g., Bottom 70 and Top 20). Figure 3 presents a visual depiction of the MPCs by decile. For each regression, Consumption is the dependent variable and the independent variable is Disposable Income. Consumption and Disposable Income data were collected from the Consumer Expenditure Survey (CEX) from 1984-2011.

Regression Results

From this work, four general conclusions may be drawn. First, in general, the regression coefficients are significant and suggest that the MPC of the high income group is less than the MPC of the low income group. The bar graph of Figure 3 (see below) drawn from the results presented in Table 3 (see below) provides a visual report of the MPCs going (left to right) from the lowest decile (first) to the highest decile (tenth). The pattern depicted in the bar graph appears to support the assumption that the MPC of the high income group is smaller than that of the low income group.

Second, we compare the MPC of the top decile, which represents Saez's top income group, to the remaining deciles. Saez reports that the share of income (including capital gains) for this top group has grown from 32.31% in 1953 to 50.47% in 2015. In Table 3, when comparing the MPC of the top decile to the bottom nine deciles, the top decile's MPC is less than the MPC of eight of the bottom nine deciles. The eighth decile is the one exception. We suggest that this eighth decile may represent the "wealthy hand-to-mouth (HtM)" or households with a highly disproportionate level of illiquid assets (e.g., retirement accounts) when compared to liquid wealth such as cash (Kaplan, Violante, Weidner, 2014). Viewed as "peculiar" in nature, HtM behavior is often difficult to capture in traditional spender-saver modeling and results can be misleading. Kaplan (et al, 2014) argues that wealthy HtM households are financially fragile, peak at an average age in the early 40s based upon head-of-household data, and their HtM status is more transient than other households. Therefore, it is possible that wealthy HtM may be able to utilize illiquid assets, if needed, to protect against large negative shocks to disposable income, exacerbating the financial fragility of the group and leading to misleading results when employing simpler MPC models. When creating a household model based on two assets (e.g., low-return liquid assets and high –return illiquid assets), Kaplan discovered this group displays large MPCs out of transitory income but small MPCs out of news concerning future income. Perhaps the net effect of these two forces on the MPC offer some explanation of the very low estimate of the 8^{th} decile's MPC.

Third, again from the point of view that the high income group is the top decile (the tenth decile) and the low income group is the bottom nine deciles, Table 3 shows that the MPC of the high income group is less than the average MPC of the low income group. Specifically the MPC of the top decile is 0.514 while the average MPCs of the bottom nine deciles is 0.651. This suggests that taking $100 from the bottom 90% of the income distribution and giving it to the top 10% would reduce consumption spending on average by $13.70 for every $100 transferred. If one draws the dividing line between high and low income groups at the 8^{th} decile and compares the average MPC of deciles 1-7 to that of 8-10, one finds the divergence is even greater. The average MPC of deciles 8-10 is 0.434 while that of deciles 1-7 is 0.724. In this case taking $100 from the low income group and giving it to the high income group would reduce consumption spending by $29.00 for every $100 transferred.

TABLE 3
RESULTS OF LEAST SQUARES ANALYSIS FOR CONSUMPTION[a]

	MPC by Deciles[b]												
Decile	**Decile1**		**Decile2**		**Decile3**		**Decile4**		**Decile5**		**Decile 6**	
(Constant)	6.94 (75.2)	***	-8.48 (78.4)	***	70.43 (93.1)	**	-21.54 (108.5)	***	7.92 (136.9)	***	52.49 (142.1)	**
Disposable Income	0.68 (0.08)	***	0.67 (0.08)	****	0.79 (0.16)	**	0.85 (0.23)	****	0.77 (0.23)	***	0.57 (0.24)	***
N	26		26		26		26		26		26	
R-Squared	0.822		0.808		0.780		0.677		0.474		0.282	
p-value (F)	0.000		0.001		0.024		0.000		0.003		0.026	
Durbin Watson	2.380		2.315		2.143		2.346		2.510		2.413	
Decile	**Decile7**		**Decile8**		**Decile9**		**Decile10**		**Top 20**		**Bottom 70**	
(Constant)	46.302 (142.1)	***	-5.104 (180.1)	***	10.962 (203.6)	**	75.076 (86.9)		-562.42 (171.1)	**	-46.733 (632.7)	**
Disposable Income	0.738 (0.31)	**	0.211 (0.12)	*	0.577 (0.01)	***	0.514 (0.18)	***	0.579 (0.01)	**	0.687 (0.20)	***
N	26		26		26		26		26		26	
R-Squared	0.480		0.189		0.981		0.295		0.887		0.514	
p-value (F)	0024		0.098		0.000		0.009		0.000		0.002	
Durbin Watson	2.617		2.258		2.411		2.411		2.005		2.413	

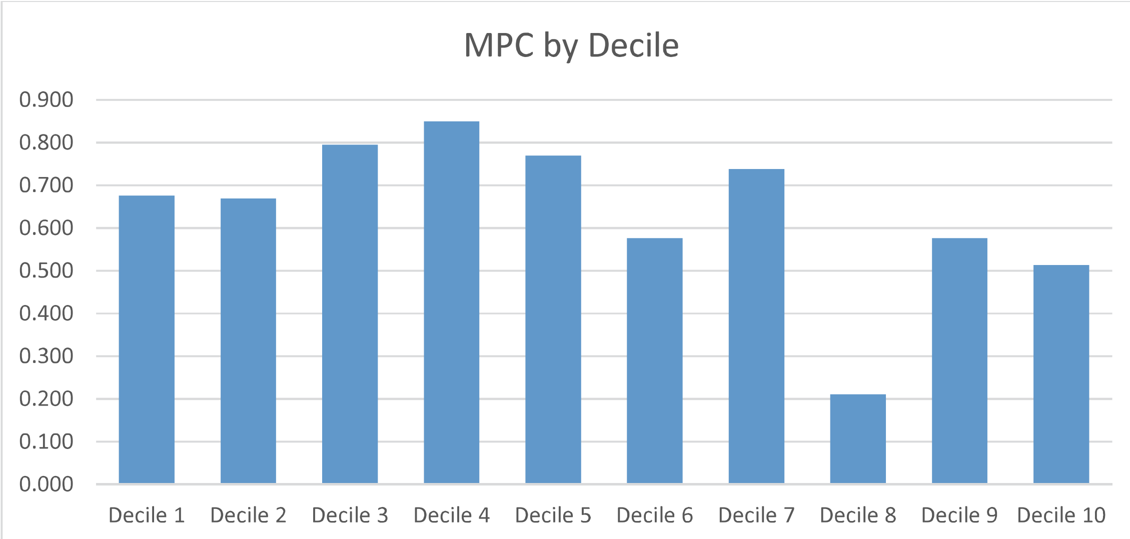
[a] Standard errors in parentheses.
[b] Corrected for heteroscedasticity and serial correlation
****p<0.001, ***p<0.010, **p<0.050, *p<0.100

Fourth, we group the top two deciles (nine and ten, e.g., Top 20) as the top income group and the bottom seven deciles (one through seven, e.g., Bottom 70) as the bottom income group. Running regressions of each group's consumption on disposable income, we find significant estimates for each group's MPC. The high income group's MPC of 0.579 is less than the low income group's MPC of 0.687.

Several features of the regressions are curious. First, the eighth decile's extremely low R^2, p, and MPC stand out in marked contrast to the behavior in the other deciles. The eighth decile could be viewed as a cusp group most impacted by saving for college and retirement. Ashford (2016) reports that high-income parents are assuming a large portion of their children's' college debt and choosing to delay retirement. As noted by Gersema (2016), the U.S. Government Accountability Office calculates that higher-income parents may be experiencing a snowball effect: middle age parents and seniors held approximately \$400 billion in college debt and roughly one quarter of that was their children's college debt. By employing tactics such as delaying retirement, liquefying assets, and securing secondary mortgages, this cusp groups' actions may be depicted in the MPC represented in Table 3.

Second, we anticipated that the MPC of the lowest deciles would be much closer to one as poverty forced the members of these deciles to spend nearly every additional dollar of income. However, Ashford (2016) suggests that parents in the lowest deciles are also attempting to take on the student loan obligations of their children as well.

FIGURE 3
MPC BY DECILE



SECTION 5: SUMMARY AND CONCLUSIONS

Our model of aggregate demand incorporates income distribution by modeling consumption as the sum of the consumption of two income groups, the high income group's consumption plus the low income group's consumption, in which the high income group's MPC is lower than the low income group's MPC. Analysis of our aggregate demand model reveals that a change of income distribution can affect aggregate demand in two significant ways. First, rising income inequality decreases aggregate demand hence providing a headwind on the economy's GDP. This GDP headwind is reinforced by the steepening of the aggregate demand due to rising income inequality. Second, rising income inequality can hinder aggregate demand's recovery from a recession. Specifically rising income inequality leads to smaller increases in aggregate demand when autonomous aggregate expenditures rise (whether from an increase in investment or consumption) or government runs expansionary policy (whether by raising autonomous government spending or the money supply). The key is the reduction in the multiplier caused by a less equal income distribution.

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APPENDIX A

Differentiating (11) with respect to the components of AE_0, w, M^s_0, and P_0 yields equations (A1) and (A6) whose results are presented in Table 1. These are the usual, standard results. For convenience equations (11) and (4.1) are repeated here.

$$y^d = \frac{AE_0+(\frac{j}{v})\frac{M_0^S}{P_0}}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{je}{v})} \tag{11}$$

$$AE_0 \equiv (c_0 + i_0 + g_0 - \mathrm{MPC}\ t_0 + \mathrm{MPWw}) \tag{4.1}$$

Shifting y^d: Effect of a changes in AE_0, Ms0, and t_1 on y^d_2

$$\delta y^d/\delta c_0 = \frac{1}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})} > 0 \text{ (}c_0\text{ only appears in numerator of (9.1), so to for } i_0,\ g_0\text{)} \tag{A1}$$

$$\delta y^d/\delta t_0 = \frac{-MPC}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})} < 0 \quad \text{(as } t_0 \text{ only in numerator of (9.1))} \tag{A2}$$

$$\delta y^d/\delta w = \frac{MPW}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})} > 0 \text{ (as w only in numerator of (9.1))} \tag{A3}$$

$$\delta y^d/\ \delta M^s_0 = \frac{\frac{j}{vP_0}}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})} > 0 \quad \text{(as } M^s_0 \text{ only in numerator of (9.1))} \tag{A4}$$

$$\delta y^d/\delta t_1 = \frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})\ (\delta\{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{je}{v})\}/\delta t_1)}{(\ 1-\mathrm{MPC}(1-t_1)+MPG+(\frac{je}{v})\ )^2} = \frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})\ MPC}{(\ 1-\mathrm{MPC}(1-t_1)+MPG+(\frac{je}{v})\ )^2} < 0 \tag{A5}$$

(t_1 only in denominator of (11))

Slope of y^d: Effect on y^d of a change in P_0

$$\delta y^d/\delta P_0 = \frac{-(\frac{j}{v})\frac{M_0^S}{P_0^2}}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{je}{v})} < 0 \quad \text{(as } P_0 \text{ only appears in numerator of (11))} \tag{A6}$$

Note for future reference that the factor $\frac{1}{1-\mathrm{MPC}(1-t_1)+MPG+(\frac{\mathrm{ej}}{\mathrm{v}})}$ enters each of (A1) – (A6).

APPENDIX B

For convenience recall the following relationships from Section 3.

$$y^d{}_1 = \frac{AE_0 + (\frac{j}{v})\frac{M_0^S}{P_0}}{1 - \Sigma + MPG + (\frac{ej}{v})} \tag{20}$$

$$c_0 \equiv c_{oh} + c_{ol} \tag{15.1}$$

$$MPWw \equiv MPW_h w_h + MPW_l w_l \tag{15.2}$$

$$\mu \equiv MPC_h t_{0h} + MPC_l t_{0l} \tag{15.3}$$

$$\Sigma \equiv MPC_h \alpha(1\text{-}t_{1h}) + MPC_l(1\text{-}\alpha)(1\text{-}t_{1l}) \tag{15.4}$$

$$AE_0 \equiv (c_0 + i_0 + g_0 - \mu + MPWw) \tag{17}$$

Shifting y^d

Effect of change in c_0 on $y^d{}_1$

$$\frac{\delta y_1^d}{\delta c_0} = \frac{\left\{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta AE_0}{\delta c_0}\right\} - \{AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]}{\delta c_0}\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} = \frac{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta AE_0}{\delta c_0}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2}$$ as c_0 appears neither in $(\frac{j}{v})\frac{M_0^S}{P_0}$ nor in $\left[1 - \Sigma + MPG + \left(\frac{je}{v}\right)\right]$ (B1)

(B1) becomes (B2) by noting $\frac{\delta AE_0}{\delta c_0} = 1$ and canceling a $\left[1 - \Sigma + MPG + \left(\frac{je}{v}\right)\right]$ from numerator and denominator

$$\frac{\delta y_1^d}{\delta c_0} = \frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)} > 0 \tag{B2}$$

Note that since i_0, g_0, w, t_{0h}, and t_{0l} all enter (20)'s numerator in a way similar to c_0, the same derivation used for c_0 produces the signs $\frac{\delta y_1^d}{\delta i_0} > 0$, $\frac{\delta y_1^d}{\delta g_0} > 0$, $\frac{\delta y_1^d}{\delta w} > 0$, $\frac{\delta y_1^d}{\delta t_{0h}} < 0$, and $\frac{\delta y_1^d}{\delta t_{0l}} < 0$ where these derivatives are denoted as (B2.2) through (B2.6) respectively.

Note also that the term $\frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)}$ enters all these derivatives a fact used later when examining the effects of a change in α.

Effect of a changes in $M^s{}_0$ on $y^d{}_1$

The effect of an increase money stock on aggregate demand and hence the economy is found by differentiating equation (20)'s expression for $y^d{}_1$ with respect to $M^s{}_0$ resulting in

$$\frac{\delta y_1^d}{\delta M_0^S} = \left(\frac{1}{1-\Sigma+MPG+\left(\frac{ej}{v}\right)}\right)\left(\frac{j}{vP_0}\right) > 0 \tag{B3}$$

Note also that the term $\frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)}$ enters this derivative (B3).

Effect of a changes in tax rates t_{ih} and t_{il} on y^d_1

The effect of an increase in the tax rate on the high group t_{1h} on aggregate demand and hence the economy is found by differentiating equation (20.1)'s expression for y^d_1 with respect to t_{1h} resulting in $\delta y_1^d/\delta t_{1h}=$ (B4)

$$\frac{\left\{\left[1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta\left(AE_0+\left(\frac{j}{v}\right)\frac{M_0^s}{P_0}\right)}{\delta t_{1h}}\right\}-\left\{\left[AE_0+\left(\frac{j}{v}\right)\frac{M_0^s}{P_0}\right]\left[\frac{\delta\left(1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+\left(\frac{je}{v}\right)\right)}{\delta t_{1h}}\right]\right\}}{\left[1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+\left(\frac{je}{v}\right)\right]^2}$$

(B4) becomes (B5) as t_{1h} does not appear $AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}$

$$\delta y_1^d/\delta t_{1h}=\frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})\ \delta\{1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+(\frac{je}{v})\}/\delta t_{1h})}{(\ 1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+(\frac{je}{v})\ )^2}=$$

$$\frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})\ (-(MPC_h\alpha)\delta(-t_{1h})/\delta t_{1h})}{(\ 1-[MPC_h\alpha(1-t_{1h})+MPC_l(1-\alpha)(1-t_{1l})]+MPG+(\frac{je}{v})\ )^2} \qquad \text{(B5)}$$

(B5) becomes (B6) by recalling $\Sigma \equiv MPC_h\alpha(1-t_{1h})+\ MPC_l(1-\alpha)(1-t_{1l})$

(B6) $$\delta y_1^d/\delta t_{1h}=\frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})(MPC_h\alpha)}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2}\rightarrow\ \frac{(-)[+][+]}{(+)}<0 \qquad \text{(B6)}$$

By similar work for δt_{1l}, (B7) gives the effect of a change of the low income group's tax rate:

$$\delta y_1^d/\delta t_{1l}=\frac{-(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})(MPC_l(1-\alpha))}{(\ 1-\Sigma+MPG+(\frac{je}{v})\ )^2}\ <0 \qquad \text{(B7)}$$

An increase in either tax rate shifts aggregate demand to the left
Note also that the term $\frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)}$ enters (B6) and (B7).

Slope of y^d_1: Effect on y^d_1 of a change in P_0

Differentiating equation (20)'s expression for y^d_1 with respect to P_0 which yields equation (B8)

$$\frac{\delta y_1^d}{\delta P_0}=\frac{\left\{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})}{\delta P_0}\right\}-\{AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]}{\delta P_0}\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} \qquad \text{(B8)}$$

(B8) becomes (B9) as P_0 only appears in (20)'s numerator term $(\frac{j}{v})\frac{M_0^S}{P_0}$

(B9) $$\frac{\delta y_1^d}{\delta P_0}=\ \frac{-(\frac{j}{v})\frac{M_0^S}{P_0^2}}{1-\Sigma+MPG+(\frac{je}{v})}<0 \qquad \text{(B9)}$$

Equation (B9) shows that the slope of the aggregate demand is negative as equation (B9) is the reciprocal of the slope of the aggregate demand. Note again the common thread: the term $\frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)}$ enters (B9).

Effects of a change in income distribution on y^d_1

Direct Effects of a change in income distribution on y^d_1: shift and slope shift: demonstration that $\frac{\partial y_1^d}{\partial \alpha} < 0$

Differentiating equation (20)'s expression for y^d_1 with respect to α which yields equation (B10)

$$\frac{\partial y_1^d}{\partial \alpha} = \frac{\left\{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]\frac{\delta(AE_0+(\frac{j}{v})\frac{M_0^S}{P_0})}{\delta\alpha}\right\} - \{AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]}{\delta\alpha}\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} \tag{B10}$$

As α is only found in Σ, (B10) becomes (B11)

$$\frac{\partial y_1^d}{\partial \alpha} = \frac{-\{AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{\frac{\delta[-\Sigma]}{\delta\alpha}\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} = \frac{\{AE_0+\left(\frac{j}{v}\right)\frac{M_0^S}{P_0}\}\{MPC_h(1-t_{1h})-MPC_l(1-t_{1l})\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} \rightarrow \frac{\{+\}\{-\}}{+} < 0 \tag{B11}$$

since $\frac{\delta[\Sigma]}{\delta\alpha} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}) < 0$ for the standing assumptions that $MPC_h < MPC_l$ and income tax system is progressive $t_{1h} > t_{1l}$.

Slope: demonstration that $\frac{\delta(\frac{\delta y_1^d}{\delta P_0})}{\delta\alpha} > 0$

The effect of α on the slope of y^d_1 is found by differentiating (B9)'s expression for $\delta y^d_1/\delta P_0$ with respect to α. Thus we have (B12) as α only appears in (B4)'s denominator via Σ

$$\frac{\delta(\frac{\delta y_1^d}{\delta P_0})}{\delta\alpha} = \frac{-\{-(\frac{j}{v})\frac{M_0^S}{P_0^2}\}\{\frac{\delta\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]}{\delta\alpha}\}}{(1-\Sigma+MPG+(\frac{je}{v}))^2} \tag{B12}$$

Noting that α only appears in Σ, (B12) becomes (B13)

$$\frac{\delta(\frac{\delta y_1^d}{\delta P_0})}{\delta\alpha} = \frac{-\{-(\frac{j}{v})\frac{M_0^S}{P_0^2}\}\{\frac{\delta(-\Sigma)}{\delta\alpha}\}}{(1-\Sigma+MPG+(\frac{je}{v}))^2} \tag{B13}$$

(B13) becomes (B14) by recalling that $= \delta\Sigma/\delta\alpha = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}) < 0$.

$$\frac{\delta(\frac{\delta y_1^d}{\delta P_0})}{\delta\alpha} = \frac{-\{(\frac{j}{v})\frac{M_0^S}{P_0^2}\}\{MPC_h(1-t_{1h})-MPC_l(1-t_{1l})\}}{(1-\Sigma+MPG+(\frac{je}{v}))^2} = \frac{-\{(+)(+)\}\{-\}}{+} > 0 \tag{B14}$$

Indirect Effect of a change in income distribution (↑α) on y^d_1

Differentiating equation (B2)'s expression for $\frac{\delta y_1^d}{\delta c_0}$ with respect to α yields equation (B15)

$$\frac{\delta\frac{\delta y_1^d}{\delta c_0}}{\delta\alpha} = \frac{\left\{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]\left(\frac{\delta 1}{\delta\alpha}\right)\right\} - (1)\left\{\frac{\delta\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]}{\delta\alpha}\right\}}{\left[1-\Sigma+MPG+\left(\frac{je}{v}\right)\right]^2} \quad \text{(B15)}$$

(B15) become (B16) by noting $\frac{\delta 1}{\delta\alpha}$ = 0 and that $\frac{\delta\Sigma}{\delta\alpha} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}) < 0$

$$\frac{\delta\frac{\delta y_1^d}{\delta c_0}}{\delta\alpha} = \frac{MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})}{(1-\Sigma+MPG+(\frac{ej}{v}))^2} \text{ and signs as } \frac{(-)}{(+)} < 0. \quad \text{(B16)}$$

The effects of increased inequality ↑α on magnitude on y^d_1 shifts and y^d_1 slope

Notably the work of Appendix B reveals the magnitude of a change in income distribution's effect on aggregate demand is reduced due to income distribution's impact on the multiplier. Inspecting Appendix B's relationships (B2) $\frac{\delta y_1^d}{\delta c_0}$ (as well as (B2)'s related derivatives(B2.2)-(B2.6) for $\frac{\delta y_1^d}{\delta i_0}$, $\frac{\delta y_1^d}{\delta g_0}$, $\frac{\delta y_1^d}{\delta w}$, $\frac{\delta y_1^d}{\delta t_{0h}}$, and $\frac{\delta y_1^d}{\delta t_{0l}}$), (B3) $\frac{\partial y^d}{\delta M_0^s}$, (B6) & (B7) $\delta y^d/\delta t_{1h \text{ or } l}$, and (B9) $\delta y^d_1/\delta P_0$ one sees that each contains the term

$$\frac{1}{1-\Sigma+MPG+\left(\frac{je}{v}\right)} \quad \text{(B17)}$$

the multiplier. Since α enters the (B17) via the denominator and only through Σ, the effect of α on Σ determines the effect of income distribution on the multiplier. Now Σ falls as α rises as seen by recalling equation (16.4) $\Sigma \equiv MPC_h\alpha(1\text{-}t_{1h}) + MPC_l(1\text{-}\alpha)(1\text{-}t_{1l})$ and differentiating Σ with respect to α yields (B18).

$$\frac{\delta[\Sigma]}{\delta\alpha} = MPC_h(1-t_{1h}) - MPC_l(1-t_{1l}); \quad \text{(B18)}$$

which is less than zero because $MPC_h < MPC_l$ by assumption and $t_{1h} > t_{1l}$ in a progressive tax system. Hence an increase in α reduces Σ which results in the multiplier falling as shown in (B19)

$$\frac{\delta Multiplier}{\delta\alpha} = \frac{MPC_h(1-t_{1h}) - MPC_l(1-t_{1l})}{(1-\Sigma+MPG+(\frac{ej}{v}))^2} \rightarrow \frac{(-)}{(+)} < 0. \quad \text{(B19)}$$

The multiplier falls when income distribution becomes more unequal i.e. ↑α.
So in each of (B2) $\frac{\delta y_1^d}{\delta c_0}$ (as well as (B2)'s related derivatives (B2.2) through (B2.6) for $\frac{\delta y_1^d}{\delta i_0}$, $\frac{\delta y_1^d}{\delta g_0}$, $\frac{\delta y_1^d}{\delta w}$, $\frac{\delta y_1^d}{\delta t_{0h}}$, and $\frac{\delta y_1^d}{\delta t_{0l}}$), (B3) $\frac{\partial y^d}{\delta M_0^s}$, (B6) & (B7) $\delta y^d/\delta t_{1h \text{ or } l}$, and (B9) $\delta y^d_1/\delta P_0$ the magnitude of income distribution's effect is reduced by increased α as is confirmed by (B16) $\frac{\delta(\frac{\delta y}{\delta c_0})}{\delta\alpha} < 0$ as well as (B14) $\frac{\delta(\frac{\delta y^d}{\delta P})}{\delta\alpha} > 0$ since the slope of aggregate demand is the reciprocal of (B14).