Revisiting the Marriage of Monopolistic Competition and Factor Proportions Theories

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This paper reconsiders the factor proportions-driven model of trade under a monopolistic competition framework when cost functions are non-homothetic. The pattern of trade is fully analyzed for a two-country, two-sector, and two-factor monopolistic competition model with transport costs. The main results of this transformed cost assumption that differ from previous literature include: (a) the average firm size in relatively capital-abundant countries is smaller; (b) controlling for industry demand, capital-abundant countries support a larger number of varieties in equilibrium; and (c) capital-abundant countries use more capital-intensive techniques in every sector. This model also generates many features of modern trade that cannot be solely explained by traditional horizontal differentiation models using a single factor or even two factors, assuming a homothetic cost function: (1) capital-abundant countries export higher priced varieties; (2) varieties produced using higher capital-intensive techniques have higher prices; (3) capital accumulation leads to increased relative prices over time; and (4) the higher priced manufacturing goods sold by richer countries also capture larger market shares relative to lower priced exports.

Keywords: Heckscher-Ohlin, non-homothetic cost, monopolistic competition, extensive and intensive margins

INTRODUCTION

The Heckscher-Ohlin (H-O) and monopolistic competition models are two of the most important pillars explaining international trade. Integrating the two models provides a more robust estimation of trade patterns than any of these models alone (Romalis, 2004). Most of the literature that integrates Dixit and Stiglitz's (1977) model of monopolistic competition with a two-factor H-O model assumes a homothetic cost function where both fixed costs and marginal costs use labor and capital in the same proportion. This assumption is highly stylized, as Helpman and Krugman (1985, pp 143) note "it [the cost function] implies that the relative factor intensity in activities that generate fixed costs are the same as in activities that generate variable costs. Thus, it does not allow for the existence of inputs, like buildings, sights, large scale equipment which generate mainly fixed costs and contribute negligibly to variable costs."

In this paper, I integrate the H-O and the monopolistic competition models using a cost structure that relaxes this rigidity by assuming fixed costs are more capital intensive.[1] The key difference from previous papers that integrate the H-O model with monopolistic competition models (for example Helpman and Krugman (1985) and Romalis (2004)) is how the factor proportions enter the model. These previous papers, working with two-factor monopolistic competition models, assume that the fixed cost for each industry uses factors in the same proportion as the variable costs do. In the current paper, however, I assume that the fixed cost uses relatively more capital.[2] This means that the capital-abundant country has a comparative advantage in producing more varieties of goods in each sector. Also, the capital-abundant North tends to specialize in the high markup sectors, where it produces many varieties in small quantities and charges a higher price for each variety due to higher marginal costs driven by higher labor costs.[3]

The main results of this transformed cost assumption that differ from previous literature include inter-country differences in the number of firms, their average sizes, and production techniques: (a) the average firm size in relatively capital-abundant countries is smaller; (b) controlling for demand, capital-abundant countries support a larger number of varieties in equilibrium; and (c) capital-abundant countries use more capital-intensive techniques in every sector. This model also predicts opposite effects of trade liberalization on these three variables across countries that are relatively capital and labor abundant. Additionally, the current model also generates many salient features of modern trade that cannot be explained solely by traditional horizontal differentiation models using a single factor or even two factors, assuming a homothetic cost function. The model predictions include: (1) high wage countries with higher capital endowments export relatively higher priced varieties; (2) countries that use more capital-intensive techniques in a sector charge higher relative prices for their products; (3) countries that accumulate capital increase their relative prices over time in all sectors; and (4) not only do rich countries export manufactured goods at higher prices but they also export larger quantities of many of these products.[4]

Empirical work has shown that differences in output mix (Romalis, 2004) and techniques (Davis and Weinstein, 2001; Xiang, 2007) both account for differences in countries' factor endowments and that the latter channel outweighs the former when endowment changes occur in a country (Blum, 2010; Nishioka, 2012). Similar to Romalis (2004), this paper allows the breakdown of factor price equalization due to the presence of trade costs in a monopolistically competitive framework, while allowing for a more realistic technology. This not only generates the differences in techniques observed across countries, but also allows average firm size to vary across countries, while correctly predicting the patterns of comparative advantage that are consistent with the factor endowment differences. Thus, this simple transformation can potentially update many of the results in applications of the monopolistic competition trade literature using an H-O framework---including the heterogeneous firm trade literature, the economic geography literature, and the home market effect literature---and improve the empirical performance of these models in lieu of its more realistic cost assumption.

The paper is organized as follows: In section 2.1, I develop the model, which first solves for the closed economy equilibrium and then moves onto an open economy equilibrium in section 2.2. Section 2.3 puts forth the predictions of the model and section 3 concludes.

A MODEL OF HORIZONTAL PRODUCT DIFFERENTIATION WITH NON-HOMOTHETIC COST

I consider here a two-country world with two factors and two differentiated product industries, assuming that the countries are identical in terms of preferences and technologies. The countries differ in terms of relative factor endowments, which are allowed to move freely across the sectors in a given country, but not across countries.[5]

The Closed Economy

In this section, I solve for firm sizes, number of firms, and techniques used in an industry for a closed economy with labor and capital endowments of L and K.

Representative Consumer's Problem

Demand in the economy is generated by a representative consumer whose utility depends on consuming outputs of n_i different varieties from each of the two differentiated product industries $i = 1,2,$

which differ in the degree of differentiability of their products. I assume that the upper tier of the utility function is Cobb-Douglas,

$$U = M_1^{b_1} M_2^{b_2}, \tag{1}$$

where the share of income spent on each industry i is given by b_i. The CES sub-utility function for each industry i is,

$$M_i = \left(\sum_{j=1}^{n_i} q_{ij}^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{2}$$

where M_i denotes the composite consumption from n_i varieties supplied by industry i. The quantity of each variety j consumed is denoted by q_{ij}. Varieties in both the industries are produced by monopolistically competitive firms and the degree of product differentiation between the varieties in each industry is given by the respective industry's elasticity of substitution σ_i. In equilibrium, all firms are identical, and each variety is produced by only one monopolistically competitive firm. Therefore, I drop the index j for each individual firm in an industry.

If Y is the total income of the economy, then from (1), $b_i Y$ is the total expenditure on industry i commodities. Denoting by p_i, the price paid for each variety, the maximization of the sub-utility (2) yields the following demand for each firm,

$$q_i^D = b_i Y p_i^{-\sigma_i} G_i^{\sigma_i - 1}, \tag{3}$$

where G_i is the price index defined for the composite consumption index in industry i. With n_i symmetric firms in industry i, we have,

$$G_i^{1-\sigma_i} = n_i p_i^{1-\sigma_i}. \tag{4}$$

Equilibrium in an Industry i

Representative Firm Problem and Technology. There are two factors of production, labor and fixed capital. Capital is used only in fixed costs and is paid a rental rate r. Labor is used only as variable input in production and gets a wage w. The labor requirement per unit of output in industry i is c_i, and capital required to set up the firm is F_i. This implies the cost function,[6,7]

$$C_i = \underbrace{c_i w q_i}_{\text{Variable Costs}} + \underbrace{r F_i}_{\text{Fixed Costs}}. \tag{5}$$

Given this cost function, a representative firm sets its price to maximize profits,

$$Max\ \pi_i = p_i q_i - c_i w q_i - r F_i. \tag{6}$$

As in the Dixit-Stiglitz (1977) model of monopolistic competition, each identical firm takes the industry price index G_i as a given while maximizing (6) subject to the identical demand (3). The optimized price is a constant mark-up over marginal costs, $p_i = \frac{\sigma_i}{\sigma_i - 1} c_i w$ and reflects closely the cost of labor in a country.

Zero Profits and Free Entry. Replacing the optimal price from the first-order condition, the operating profit is

$$(p_i q_i - c_i w q_i) = \frac{p_i q_i}{\sigma_i}. \tag{7}$$

Free entry and exit imply that profits (prof max) for all firms in the industry must be zero in equilibrium. Then from (6) and (7) the quantity supplied by each producer is,

$$q_i^s = (\sigma_i - 1)\frac{F_i}{c_i}\frac{r}{w}. \tag{8}$$

Firm scale in (8) increases in fixed costs and falls in variable costs.

Number of Firms. Given free entry and the total income spent on the industry's varieties $b_i Y$, we can solve for the number of firms with non-negative profits in an industry. The homogeneity of firms means each of the n_i operating firms earns $p_i q_i^s$ amount of revenue. Therefore, the total number of firms in an industry is $n_i = \frac{b_i Y}{p_i q_i^s}$, and substituting for optimal price and scale (8) we get,

$$n_i = \frac{b_i Y}{\sigma_i}\frac{1}{F_i r}. \tag{9}$$

From (9), the number of firms that can operate in an industry increases as demand increases $(b_i Y)$, the products become more differentiable $(i.e. \text{ low substitution elasticity } \sigma_i)$ and the cost of starting a business in that industry falls $(F_i r)$.

Production Techniques. The capital intensity of a firm (or its technique) is defined as the ratio of the amount of capital used by a firm per unit of labor it uses, i.e. $\frac{F_i}{c_i q_i^s}$. Substituting the optimal quantity supplied from (8) we get firm technique in industry i as,

$$\frac{K_i}{L_i} = \frac{w}{(\sigma_i - 1)r}. \tag{10}$$

Because fixed and variable cost components differ in factor intensities, optimal technique depends on firm scale, and firm scale depends on factor prices through (8).

General Equilibrium in the Closed Economy

Aggregate labor demand for a firm in industry i is given by $c_i q_i^s$, so the total labor demand in an industry is, $L_i^D = n_i c_i q_i^s$. Substituting n_i from (9) and q_i^s from (8), we can solve for aggregate labor demand in each industry L_i^D as,

$$L_i^D = \frac{b_i Y}{w}\frac{\sigma_i - 1}{\sigma_i}.$$

Summing L_i^D across the two sectors $i = 1, 2$, and assuming $\bar{L}$ as the total labor supply in the country, the labor market clearing condition is

$$\bar{L} = \frac{Y}{w}\left[b_1\frac{\sigma_1 - 1}{\sigma_1} + b_2\frac{\sigma_2 - 1}{\sigma_2}\right].$$

Denoting the constant terms within the square brackets as A, the equilibrium wage rate is

$$w = A\frac{Y}{\bar{L}}.$$

The total capital demand in an industry i is $K_i^D = n_i F_i$. Aggregating across the two sectors, the total capital demand is $\overline{K} = \frac{Y}{r} B$, where $B = \frac{b_1}{\sigma_1} + \frac{b_2}{\sigma_2}$. This gives the equilibrium rental rate as $r = \frac{Y}{\overline{K}} B$. Combining the results for equilibrium wage and rental rates, we get

$$\frac{w}{r} = \frac{\overline{K}}{\overline{L}} \frac{A}{B}, \tag{11}$$

showing that the relative factor price in a country is inversely proportional to the relative factor endowments.

Comparison With Traditional Increasing Returns to Scale Monopolistic Competition Models

Most of the literature using the Dixit-Stiglitz (1977) model of monopolistic competition with two factors assumes a homothetic cost function of the Cobb-Douglas type,

$$C_i = (c_i q_i + F_i) w^{\alpha_i} r^{1-\alpha_i},$$

where both fixed costs and marginal costs use labor and capital in the same proportion of $\frac{\alpha_i}{1-\alpha_i}$. The results comparing the equilibrium number of firms and optimal firm scales in the traditional model and the present model is given below,

$$n_i = \frac{b_i Y}{F_i \sigma_i} \left(\frac{1}{w^{\alpha_i} r^{1-\alpha_i}} \right); \quad q_i^s = (\sigma_i - 1) \frac{F_i}{c_i}. \qquad \text{(Traditional Model)}$$

$$n_i = \frac{b_i Y}{F_i \sigma_i} \left(\frac{1}{r} \right); \quad q_i^s = (\sigma_i - 1) \frac{F_i}{c_i} \frac{r}{w}. \qquad \text{(Present Model)}$$

In the traditional model, the firm size is exogenous and identical across countries for any given industry. In the present model, firm size is endogenously determined by the factor price ratio in a country. Starting a business is more reliant on fixed capital costs in this model and, therefore, the rental rate of capital (r) plays a more significant role in determining the number of firms in the present model compared to the traditional two-factor model.

Open Economy

This section solves for the equilibrium when two economies North (N) and South (S) trade, assuming preferences and technologies are identical as in the closed economy above and N is relatively more capital abundant than S. All of S's variables are denoted with a star, '$*$'. Total income in each country is given as the sum of factor payments, so that $Y = r\overline{K} + w\overline{L}$ for N, and similarly Y^* for S. The number of industry i varieties produced in N and S is given by n_i and n_i^* and we solve for the equilibrium free entry n and n^* in a trade equilibrium.[8]

Equilibrium in an Industry i

Using the utility function defined earlier (equations 1 and 2), the demand facing each N country firm is the sum of the domestic and foreign demands,[9]

$$q_i^D = b_i p_i^{-\sigma_i} \left(\underbrace{Y G_i^{\sigma_i - 1}}_{\text{Home Demand}} + \underbrace{Y^* \tau_i^{1-\sigma_i} G_i^{*\sigma_i - 1}}_{\text{Demand from } S} \right) \tag{13}$$

where τ_i is the iceberg trade costs in sector i, and the terms G_i and $\boldsymbol{G_i^*}$ are, respectively, the industry i price indexes in the N and S and are defined as

$$G_i^{1-\sigma_i} = n_i p_i^{1-\sigma_i} + n_i^* p_i^{*1-\sigma_i} \tau_i^{1-\sigma_i} \tag{14}$$

$$G_i^{*1-\sigma_i} = n_i^* p_i^{*1-\sigma_i} + n_i p_i^{1-\sigma_i} \tau_i^{1-\sigma_i} \tag{15}$$

The expressions for the optimal firm scale $q_i^s = (\sigma_i - 1)\frac{F_i r}{c_i w}$ and technique $\frac{K_i}{L_i} = \frac{1}{(\sigma_i - 1)}\frac{w}{r}$ used remain identical to the closed economy situation, but their optimal values change as trade affects optimal $\frac{w}{r}$.

Quantity supplied (8) by each firm must be equal to the quantity demanded (13) from each firm for markets to clear. Therefore, equating zero profits scale q^s to aggregate demand,

$$p_i q_i^S = b_i Y \left(\frac{p_i}{G_i}\right)^{1-\sigma_i} + b_i Y^* \left(\frac{p_i \tau}{G_i^*}\right)^{1-\sigma_i}. \tag{16}$$

Replacing the equilibrium quantity supplied by each firm in N and S from equation (8) into equation (16) and dividing gives,

$$\frac{p_i q_i^S}{p_i^* q_i^{S*}} = \tilde{r} = \frac{\tilde{p}_i^{1-\sigma_i} \tilde{Y}\left[1 + \left(\frac{G_i}{G_i^*}\right)^{1-\sigma_i} \tau_i^{1-\sigma_i} \frac{1}{\tilde{Y}}\right]}{\left(\frac{G_i}{G_i^*}\right)^{1-\sigma_i}\left[1 + \left(\frac{G_i^*}{G_i}\right)^{1-\sigma_i} \tau_i^{1-\sigma_i} \tilde{Y}\right]} \tag{17}$$

where a tilde ('~'), represents the ratio of an N firm's variable relative to its Southern counterpart. Simplifying the value of $\left(\frac{G_i}{G_i^*}\right)^{1-\sigma_i}$ using (G1) and (G*) yields,

$$\left(\frac{G_i}{G_i^*}\right)^{1-\sigma_i} = \frac{\tilde{n}_i \tilde{p}_i^{1-\sigma_i} + \tau^{1-\sigma_i}}{1 + \tilde{n}_i \tilde{p}_i^{1-\sigma_i} \tau^{1-\sigma_i}} \tag{18}$$

Solving (17) and (18) simultaneously, we get $\frac{n_i}{n_i^*}$ as,

$$\frac{n_i}{n_i^*} = \tilde{n}_i = \frac{\frac{\tilde{p}_i^{1-\sigma_i}}{\tilde{r}}\left(\tau^{2-2\sigma_i} + \tilde{Y}\right) - \tau^{1-\sigma_i}(1 + \tilde{Y})}{\tilde{p}_i^{1-\sigma_i}\left(1 + \tau^{2-2\sigma_i}\tilde{Y}\right) - \frac{\tilde{p}^{2-2\sigma_i}}{\tilde{r}}\tau^{1-\sigma_i}(1 + \tilde{Y})} \tag{19}$$

For the following analysis, notation is simplified by defining the expression $\tilde{p}_i^{\sigma_i - 1}\tilde{r}$ as $\tilde{\rho}_i$

Using (19), we can derive conditions when intra-industry trade exists in industry i. For example, for $n_i^* = 0$,[10]

$$\tilde{\rho}_i = \tilde{p}_i^{\sigma_i - 1}\tilde{r} \leq \underline{\rho_i} = \left(\frac{\tau_i^{1-\sigma_i}(\tilde{Y} + 1)}{\tilde{Y}\tau_i^{2-2\sigma_i} + 1}\right) \tag{20}$$

This happens when the N specializes completely so that the aggregate world expenditure in a sector goes fully to the N's producers, $n_i p_i q_i^S = b_i(Y + Y^*)$. Equation (20) shows that given σ_i, if relative price $\tilde{p}_i$ (or relative wage $\tilde{w}$) and relative rental rate $\tilde{r}$ are not too high, then the N could become the sole producer in the industry. Similarly, $n_i = 0$ and S captures the entire market if,

$$\tilde{\rho}_i = \tilde{p}_i^{\sigma_i - 1}\tilde{r} \geq \overline{\rho_i} = \frac{\tilde{Y} + \tau_i^{2-2\sigma_i}}{\tau_i^{\sigma_i - 1}(\tilde{Y}+1)} \tag{21}$$

i.e. if N's relative price is high enough, its rental rate is high enough, and the industry is less differentiable (i.e. σ_i is high), then S becomes the sole producer in an industry.

Therefore when $\underline{\rho} < \rho < \bar{\rho}$, both N and S produce in a sector and we have intra-industry trade in the sector. Appendix A.2 further proves that $\underline{\rho_i} < \overline{\rho_i}$ when $\sigma_i > 1$ and $\tau_i > 1$.

North Firms' Share of World Revenues. The N's share in total world exports in any industry i is $v_i = \frac{n_i p_i q_i^S}{(n_i p_i q_i^S + n_i^* p_i^* q_i^{S*})}$, where each N firm's revenue is $p_i q_i$ and n_i is the total number of firms in N. Dividing the numerator and denominator by $n_i^* p_i^* q_i^{*S}$ and replacing the ratio of the equilibrium quantities supplied by each N and S firm using (8) $\tilde{q}_i^S = \frac{\tilde{r}}{\tilde{w}}$, N's share of total world trade flows in an industry is

$$v_i = \frac{\tilde{n}_i \tilde{p}_i \tilde{q}_i^S}{(1 + \tilde{n}_i \tilde{p}_i \tilde{q}_i^S)} = \frac{\tilde{n}_i \tilde{r}}{(1 + \tilde{n}_i \tilde{r})} \tag{22}$$

Substituting the value of $\tilde{n}_i$ from (19) and given the conditions for incomplete specialization, the N's share of world exports can be represented as

$$v_i = \begin{cases} 1 & \text{if } \tilde{\rho}_i \leq \underline{\rho_i} \\ \frac{Y}{W}\left[\frac{-\tilde{\rho}_i \tau_i^{1-\sigma_i}\left(\frac{Y^*}{Y}+1\right)+1+\frac{Y^*}{Y}\tau_i^{2-2\sigma_i}}{-\left(\tilde{\rho}_i+\frac{1}{\tilde{\rho}_i}\right)\tau_i^{1-\sigma_i}+\tau_i^{2-2\sigma_i}+1}\right] & \text{if } \tilde{\rho}_i \in \left[\underline{\rho_i}, \overline{\rho}_i\right] \\ 0 & \text{if } \tilde{\rho}_i \geq \overline{\rho_i} \end{cases} \tag{23}$$

where the aggregate world income is denoted as $W = Y^* + Y$. Therefore, N's share of industry production is a function of relative country sizes, trade costs, and relative factor prices.

General Equilibrium When N Is Relatively Capital Abundant $\left(\frac{K}{L} > \frac{K^*}{L^*}\right)$

Factor Price Equalization and Trade Patterns. When trade costs are assumed away under a monopolistic competitive framework, all the conditions for factor price equalization (FPE) theorem (as noted in Helpman and Krugman, 1985, chapters 1 and 7) are satisfied in this model. Namely: (i) preferences are well behaved and homothetic; (ii) free entry ensures zero profit; (iii) the number of factors does not exceed the number of goods; and (iv) under factor price equalization, all firms are identical across N and S $\left(q_i^S = (\sigma_i - 1)\frac{F_i r}{c_i w}\right)$, so that product expansion in an industry takes place only through an increase in the number of identical firms, rendering constant returns to output growth. This would imply FPE if the factor endowment differences are not too large and the industries have sufficiently different factor intensities so that the factor price equalization set is large. Since the capital intensity of the industry with σ_i as the elasticity substitution is given by $\frac{K_i}{L_i} = \frac{w}{(\sigma_i - 1)r}$, as long as two industries have sufficiently different σ_i's, the latter condition is satisfied. In that case, production costs are same and with zero trade costs, commodity prices are also equalized and the geographic pattern of production for a given product is indeterminate. The aggregate pattern of trade across industries is determinate however, so that the capital-abundant N makes more of the capital-intensive industry's goods.

Factor Price Non-Equalization Under Zero Trade Cost. When factor endowment differences are large, factor price equalization breaks down, even when trade costs are zero.

Factor Price Non-Equalization with Positive Trade Cost. The factor market clearance conditions in this two-sector model for N and S are

$$n_1F_1 + n_2F_2 = \overline{K} \qquad n_1^*F_1 + n_2^*F_2 = \overline{K}^*. \tag{24}$$

$$n_1c_1q_1^s + n_2c_2q_2^s = \overline{L} \qquad n_1^*c_1q_1^{*s} + n_2^*c_2q_2^{*s} = \overline{L}^*. \tag{25}$$

I show, using the method of contradiction that FPE cannot hold when trade costs are positive. Suppose FPE holds, even with positive trade costs. From (10), firms' techniques in each sector are identical across N and S. Suppose country sizes are also identical; then, from (19), the number of firms is equal across N and S, so clearly factor price equalization cannot clear factor markets. When the relative country sizes differ, given FPE, the bigger country has a larger number of firms in each sector. Therefore, FPE implies that a bigger country demands more labor and capital in the same proportion irrespective of its factor endowments. Thus, factor price equalization would not clear factor markets when trade costs are positive as long as relative factor abundance exists.

Factor Market Clearance Condition $\left(\frac{w^*}{r^*} < \frac{w}{r} \text{ when } \frac{K}{K^*} > \frac{L}{L^*}\right)$. When FPE breaks down, factor markets clear only when N has a higher wage rental ratio. This is because full employment of factors in the N occurs if (i) the N uses capital more intensively in each sector than the S; (ii) it sells more varieties $n > n^*$ for a given size of each sector; and/or (iii) it has larger shares of world exports in capital-intensive sectors. From equation (10), we know that condition (i) is satisfied only when N has a higher (w/r) than S. For the number of firms to be larger in the N for a given amount of a sector's output, the firm size $q_i^s = (\sigma_i - 1)\frac{F_i r}{c_i w}$ must be smaller in the N, which happens if $\frac{w}{r} > \frac{w^*}{r^*}$. For N to export larger shares in more capital-intensive sectors, the N must also have higher wage rental rates (proven in section 2.3 below). All of these results are reversed when $\frac{w^*}{r^*} > \frac{w}{r}$. Hence, only when $\frac{w^*}{r^*} < \frac{w}{r}$ can the factor markets clear in the N, given its relative capital abundance.

Relative Changes in $\frac{\tilde{w}}{\tilde{r}}$ Under Capital Accumulation and Trade Liberalization

Capital Accumulation. As a country accumulates capital, the excess capital gets used if: (i) the country uses capital more intensively in all sectors; (ii) the number of firms rise in all sectors; and/or (iii) production expands in more capital-intensive sectors. From equation (10), we know that condition (i) is satisfied only when that country's (w/r) increases. For the number of firms to expand given a sector's output, the firm size $q_i^s = (\sigma_i - 1)\frac{F_i r}{c_i w}$ must fall, which happens if (w/r) rises. For the country to expand in more capital-intensive sectors, (w/r) must rise, lowering its share in the more homogeneous sectors (proven in section below). Therefore, as a country becomes relatively more capital abundant, its wage rental ratio relative to the other must increase to accommodate the higher capital resources.

Trade Liberalization. In autarky, the capital-abundant country has a higher wage rental ratio, $\frac{w}{r} = \frac{\overline{K}}{\overline{L}}\frac{A}{B}$ (see 11). But from the discussion above, factor prices equalize under free trade.[11] Therefore, the current model predicts factor price convergence under trade liberalization. In autarky, capital is relatively cheaper in the N. Therefore, when trade opens, it is relatively easier in the N to expand in the more capital-intensive sectors and increase the extensive margin more in all sectors, and the reverse is true in the N due to its cheaper labor. Therefore, trade liberalization increases demand for capital resources more in the N and labor resources more in the S, decreasing pressure on (w/r) in the N and the opposite in the S. The empirical evidence of this prediction is mixed, though.[12]

We can summarize the preceding equilibrium factor price results in the following lemma:

Lemma 1:

From the general equilibrium analysis in this model, factor markets are cleared if,

(i) the relatively capital-abundant N has a higher wage rental ratio, i.e. $\tilde{w} > \tilde{r}$;

(ii) a country's relative (w/r) rises with capital accumulation; and

(iii) trade liberalization leads to a convergence in factor price ratios.

Implications on World Trade Patterns

North's Comparative Advantage

The cross-industry comparative advantage due to differences in factor prices is driven by the substitution elasticity σ_i of the industry (i.e., the degree of product differentiability). Since the capital-abundant country N, has a higher $\tilde{w} > \tilde{r}$, N's comparative advantage declines as $\tilde{w}^{\sigma_i-1}\tilde{r}$ rises with σ_i. To show this formally, I hold relative country sizes and $\tau_i^{\sigma_i-1}$ constant and show that $\frac{\partial v_i}{\partial \tilde{p}_i}\Big|_{\tilde{Y},\tau_i^{\sigma_i-1}} < 0$ (proof in appendix A.3) where $\tilde{p}_i = \tilde{w}^{\sigma_i-1}\tilde{r}$. Since labor costs are higher in N and $\tilde{w} > \tilde{r}$, N's share of world production in the industry with a smaller substitution elasticity (i.e., the manufacturing sector) rises as $\tilde{p}$ falls, with goods becoming more differentiable. In this two-sector world, trade balance implies that N has a comparative advantage in the more differentiable sector and enjoys a higher share of the world output in this sector, even when it has higher relative prices.

Differences in Prices and Techniques Used

From (6), the profit maximizing price is $p = \frac{\sigma}{\sigma-1}cw$. Given N has higher per capita capital stocks and higher wages, $\frac{p}{p^*} = \tilde{w} > 1$.[13] Also, from (10) the capital intensity for any N firm is given by $\frac{K}{L} = \frac{1}{(\sigma-1)}\frac{w}{r}$. This gives the relative technique used, $\widetilde{\left(\frac{K}{L}\right)} = \frac{\tilde{w}}{\tilde{r}}$. Therefore, the capital-abundant N uses a more capital-intensive technique in every sector (higher $\widetilde{\left(\frac{K}{L}\right)}$), as $\tilde{w} > \tilde{r}$. So, the N charges higher prices for varieties within the same sector and also produces these varieties using a more capital-intensive technique.

The intuition is that both N and S firms use equal amounts of capital to start a business, but N firms have a smaller scale with less labor usage. Therefore, N firms use more capital per unit of output produced. Starting with Davis and Weinstein (2001), many empirical studies have shown that capital-abundant countries use more capital-intensive techniques. Xiang (2007) shows that the distribution of industry capital intensities for the N first-order stochastically dominates industry capital intensities of the S. Blum (2010) and Nishioka (2012) also show that when countries accumulate capital, they move to more capital-intensive techniques in all industries.

Relative Intensive and Extensive Margins

The N has a smaller average firm size in each sector, which implies that it has a larger number of firms for the given size of a sector. This is because the equilibrium firm size in any sector is $q^s = (\sigma-1)\frac{F}{c}\frac{r}{w}$, giving the relative size of the average firm across N and S as $\frac{\tilde{r}}{\tilde{w}} < 1$.

Hummels and Klenow (2005) find evidence of this from country-level trade flow data showing that economies that are larger but poorer (lower capital endowments in the current model) expand more on the intensive margin than the extensive margin.

Effects of Capital Accumulation on Trade Patterns

As countries accumulate capital, the average size of firms declines in all sectors. This is because, from (8) the equilibrium firm size in any sector is, $q^s = (\sigma - 1)\frac{F}{c}\frac{r}{w}$ and from Lemma 1(ii) a country's relative $\frac{w}{r}$ must rise with capital accumulation. This also implies controlling for industry size, the number of firms must increase with capital accumulation.

The effect of capital accumulation on the number of firms is not homogeneous across all industries in the open economy version of the current model. Under the trade equilibrium, different industries' shares could be changing differently for countries that are accumulating capital. Depending on which industries have a comparative advantage in the capital-accumulating country, the aggregate number of firms in an industry could be rising or falling. However, since the firm size must go down in all industries, controlling for the industry size, a country that accumulates capital must experience an increase in the number of firms.

Effects of Trade Liberalization on Trade Patterns

From *Lemma 1* (iii), we know that trade liberalization has a converging effect on factor prices. Capital-abundant countries see their relative wage rental ratios fall, and labor-abundant countries see their relative wage rental ratios rise. Since firm size from (8) is, $q^s = (\sigma - 1)\frac{F}{c}\frac{r}{w}$, for both capital- and labor-abundant countries, a rise in the (r/w) ratio in the N implies that average firm sizes grow in the N when it liberalizes trade; a rise in the (w/r) ratio in the S implies that the average firm sizes decline in the S when trade liberalizes. Once the industry size is controlled, since average firm size is increasing in the N, the number of firms must be decreasing. For the S, average firm size falls; so, controlling for industry size, the number of firms should rise with trade liberalization. Therefore, when a country liberalizes trade, (i) across all sectors the average size of firms rises in the capital abundant N and falls in the S; and, (ii) controlling for industry size, the number of firms must fall in the N and rise in the S.

CONCLUSION

In this paper, I have incorporated a non-homothetic cost function in a factor proportions-driven monopolistic competition model. The pattern of trade is fully analyzed for a two-country, two-sector, and two-factor monopolistic competition model with transport costs. This simple transformation of the cost function produces some novel results that mimic many real-world trade patterns that the standard two-factor monopolistic competition model with a homothetic cost function assumption cannot produce. Therefore, I believe incorporating this cost assumption in the myriad other applications of the monopolistic competition model would be able to better predict or correctly replicate many other nuances of trade patterns. Applications of this altered cost can readily be used to study patterns of home market effects across countries differing in factor endowments or the differences in the patterns of firm-level trade when firm in rich and poor countries trade.

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ENDNOTES

1. This is not the first paper to assume fixed costs are more capital intensive. A cost structure identical to the one we use here has been used by the footloose capital models in the `new economic geography' literature. This literature, however, avoids all Heckscher-Ohlin motives of trade by incorporating a costlessly traded homogeneous good and identical factor proportions across countries. A general version of this non-homothetic cost structure is also considered by Flam and Helpman (1987), but in a different context than we use here.
2. Instead of capital being used more intensively than labor in fixed costs, one might also assume skilled labor or human capital as the factor used more intensively in fixed costs, as starting a new product is R&D intensive, and R&D is likely more human capital intensive. For pedagogical simplicity, we call the second factor capital.
3. Note that a similar result would emerge for the human capital-abundant North if fixed costs were considered to be R&D costs, which are more human capital intensive.
4. These findings are in line with the findings in Schott (2004), Hummels and Klenow (2005), Hallak and Schott (2010), and Khandelwal (2010).
5. We assume firms are homogeneous since all our stylized facts can be derived by analyzing the average firm results from a heterogeneous firm model.
6. Non-homotheticity of the two-factor cost function of the type assumed here has also been used by Martin and Rogers (1995). However, Martin and Roger's (1995) model assumes factor price equalization due to the presence of a costlessly traded homogeneous product industry. Heckscher-Ohlin motive for trade is avoided by assuming that countries have the same factor proportions, but differ only in size. Flam and Helpman (1987) also assume a non-homothetic cost function in a Dixit-Stiglitz model, but for a different purpose.
7. For analytical tractability, we have assumed here the simplest cost function that makes the fixed costs more capital intensive and the variable costs labor intensive. The standard approach following Helpman and Krugman (1985) is to assume fixed and variable costs have the same factor intensities. The resulting differences in equilibrium variables of the model are highlighted in section 2.1.4. Note also that none of the results derived below depend specifically on this assumption, and we show in the appendix (A.1) that all the results continue to hold even when fixed and variable costs include both factors, while maintaining that fixed costs are sufficiently more capital intensive than marginal costs.
8. We simplify the analysis by ruling out home market effects, which is the tendency of a differentiated product industry to concentrate in a larger country, and the effect varies due to changing trade costs and product differentiability across sectors. To abstract from home market effects in sector , all the relevant partial effects are derived by assuming that relative country sizes $\left(\frac{Y}{Y^*}\right)$ and effective iceberg trade costs in a sector $(\tau_i^{\sigma_i-1})$ are constants.
9. This open economy solution for the number of firms and industry shares follows the analysis by Romalis (2004) for the equilibrium without factor price equalization under the changed cost function.
10. This is derived from (19) so that $\frac{n_i}{n_i^*} \to \infty$.
11. Assuming that factor endowments are not too different.
12. For positive evidence, see Ben-David (1993, 1996) and Sachs and Warner (1995). Slaughter (2001), among others, provides evidence against this prediction.
13. This is true as long as richer countries have higher labor costs. As long as the intensity of labor use in the variable cost is large enough, the result also holds for a more general marginal cost using both labor and capital.

REFERENCES

Ben-David, D. (1993). Equalizing Exchange: Trade Liberalization and Income Convergence. *Quarterly Journal of Economics*, 108, 653-79.

Ben-David, D. (1996). Trade and Convergence among Countries. *Journal of International Economics*, 40, 279-98.

Blum, B. (2010). Endowments, Output, and the Bias of Directed Innovation. *Review of Economics Studies*, 77, 534-59.

Davis, D., & Weinstein, D. (2001). An Account of Global Factor Trade. *American Economic Review*, 91, 1423-54.

Dixit, A., & Stiglitz, J.E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, 67, 297-308.

Flam, H., & Helpman, H. (1987). Industrial Policy under Monopolistic Competition. *Journal of International Economics*, 22, 79-102.

Hallak, J., & Schott, P.K. (2011). Estimating Cross-Country Differences in Product Quality. *Quarterly Journal of Economics*, 126, 417-74.

Helpman, E., & Krugman, P. (1985). *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy*. Cambridge, MA: MIT Press.

Hummels, D., & Klenow, P. (2005). The Variety and Quality of a Nation's Exports. *American Economic Review*, 95, 704-23.

Khandelwal, A. (2010). The Long and Short (of) Quality Ladders. *Review of Economic Studies*, 77, 1450-76.

Martin, P., & Rogers, C.A. (1995). Industrial Location and Public Infrastructure. *Journal of International Economics*, 39, 335-51.

Nishioka, S. (2012). International Differences in Production Techniques: Implications for the Factor Content of Trade. *Journal of International Economics*, 87, 98-104.

Romalis, J. (2004). Factor Proportions and the Structure of Commodity Trade. *American Economic Review*, 94, 67-97.

Sachs, J. D., & Warner, A. (1995). Economic Reform and the Process of Global Integration. *Brookings Papers on Economic Activity*, 1, 1-118.

Schott, P. K. (2004). Across-product versus within-product specialization in international trade. *Quarterly Journal of Economics*, 119(2), 647-678.

Slaughter, M. J. (2001). Trade Liberalization and Per Capita Income Convergence: A Difference-in-Differences Analysis. *Journal of International Economics*, 55, 203-28.

Xiang, C. (2007). Diversification Cones, Trade Costs, and Factor Market Linkages. *Journal of International Economics*, 71, 448-66.

APPENDIX

Generalized Cost Function with both Factors Entering Fixed and Variable Costs

In this section I generalize the assumption in the main model that marginal costs comprise labor costs only and fixed costs comprise capital expenditures only. All the results still hold as long as the marginal cost is sufficiently more labor intensive than fixed costs and the relative wage-rental ratio is different enough.

Let us assume a non-homothetic cost function,

$$C = \underbrace{cw^{1-\alpha}r^{\alpha}q}_{\text{Variable Costs}} + \underbrace{w^{\alpha}r^{1-\alpha}F}_{\text{Fixed Costs}}, \quad \alpha < \frac{1}{2}. \tag{26}$$

and define relative prices as

$$\tilde{p} = \frac{w^{1-\alpha}r^{\alpha}}{w^{*1-\alpha}r^{*\alpha}} = \widetilde{w}^{1-\alpha}\tilde{r}^{\alpha}.$$

Thus, the N has a higher relative price if α is small and $\widetilde{w}$ is large and/or $\tilde{r}$ is not too small. A sufficient condition is that $\widetilde{w} > \left(\frac{1}{\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}}$. The zero profit condition yields quantity supplied by each producer as

$$q^{s} = (\sigma - 1)\frac{F}{c}\left(\frac{r}{w}\right)^{1-2\alpha}. \tag{27}$$

when $\alpha < \frac{1}{2}$, the intensive margin is smaller in the richer country where (r/w) is relatively smaller.

The three other mains prediction that we need to prove are:

1) *The extensive margin is higher for more capital-abundant countries.*
Given that α is less than half and $\widetilde{w} > \tilde{r}$, (27) shows that firm size is smaller in the capital-abundant country. Therefore, for a given country size, the capital-abundant countries have a larger extensive margin.
2) *Capital-abundant countries use more capital-intensive techniques in every industry.*
From (27), we know that the optimal firm size is larger in the labor-abundant country. Since capital is more expensive in the labor-abundant country, firms in the labor-abundant country also use more labor per unit of capital in their fixed costs. This means the capital-abundant country uses more capital per unit of labor used in production.
3) *Countries that accumulate capital over time get higher relative prices for their products.*
This result depends on the general equilibrium solution of factor prices and the proof of factor price divergence with factor endowment differences. The generalized cost function such as (26) does not give closed-form solutions for the aggregate factor demands in each sector to show that factor prices must diverge as factor endowments diverge.
4) Given that all the reasons why the equilibrium wage-rental ratio needs to rise when a country accumulates capital remain identical as outlined in the general equilibrium section for the simplified cost structure (see Section 2.2.2), capital accumulation leads to increased wage-rental ratio in this general case, too, and this would imply higher relative prices of products as long as $\alpha < 1/2$, (i.e., variable costs are more labor intensive).

Proof that $\bar{\rho} > \underline{\rho}$

Note that both $\underline{\rho} = \left(\frac{\tau^{1-\sigma}(1+\frac{Y^*}{Y})}{\tau^{2-2\sigma}+\frac{Y^*}{Y}}\right) > 0$ and $\bar{\rho} = \frac{\frac{Y^*}{Y}\tau^{2-2\sigma}+1}{\tau^{1-\sigma}(1+\frac{Y^*}{Y})} > 0$. Therefore, $< \bar{\rho}$ if $\frac{\bar{\rho}}{\underline{\rho}} > 1$. But,

$$\frac{\bar{\rho}}{\underline{\rho}} = \frac{\left(\frac{Y^*}{Y}\tau^{2-2\sigma}+1\right)\left(\tau^{2-2\sigma}+\frac{Y^*}{Y}\right)}{\left[\tau^{1-\sigma}(1+\frac{Y^*}{Y})\right]^2}$$

$$= \frac{\tau^{2-2\sigma}\left(\frac{Y^*}{Y}\right)^2+\tau^{2-2\sigma}+\left(1+\tau^{4-4\sigma}\right)\frac{Y^*}{Y}}{\tau^{2-2\sigma}\left(\frac{Y^*}{Y}\right)^2+\tau^{2-2\sigma}+2\tau^{2-2\sigma}\frac{Y^*}{Y}} \tag{28}$$

The first two term in the numerator and denominator are identical, therefore $\frac{\bar{\rho}}{\underline{\rho}} > 1$ if $(1 + \tau^{4-4\sigma}) > 2\tau^{2-2\sigma} \Rightarrow (1 - \tau^{2-2\sigma}) > 0$. Given that trade costs are positive $\tau > 1$, and given $\sigma > 1$, the last condition is always satisfied.

Proof that $\frac{dv}{d\tilde{\rho}} \leq 0$

$$\frac{\partial v}{\partial \tilde{\rho}} = \frac{Y}{W}\left[\begin{array}{c}\frac{\left[-\tau^{1-\sigma}\left(\frac{Y^*}{Y}+1\right)\right]\left[-\left(\tilde{\rho}+\frac{1}{\tilde{\rho}}\right)\tau^{1-\sigma}+\tau^{2-2\sigma}+1\right]}{\left[-\left(\tilde{\rho}+\frac{1}{\tilde{\rho}}\right)\tau^{1-\sigma}+\tau^{2-2\sigma}+1\right]^2} \\ -\frac{\left[\frac{1}{\tilde{\rho}^2}\tau^{1-\sigma}-\tau^{1-\sigma}\right]\left[-\tilde{\rho}\tau^{1-\sigma}\left(\frac{Y^*}{Y}+1\right)+1+\frac{Y^*}{Y}\tau^{2-2\sigma}\right]}{\left[-\left(\tilde{\rho}+\frac{1}{\tilde{\rho}}\right)\tau^{1-\sigma}+\tau^{2-2\sigma}+1\right]^2}\end{array}\right] \quad \text{if } \tilde{\rho} \in \left[\underline{\rho}, \bar{\rho}\right]$$

Simplifying yields,

$$\frac{\partial v}{\partial \tilde{\rho}} = -\frac{Y}{W}\left[\frac{\left[\tilde{\rho}^2\tau^{1-\sigma}\left(\frac{Y^*}{Y}+1\right)\right]\left[\left(\tau^{2-2\sigma}+\frac{Y}{Y^*}\right)\tilde{\rho}^2-2\tau^{2-2\sigma}\left(\frac{Y}{Y^*}+1\right)\tilde{\rho}+\tau^{2-2\sigma}\frac{Y}{Y^*}+1\right]}{\left[-\left(\tilde{\rho}+\frac{1}{\tilde{\rho}}\right)\tau^{1-\sigma}+\tau^{2-2\sigma}+1\right]^2}\right] \tag{29}$$